

KING FAHD UNIVERSITY OF PETROLUEM AND MINERALS
Faculty of Science, Prep-Year Math Program
Math 001 - Term 041

CLASS TEST 2

Code 1

Name:

ID#:

Sr. #:

Section:

[Provide neat and complete solution. Show all necessary steps for full credit.]

Question1

If $f(x) = 2x^2 + 5$ and $g(x) = 3x + a$, find a so that the graph of $f \circ g$ crosses the y -axis at 23.

4 points

$$(f \circ g)(x) = f(g(x)) = f(3x + a) = 2(3x + a)^2 + 5$$

$$f \circ g \text{ crosses the } y\text{-axis when } x = 0 \Rightarrow 2a^2 + 5 = 23 \Rightarrow 2a^2 = 18 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$$

Question2 Given the equation of a circle $2x^2 + 2y^2 + 8x - 4y + 2 = 0$.

a) Write this equation in the standard form.

3 points

$$x^2 + y^2 + 4x - 2y = -1$$

$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$$

$$(x + 2)^2 + (y - 1)^2 = 4$$

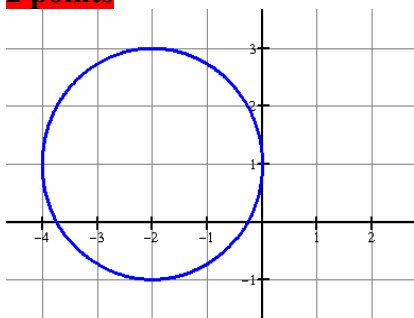
b) Find the center and the radius of this circle.

1 point

$$C : (-2, 1), r = 2$$

c) Determine whether the circle is tangent to the x -axis, y -axis or both.

2 points



so, y -axis only

Question3

Given the function $f(x) = 4x - x^2, x \leq 2$.

d) Write $f^{-1}(x)$.

4 points

let $f(y) = x \Rightarrow 4y - y^2 = x$, complete the square for y

$$\Rightarrow y^2 - 4y = -x, \text{ add 4 to both sides}$$

$$\Rightarrow (y - 2)^2 = -x + 4$$

$$\Rightarrow y - 2 = \pm\sqrt{-x + 4}$$

$$\Rightarrow y = 2 \pm \sqrt{-x + 4}, y \leq 2$$

$$\Rightarrow y = 2 - \sqrt{-x + 4}$$

$$\Rightarrow f^{-1}(x) = 2 - \sqrt{-x + 4}$$

e) State the **domain** and **range** of f^{-1}

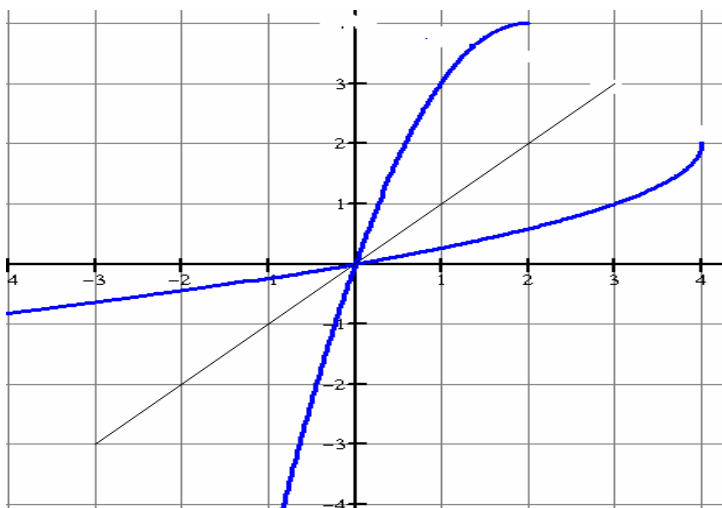
2 points

$$D_{f^{-1}} : -x + 4 \geq 0 \Rightarrow x \leq 4 \Rightarrow D_{f^{-1}} = (-\infty, 4]$$

$$R_{f^{-1}} = D_f = (-\infty, 2]$$

f) Sketch the graphs of f and f^{-1} in the same coordinate system.

Bonus(+3)



Question 4 If the line L passes through the vertex of the quadratic function $f(x) = -2x^2 + 4x + 1$ and is perpendicular to the line $3y - 2x + 1 = 0$. Write the equation of the line L .

5 points

$$* h = \frac{-b}{2a} = 1, k = f(h) = f(1) = 3 \Rightarrow \text{The vertex is } (h, k) = (1, 3)$$

$$* 3y - 2x + 1 = 0 \Rightarrow 3y = 2x - 1 \Rightarrow y = \frac{2}{3}x - \frac{1}{3} \Rightarrow m = \frac{2}{3}$$

$$* \text{The slope of the required line is } -\frac{3}{2}$$

$$* \text{Eq. : } y - 3 = -\frac{3}{2}(x - 1)$$

Question 5 The graph of the equation $y = |x - 2| + 1$ is reflected across the y -axis, then shifted 2 units left, then shifted 1 unit down. The equation of the new graph is $y = |ax + b| + c$. Find the value of $a + b + c$.

5 points

$$y_1 = |-x - 2| + 1$$

$$y_2 = |-(x + 2) - 2| + 1 = |-x - 4| + 1 =$$

$$y_{\text{new}} = |-x - 4| + 1 - 1 = |-x - 4|$$

Hence,

$$a = -1, b = -3, c = 0 \Rightarrow a + b + c = -5$$

Note :

$$y_{\text{new}} = |-x - 4| = |-(x + 4)| = |x + 4|$$

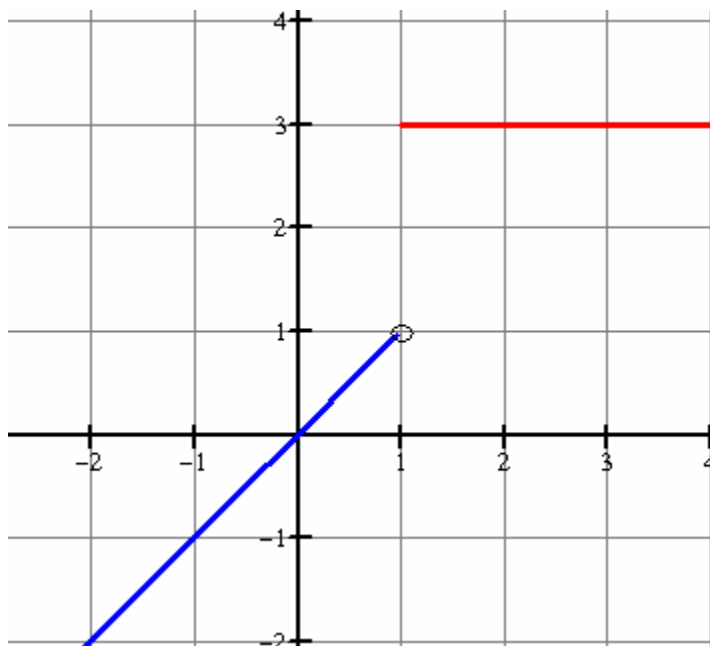
Hence,

$$a = 1, b = 3, c = 0 \Rightarrow a + b + c = 5$$

One solution is enough.

Question 6

a) If $g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x \geq 1 \end{cases}$, then find the range of $g(x)$.

3 points

$$R = (-\infty, 1) \cup \{3\}$$

b) If $h(x) = [x]$, where $[x]$ is the greatest integer of less than or equal to x , then find the value of $h(-0.5) + h^2(\sqrt{8})$.

4 points

$$[-0.5] + ([2.828])^2 = -1 + (2)^2 = -1 + 4 = 3$$

Question 7 If $f(x) = x^2 + 1$ and $g(x) = |x| - 4$, find:

a) $\frac{f(1+h) - f(1)}{h}$

3 points

$$\frac{f(1+h) - f(1)}{h} = \frac{(1+h)^2 + 1 - 2}{h} = \frac{1 + 2h + h^2 - 1}{h} = \frac{2h + h^2}{h} = \frac{h(2+h)}{h} = 2 + h$$

b) $(f \circ g)(2) + (g \circ g)(-1)$

3 points

$$f(g(2)) + g(g(-1)) = f(-2) + g(-3) = 5 - 1 = 4$$

c) The Domain of $\left(\frac{f}{\sqrt{g}}\right)(x)$

3 points

$$\frac{f}{\sqrt{g}} = \frac{x^2 + 1}{\sqrt{|x| - 4}}$$

$$|x| - 4 > 0 \Rightarrow |x| > 4 \Rightarrow x < -4 \text{ or } x > 4$$

$$\therefore D = (-\infty, -4) \cup (4, \infty)$$

Question 8 If the quadratic function $f(x) = 2x^2 + bx + c$ decreases on $(-\infty, 2]$ and has x -intercept $(3, 0)$ find the value of $b + c$

5 points

$$* h = 2 = \frac{-b}{2a} = \frac{-b}{2(2)} = \frac{-b}{4} \Rightarrow b = -8$$

$$* f(3) = 0 \Rightarrow 2(9) - 24 + c \Rightarrow c = 6$$

$$* b + c = -8 + 6 = -2$$

Question 9

Identify the equation or the set of ordered pair that define y as a function of x .

3 points

a) $\{(2,5), (-4,3), (-2,5)\}$

function

b) $|x| + y^3 = 1$

function