

**King Fahd University of Petroleum and
Minerals
College of Sciences
Prep-Year Math Program**

KEY

Math 002 Exam II

Term 021 (2002-2003)

Sunday, December 22, 2002

Time Allowed: 90 Minutes

KEY

Student's Name: _____

ID #: _____ **Section #:** _____

This exam consists of Two parts

Part I : Multiple Choice: Bubble the correct answer on the OMR sheet.

Part II : Written Questions: Provide neat and complete solutions.

Show all necessary steps for full credit.

Calculators, Pagers, or Mobiles are NOT allowed during this exam.

Question	Points	GRADER
Part I: MCQ (1 - 6)	12	
Part II: Written		
1	5	
2	3	
3	5	
4	3	
5	4	
6	4	
7	4	
8	4	

Total

44

Part I: (12-points) Multiple Choice Questions (MCQ).
Bubble the Correct Answer in the OMR Sheet.

1. The expression $2 \csc x \cos \frac{x}{2}$ simplifies to

(a) $\csc \frac{x}{2}$

(b) $\sec \frac{x}{2}$

(c) $\tan \frac{x}{2}$

(d) $\cot \frac{x}{2}$

2. Which one of the following statements is FALSE?

(a) $\csc \left(\frac{\pi}{2} + \theta \right) = -\sec \theta$

(b) $\tan \left(\frac{\pi}{2} + \theta \right) = -\cot \theta$

(c) $\sec \left(\frac{\pi}{2} + \theta \right) = -\csc \theta$

(d) $\cot \left(\frac{\pi}{2} + \theta \right) = -\tan \theta$

3. The exact value of $\sin^{-1} \left(\sin \frac{9\pi}{5} \right)$ is

(a) $-\frac{\pi}{5}$

(b) $\frac{\pi}{5}$

(c) $\frac{9\pi}{5}$

(d) $-\frac{2\pi}{5}$

4. Which one of the following statements is always TRUE for any two nonzero vectors \mathbf{u} and \mathbf{v} and any nonzero real number k ?

- (a) The vector $\frac{-\mathbf{u}}{\|\mathbf{u}\|}$ is a unit vector
- (b) The vectors \mathbf{u} and $k\mathbf{u}$ have the same direction
- (c) $\|\mathbf{u} + \mathbf{v}\| < \|\mathbf{u}\| + \|\mathbf{v}\|$
- (d) $\|k\mathbf{v}\| = k\|\mathbf{v}\|$

5. The value of $\frac{\tan \frac{7\pi}{12} - \tan \frac{3\pi}{4}}{1 + \tan \frac{7\pi}{12} \tan \frac{3\pi}{4}}$ is equal to

- (a) $-\tan \frac{\pi}{6}$
- (b) $\tan \frac{\pi}{6}$
- (c) $-\tan \frac{4\pi}{3}$
- (d) $\tan \frac{4\pi}{3}$

6. The number of solutions of the equation $2 - 2|\cos x| = 1$ with $0 \leq x \leq \frac{3\pi}{2}$ is equal to

- (a) 3
- (b) 4
- (c) 1
- (d) 2

Part II: Written Questions.

[Provide neat and complete solution. Show all necessary steps for full credit.]

1. (5-points) Given the function $f(x) = -2 \sin\left(2x - \frac{\pi}{4}\right)$.

(a) Find the period of $f(x)$.

$$\text{The period} = \frac{2\pi}{2} = \pi$$

--- 1 point

(b) Find the phase shift of the graph of $f(x)$.

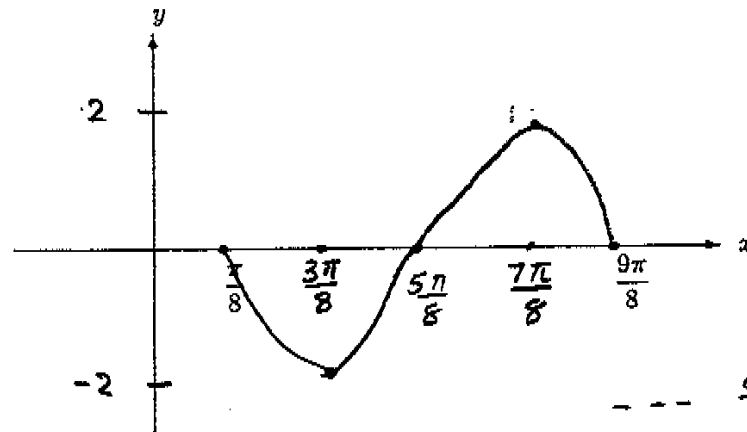
$$\text{The phase shift} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$$

--- 1 point

(c) Find the range of $f(x)$.

$$\text{The range} = [-2, 2]$$

--- 1 point

(d) Sketch the graph of $f(x)$ over the interval $\left[\frac{\pi}{8}, \frac{9\pi}{8}\right]$.

--- 2 points

2. (3-points) If $0 \leq \alpha < 2\pi$, find the exact value of $\sec \frac{\alpha}{2}$ given that $\cos \alpha = \frac{4}{5}$ and α is in quadrant IV.

$$\frac{3\pi}{2} < \alpha < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{\alpha}{2} < \pi \Rightarrow \alpha \in \text{Q II}$$

--- 1 point

$$\Rightarrow \sec \frac{\alpha}{2} = \frac{1}{\cos \frac{\alpha}{2}} = -\sqrt{\frac{2}{1 + \cos \alpha}}$$

--- 1 point

$$\Rightarrow \sec \frac{\alpha}{2} = -\sqrt{\frac{2}{1 + \frac{4}{5}}} = -\sqrt{\frac{10}{9}} = -\frac{\sqrt{10}}{3}$$

--- 1 point

3. (5-points) Given the vectors $u = -2i + 3j$ and $v = i + 5j$.

(a) Find a vector of length 3 in the opposite direction of the vector u .

$$\text{The required vector} = \frac{-3u}{\|u\|} \quad \dots \text{ 1 point}$$

$$= \frac{6i - 9j}{\sqrt{4+9}}$$

$$= \frac{6}{\sqrt{13}}i - \frac{9}{\sqrt{13}}j \quad \dots \text{ 1 point}$$

(b) Find the measure of the smallest angle between the vectors u and v .

Let α be the required angle \Rightarrow

$$\cos \alpha = \frac{u \cdot v}{\|u\| \|v\|} \quad \dots \text{ 1 point}$$

$$= \frac{(-2)(1) + (3)(5)}{\sqrt{4+9} \sqrt{1+25}} = \frac{13}{\sqrt{13} \sqrt{26}} = \frac{13}{13\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \dots \text{ 1 point}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \quad (\text{or } 45^\circ) \quad \dots \text{ 1 point}$$

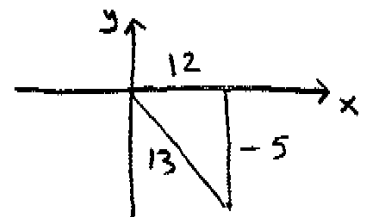
4. (3-points) Solve $\sin^{-1}x - \tan^{-1}\left(-\frac{5}{12}\right) = \pi/2$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{2} + \tan^{-1}\left(-\frac{5}{12}\right)$$

$$\Rightarrow \sin(\sin^{-1}x) = \sin\left(\frac{\pi}{2} + \tan^{-1}\left(-\frac{5}{12}\right)\right) \quad \dots \text{ 1 point}$$

$$\Rightarrow x = \cos\left(\tan^{-1}\left(-\frac{5}{12}\right)\right) \quad \dots \text{ 1 point}$$

$$= \frac{12}{13} \quad \dots \text{ 1 point}$$



5. (4-points) Find the standard form of the equation of the ellipse that has foci at $(-3, 0)$ and $(-3, 6)$ and vertices at $(-3, -2)$ and $(-3, 8)$.

\Rightarrow The major axis is parallel to the y-axis } ... 1 point
and the center is at $(-3, \frac{0+6}{2}) = (-3, 3)$

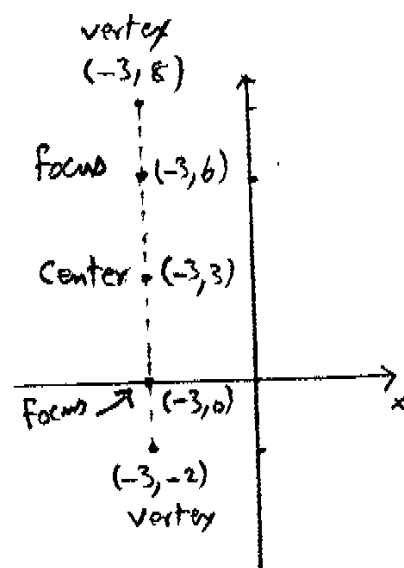
$$a = 8 - 3 = 5 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \dots 1 \text{ point}$$

$$c = 3 - 0 = 3$$

$$b^2 = a^2 - c^2 = 25 - 9 = 16 \dots 1 \text{ point}$$

\Rightarrow The required equation is :

$$\frac{(x+3)^2}{16} + \frac{(y-3)^2}{25} = 1$$



6. (4-points) Verify the identity $\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \csc x - \cot x$, $0 < x < \frac{\pi}{2}$.

$$\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \sqrt{\frac{(1 - \cos x)^2}{1 - \cos^2 x}} = \sqrt{\frac{(1 - \cos x)^2}{\sin^2 x}} \dots 1 \text{ point}$$

$$= \frac{1 - \cos x}{\sin x}, \text{ (since } 0 < x < \frac{\pi}{2} \text{)} \dots 1 \text{ point}$$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} \dots 1 \text{ point}$$

$$= \csc x - \cot x \dots 1 \text{ point}$$

as required.

7. (4-points) Find the vertex, focus, and directrix of the parabola given by the equation $6y - 3x^2 - 12x + 4 = 0$.

Put the equation in the standard form:

$$3x^2 + 12x = 6y + 4 \Rightarrow 3(x^2 + 4x) = 6y + 4 \Rightarrow$$

$$3(x+2)^2 = 6y + 4 + 12 \Rightarrow 3(x+4)^2 = 6y + 16$$

$$\Rightarrow 3(x+2)^2 = 6(y + \frac{8}{3}) \Rightarrow$$

$$(x+2)^2 = 2(y + \frac{8}{3}) \quad \dots 1 \text{ point}$$

Compare with $(x-h)^2 = 4p(y-k) \Rightarrow$

$$\text{The coordinates of the vertex are } (-2, -\frac{8}{3}) \dots 1 \text{ point}$$

$$\text{and } 4p = 2 \Rightarrow p = \frac{1}{2} \Rightarrow$$

$$\text{The coordinates of the focus are}$$

$$(-2, -\frac{8}{3} + \frac{1}{2}) = (-2, -\frac{13}{6}) \dots 1 \text{ point}$$

$$\text{The equation of the directrix is}$$

$$y = -\frac{8}{3} - \frac{1}{2} = -\frac{19}{6} \dots 1 \text{ point}$$

8. (4-points) Solve $2 \sin x - \cos 2x = \frac{1}{2}$, where $0 \leq x < \pi$.

$$\Rightarrow 2 \sin x - (1 - 2 \sin^2 x) = \frac{1}{2} \quad \dots 1 \text{ point}$$

$$\Rightarrow 4 \sin x - 2 + 4 \sin^2 x = 1$$

$$\Rightarrow 4 \sin^2 x + 4 \sin x - 3 = 0$$

$$\Rightarrow (2 \sin x - 1)(2 \sin x + 3) = 0 \quad \dots 1 \text{ point}$$

$$\Rightarrow \sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -\frac{3}{2} \quad \dots 1 \text{ point}$$

$$\Rightarrow \text{The solutions in the interval } 0 \leq x < \pi$$

$$\text{are: } \frac{\pi}{6} \text{ and } \frac{5\pi}{6} \quad \dots 1 \text{ point}$$