

KING FAHD UNIVERSITY OF PETROLUUM AND MINERALS

Diploma Math Program

Math 003 - Term 041

CLASS TEST 2 - Code 1

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Name:

ID

Section:

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[Provide neat and complete solution. Show all necessary steps for full credit.]

**Question1**

Given the equation of a circle  $2x^2 + 2y^2 + 8x - 4y + 2 = 0$ .

a) Write this equation in the standard form.

**3 points**

$$x^2 + y^2 + 4x - 2y = -1$$

$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$$

$$(x + 2)^2 + (y - 1)^2 = 4$$

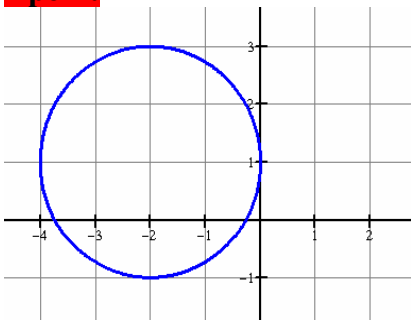
b) Find the center and the radius of this circle.

**1 point**

$$C : (-2, 1), r = 2$$

c) Determine whether the circle is tangent to the  $x$  - axis,  $y$  - axis or both.

**1 point**



so,  $y$  - axis only

## Question 2

Given the quadratic function  $f(x) = -2x^2 + 4x + 1$ .

a) Find the vertex of the function  $f(x)$ .

**2 points**

$$* h = \frac{-b}{2a} = 1, k = f(h) = f(1) = 3 \Rightarrow \text{The vertex is } (h, k) = (1, 3)$$

b) Find the equation of the line that passes through the vertex of the function  $f(x)$  and perpendicular to the line  $3y - 2x + 1 = 0$ . Write the answer in the form  $y = mx + b$ .

**3 points**

$$* 3y - 2x + 1 = 0 \Rightarrow 3y = 2x - 1 \Rightarrow y = \frac{2}{3}x - \frac{1}{3} \Rightarrow m = \frac{2}{3}$$

$$* \text{The slope of the required line is } -\frac{3}{2}$$

$$* \text{Eq.: } y - 3 = -\frac{3}{2}(x - 1)$$

c) Find  $\frac{f(1+h) - f(1)}{h}$ . Write the answer in the simplest form.

**3 points**

$$\begin{aligned} \frac{f(1+h) - f(1)}{h} &= \frac{-2(1+h)^2 + 4(1+h) + 133}{h} \\ &= \frac{-2(1+2h+h^2) + 4 + 4h - 2}{h} \\ &= \frac{-2 - 4h - 2h^2 + 4 + 4h - 2}{h} \\ &= \frac{-2h^2}{h} - 2h \end{aligned}$$

### Question 3

Given the function  $f(x) = 4x - x^2$ ,  $x \leq 2$ .

d) Find  $f^{-1}(x)$ .

**4 points**

$$\begin{aligned} \text{let } f(y) = x &\Rightarrow 4y - y^2 = x, \text{ complete the square for } y \\ &\Rightarrow y^2 - 4y = -x, \text{ add 4 to both sides} \\ &\Rightarrow (y - 2)^2 = -x + 4 \\ &\Rightarrow y - 2 = \pm\sqrt{-x + 4} \\ &\Rightarrow y = 2 \pm \sqrt{-x + 4}, y \leq 2 \\ &\Rightarrow y = 2 - \sqrt{-x + 4} \\ &\Rightarrow f^{-1}(x) = 2 - \sqrt{-x + 4} \end{aligned}$$

e) State the **domain** and **range** of  $f^{-1}(x)$

**2 points**

$$\begin{aligned} D_{f^{-1}} : -x + 4 \geq 0 &\Rightarrow x \leq 4 \Rightarrow D_{f^{-1}} = (-\infty, 4] \\ R_{f^{-1}} = D_f &= (-\infty, 2] \end{aligned}$$

### Question 4

The graph of the equation  $y = |x - 2| + 1$  is reflected across the  $x$ -axis, then shifted 2 units left, then shifted 3 unit down. If the equation of the new graph is  $y = a|x + b| + c$  the find the value of  $a, b$  and  $c$ .

**5 points**

$$\begin{aligned} y_1 &= -(|x - 2| + 1) = -|x - 2| - 1 \\ y_2 &= -|(x + 2) - 2| - 1 = -|x| + 1 \\ y_{\text{new}} &= -|x| - 1 - 3 = -|x| - 2 \\ \text{Hence,} \\ a &= -1, b = 0 \text{ and } c = -4 \end{aligned}$$

### Question 5

$$\text{Let } f(x) = \begin{cases} x|x|+1.5 & \text{if } x < 0 \\ [x-2] & \text{if } x \geq 0 \end{cases} \text{ and } g(x) = \sqrt{1-x^2}.$$

a) Find  $(f \circ f)(-1) + (f \circ g)(0)$ .

**4 points**

$$f(f(-1)) + f(g(0)) = f(0.5) + f(1) = -2 - 1 = -3$$

b) Find the domain of  $g(x)$ .

**2 points**

$$1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow |x| \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow D = [-1, 1]$$