Section 6.2 Sum, Difference and cofunction Identities

1. $\dfrac{\tan 145^\circ - \cot 55^\circ}{1 + \tan 145^\circ \cot 55^\circ} =$

A) $-\tan 10^\circ$

B) $\tan 20^\circ$

C) $\tan 15^\circ$

D) $-\tan 25^\circ$

E) $-\tan 70^\circ$

2. If $\sin\alpha = \dfrac{5}{13}$, $\dfrac{\pi}{2} < \alpha < \pi$, then $14\tan(\dfrac{\pi}{4} - \alpha) =$

A) -17

B) 34

C) -3

D) -37

E) 18

3. $\cos 3\alpha =$

A) $3\cos^3\alpha - \cos\alpha$

B) $2\cos^3\alpha - 4\cos\alpha$

C) $4\cos^3\alpha - 3\cos\alpha$

D) $3\cos^3\alpha + 4\cos\alpha$

E) $4\cos^3\alpha + \cos\alpha$

4. If $\cot\alpha = -\frac{12}{5}$, α is a Quadrant II angle and $\csc\beta = -\frac{5}{4}$, β is a Quadrant IV angle, then $\sec(\alpha - \beta) =$

A) $-\frac{65}{16}$

B) $\frac{65}{16}$

C) $\frac{65}{56}$

D) $-\frac{65}{56}$

E) $\frac{56}{65}$

5. If $\cos\alpha = \cos\beta = 1/2$, then the value of

$\cos(\alpha+\beta)\cos(\alpha-\beta)$ is

(a) $\frac{1}{2}$

(b) 1

(c) -1

(d) $-\frac{1}{2}$

(e) 0

6. $\dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha-\beta)\tan(\alpha+\beta)} =$

(a) $\tan(-2\beta)$

(b) $\tan(\alpha+\beta)$

(c) $\tan 2\alpha$

(d) $\tan(2\alpha+2\beta)$

(e) $\tan(2\alpha-2\beta)$

7. $\sin\beta\left[\dfrac{\cos(\alpha+\beta)}{\sin\beta\cos\beta}+\dfrac{\sin\alpha}{\cos\beta}\right] =$

(a) $\cos\beta$

(b) $\sin\alpha$

(c) $\cos\alpha$

(d) $\sin\beta$

(e) $\cos\alpha+\sin\beta$

8. If $\sin\theta = \frac{4}{5}$ and θ in **quadrant II**, then $\cot 2\theta$ is **equal** to

(a) $\frac{-24}{25}$

(b) $\frac{-25}{7}$

(c) $\frac{5}{3}$

(d) $\frac{7}{24}$

(e) $\frac{-3}{5}$

9. $\dfrac{\cos(x-y)}{\cos x \sin y} - \dfrac{\tan(x+y) - \tan y}{1 + \tan(x+y)\tan y} =$

(a) $\cot y$

(b) $2\tan x + \cot y$

(c) $\tan x - \tan y$

(d) $2\tan x$

(e) $\cot x - \cot y$

5

10. The exact value of $\cos\left(-\frac{7\pi}{12}\right)$ is:

(a) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(b) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(c) $\frac{\sqrt{3}-\sqrt{2}}{2}$

(d) $\frac{\sqrt{2}-\sqrt{6}}{4}$

(e) $\frac{\sqrt{2}-\sqrt{3}}{2}$

11. $\tan 15^\circ =$

(a) $\frac{\sqrt{3}}{6}$

(b) $2-\sqrt{3}$

(c) $2+\sqrt{3}$

(d) $\frac{\sqrt{2+\sqrt{2}}}{2}$

(e) $\frac{\sqrt{3+\sqrt{3}}}{3}$

6

12. Given $\sin\alpha = \frac{\sqrt{2}}{2}$, α in Quadrant I and $\cos\beta = \frac{\sqrt{3}}{2}$, β in Quadrant IV, find $\cos(\alpha - \beta)$

(a) $\frac{\sqrt{3}-\sqrt{2}}{2}$

(b) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(c) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(d) $\sqrt{2}-\sqrt{6}$

(e) $\frac{\sqrt{3}+\sqrt{2}}{2}$

13. $\frac{\sin^2 36° + \cos^2 36° + \tan 36° \cot 264°}{\tan 36° - \cot 264°} =$

A) $\frac{1}{\sqrt{3}}$

B) 1

C) $-\sqrt{3}$

D) $\sqrt{3}$

E) $-\frac{1}{\sqrt{3}}$

14. Given $\tan\alpha = -\frac{4}{3}$ where α terminates in quadrant II and $\cos\beta = \frac{12}{13}$ where β terminates in quadrant IV. then $\csc(\alpha+\beta) =$

(a) $\frac{65}{63}$

(b) $\frac{65}{33}$

(c) $-\frac{63}{65}$

(d) $-\frac{65}{33}$

(e) $-\frac{65}{63}$

15. $\dfrac{\tan\frac{31\pi}{12} + \tan\left(-\frac{\pi}{4}\right)}{1 + \tan\frac{7\pi}{12}\tan\frac{\pi}{4}} =$

(a) $\tan\frac{\pi}{3}$

(b) $\tan\frac{\pi}{6}$

(c) $-\tan\frac{\pi}{3}$

(d) $-\tan\frac{\pi}{6}$

(e) $\tan\frac{5\pi}{12}$

16. The value of $\frac{(1-\tan 12^\circ \tan 33^\circ)}{\tan 12^\circ + \tan 33^\circ}(\cos 13^\circ \cos 17^\circ - \cos 77^\circ \cos 73^\circ)$ is equal to

(a) $\frac{-\sqrt{3}}{2}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{-\sqrt{2}}{2}$

(d) $\frac{\sqrt{2}}{2}$

(e) $\frac{-1}{2}$

17. $\frac{\sin 3x}{\sin 2x} + \frac{\cos 3x}{\cos 2x} =$

a) $\frac{2\sin 5x}{\sin 4x}$

b) $\frac{5}{2}$

c) $\frac{\sin 5x}{\sin 4x}$

d) $\frac{\sin 6x}{\sin 4x}$

e) 3

18. If $\frac{1-\cos 4x}{\sin 4x} = \frac{a\tan x}{b+c(\sec^2 x)}$, then $a+b+c$ is equal to

(a) 3

(b) 2

(c) 4

(d) 5

(e) 1

19. $\sec x \cos\left(\frac{\pi}{2} - x\right)$ is equal to

(a) $\tan x$

(b) 1

(c) $\frac{1}{\cos^2 x}$

(d) $\cot x$

(e) $\sec^2 x$

20. If $\cos\alpha = \frac{15}{17}$, α in Quadrant IV, and $\sin\beta = -\frac{3}{5}$, β in Quadrant III, then $\tan(\alpha - \beta) =$

(a) $-\frac{77}{36}$

(b) $-\frac{13}{84}$

(c) $\frac{84}{77}$

(d) $-\frac{49}{36}$

(e) $\frac{77}{84}$

21. The value of the expression $\frac{1 - \tan 29^\circ \cot 59^\circ}{\tan(29^\circ) + \cot 59^\circ}$ is

A) $\frac{1}{\sqrt{3}}$

B) $\sqrt{3}$

C) 1

D) $-\frac{1}{\sqrt{3}}$

E) $-2\tan 29^\circ$

11

22. If $\cos\alpha = \frac{-3}{5}$, $\sin 2\alpha > 0$, $\sin\beta = \frac{5}{13}$ and β is in quad $\cos(\alpha+\beta) =$

(a) $\frac{-16}{65}$

(b) $\frac{56}{65}$

(c) $\frac{24}{65}$

(d) $-\frac{56}{65}$

(e) $\frac{16}{65}$

23. $\cos 3x$ is equal to

(a) $\cos x\left[1-4\sin^2 x\right]$

(b) $\cos^3 x$

(c) $\cos x$

(d) $\cos x\left[1+2\sin^2 x\right]$

(e) $3\cos x$

24. The expression $\dfrac{\cot\left(\theta - \frac{3\pi}{2}\right)}{\csc\left(\theta - \frac{\pi}{2}\right)}$ is identical to

(a) $\sin\theta$

(b) $\cos\theta$

(c) $\tan\theta$

(d) $\csc\theta$

(e) $\cot\theta$

25. $\dfrac{\tan 155° - \cot 35°}{1 + \tan 155° \cot 35°} =$

(a) $-\tan 80°$

(b) $\tan 20°$

(c) $-\tan 75°$

(d) $-\tan 25°$

(e) $\tan 15°$

26. $\sqrt{\dfrac{1-\sin 310^\circ}{1-\cos 400^\circ}}$ is equal to

(a) $\tan 20^\circ$

(b) $\cot 80^\circ$

(c) $\tan 70^\circ$

(d) $\sin 310^\circ \sec 400^\circ$

(e) a real number between $\tan 50^\circ$ and $\tan 55^\circ$

27. If we use a trigonometric identity of the difference of two angles, then the exact value of $\sin 165^\circ$ is equal to

(a) $\frac{1}{4}(\sqrt{6}-\sqrt{2})$

(b) $\frac{1}{4}(\sqrt{6}+\sqrt{2})$

(c) $\frac{1}{4}(\sqrt{3}-1)$

(d) $-\frac{1}{4}(\sqrt{6}+\sqrt{2})$

(e) $-\frac{1}{4}(\sqrt{3}+1)$

28. $\dfrac{1-\tan 21^\circ \cot 51^\circ}{\tan 21^\circ + \tan 39^\circ} =$

(a) $\dfrac{\sqrt{3}}{3}$

(b) $-\sqrt{3}$

(c) $-\dfrac{\sqrt{5}}{2}$

(d) $2\sqrt{2}$

(e) $\dfrac{\sqrt{3}}{2}$

29. $\dfrac{\sin 105^\circ}{\cos 165^\circ}$ is equal to

(a) -1

(b) $\dfrac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}$

(c) 0

(d) 1

(e) $\dfrac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$

15

30. Given $\sin\alpha = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$, and $\cos\beta = -\frac{12}{13}$, $\pi < \beta < \frac{3\pi}{2}$, then $\tan(\alpha - \beta)$ is equal to

(a) $-\frac{56}{33}$

(b) $\frac{16}{63}$

(c) $-\frac{16}{63}$

(d) $-\frac{43}{34}$

(e) $\frac{7}{6}$

31. $\sin 2x - \tan x =$

(a) $\tan x \cos 2x$

(b) $\cot x \cos 2x$

(c) $-\tan 2x \cos x$

(d) $\sin x \cos 2x$

(e) $\cos x \sin 2x$

16

32. The product $\cos 22^\circ \cdot \cot 68^\circ \cdot \csc(-22^\circ)$ simplifies to

(a) -1

(b) $\cos^2 22^\circ$

(c) $\sin^2 22^\circ$

(d) 1

(e) $-\cos^2 22^\circ$

33. If L is the distance between the two points $P_1(\cos\theta, \sin\theta)$ and $P_2(\cos 2\theta, \sin 2\theta)$, then $L^2 =$

(a) $2 - 2\cos\theta$

(b) $2 + 2\sin\theta$

(c) $2 + 2\cos 3\theta$

(d) $3 - \cos\theta$

(e) $3 - \cos 3\theta$

34. Given that $\sin\alpha = \frac{4}{5}$, α in Quadrant I, and $\tan\beta = \frac{5}{12}$, β in Quadrant III, then $\sin\left(\frac{\pi}{2} + \alpha - \beta\right)$ is equal to

(a) $-\frac{56}{65}$

(b) $-\frac{16}{65}$

(c) $-\frac{4}{13}$

(d) $\frac{56}{65}$

(e) $\frac{36}{65}$

35. 4. If $\tan\alpha = \frac{4}{3}$ and $\cos\beta = -\frac{12}{13}$ such that α and β are both in quadrant III, then the value of $\cos\left(\frac{\pi}{2} + (-\alpha + \beta)\right)$ is equal to

A) $\frac{33}{65}$

B) $\frac{63}{65}$

C) $\frac{16}{65}$

D) $\frac{56}{65}$

E) $\frac{23}{65}$

36. If $\sin\alpha = \frac{3}{5}$, α lies in second quadrant, and $\cos\beta = -\frac{5}{13}$, β lies in third quadrant, then $\sin\left(\frac{\pi}{2} - \alpha + \beta\right) =$

(a) $-\frac{16}{65}$

(b) $\frac{48}{65}$

(c) $-\frac{48}{65}$

(d) $\frac{12}{65}$

(e) $-\frac{36}{65}$

37. Given that $\sin\theta = -\frac{1}{2}$, θ in the quadrant III, and $\cos\alpha = \frac{1}{2}$, α in the quadrant IV, then $\cot(\theta + \alpha) =$

A) $-\frac{\sqrt{3}}{2}$

B) 0

C) $\frac{\sqrt{3}}{3}$

D) $-\frac{\sqrt{3}}{3}$

E) $-\sqrt{3}$

38. $\tan 105^\circ =$

a) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

b) $1+\sqrt{3}$

c) $2-\sqrt{3}$

d) $-2+\sqrt{3}$

39. The value of the expression $\sin(\frac{3\pi}{2}+\beta)\cos(\pi-\beta)+\cos(\frac{\pi}{2}-\beta)\sin\beta$ is equal to:

A) 0

B) $\frac{\pi}{2}$

C) π

D) 1

E) $\frac{3\pi}{2}$

40. If $\sin\alpha=\frac{3}{5}$, α in Quadrant I, and $\cos\beta=-\frac{5}{13}$, β in Quadrant II, then $\tan(\alpha-\beta)=$

(a) $-\frac{63}{16}$

(b) $-\frac{12}{5}$

(c) $-\frac{33}{5}$

(d) $-\frac{61}{65}$

(e) –0.75

Section 6.3 Double And Half Angle Identities

1. The value of $\dfrac{4\sin 15^\circ \cos 15^\circ}{(\cos 15^\circ + \sin 15^\circ)(\cos 15^\circ - \sin 15^\circ)}$ is

A) $2\sqrt{3}+1$

B) $2\sqrt{2}$

C) $\sqrt{2}$

D) $2\sqrt{3}$

E) $\dfrac{2\sqrt{3}}{3}$

2. If $\tan\theta = -\dfrac{3}{4}$, $\dfrac{\pi}{2} < \theta < \pi$, then $\cos(\dfrac{\pi}{2} - \dfrac{\theta}{2}) =$

A) $\dfrac{2\sqrt{5}}{5}$

B) $-\dfrac{7}{25}$

C) $\dfrac{\sqrt{10}}{10}$

D) $-\dfrac{\sqrt{10}}{10}$

E) $\dfrac{3\sqrt{10}}{10}$

3. The exact value of $\cos\left(-\frac{7\pi}{12}\right)$ is:

(a) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(b) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(c) $\frac{\sqrt{3}-\sqrt{2}}{2}$

(d) $\frac{\sqrt{2}-\sqrt{6}}{4}$

(e) $\frac{\sqrt{2}-\sqrt{3}}{2}$

4. If $\cos\theta = \frac{3}{5}$, $\sin\theta < 0$, $0 \leq \theta \leq 2\pi$, then $\tan\frac{\theta}{2}$ is equal to:

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) $-\frac{4}{3}$

(d) $\frac{4}{3}$

(e) $-\frac{2}{3}$

(5)

5. $\cos 13^0 \cos 9.5^0 - \sin 13^0 \sin 9.5^0 =$

a) $\frac{1}{2}\sqrt{\sqrt{2}-1}$

b) $\frac{1}{2}\sqrt{2-\sqrt{2}}$

● $\frac{1}{2}\sqrt{2+\sqrt{2}}$

d) $\frac{\sqrt{2}}{2}$

e) $-\frac{\sqrt{2}}{2}$

6. $\tan 15° =$

(a) $\frac{\sqrt{3}}{6}$

(b) $2-\sqrt{3}$

(c) $2+\sqrt{3}$

(d) $\frac{\sqrt{2+\sqrt{2}}}{2}$

(e) $\frac{\sqrt{3+\sqrt{3}}}{3}$

7. The expression $\frac{1}{\sin^2 x + \cos 2x}$ is identical to:

a) $\sin^2 x$

b) $\csc^2 x$

c) $\tan^2 x$

d) $\cot^2 x$

● $\sec^2 x$

(4)

8. $\frac{\sin 3x}{\sin 2x} + \frac{\cos 3x}{\cos 2x} =$

a) $\frac{2\sin 5x}{\sin 4x}$

b) $\frac{5}{2}$

c) $\frac{\sin 5x}{\sin 4x}$

d) $\frac{\sin 6x}{\sin 4x}$

e) 3

9. $-\sqrt{\frac{1+\cos 378°}{2}} =$

(a) $-\cos 189°$

(b) $\sin 189°$

(c) $-\sin 189°$

(d) $-\tan 189°$

(e) $\cos 189°$

5

16. If $\sin\theta = \frac{4}{5}$ and θ in **quadrant II**, then $\cot 2\theta$ is **equal** to

(a) $\frac{-24}{25}$

(b) $\frac{-25}{7}$

(c) $\frac{5}{3}$

(d) $\frac{7}{24}$

(e) $\frac{-3}{5}$

11. The expression $\frac{1+\cos 2x}{\sin 2x}$ is identical to:

a) $\tan x$

b) $\cot x$

c) $\csc x$

d) $\sec x$

e) $\cos x \sin x$

6

12. The exact value of the product $\sin 112.5° \cos 112.5°$ is equal to :

a) $\sqrt{2}$

b) $-\frac{1}{2}$

c) $\frac{\sqrt{2}}{2}$

d) $-\frac{\sqrt{2-\sqrt{2}}}{2}$

e) $-\frac{\sqrt{2}}{4}$

13. If $\tan\frac{\theta}{2} = -\frac{4}{3}$, $\pi < \theta < \frac{3\pi}{2}$, then $\csc\theta =$

(a) $-\frac{25}{24}$

(b) $\frac{5}{3}$

(c) $-\frac{25}{7}$

(d) $\frac{10}{3}$

(e) $-\frac{25}{12}$

14. The exact value of $\cos 157.5°$ is equal to

(a) $-\dfrac{\sqrt{2+\sqrt{2}}}{2}$

(b) $-\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(c) $\dfrac{\sqrt{2+\sqrt{2}}}{2}$

(d) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(e) $2+\sqrt{2}$

15. The exact value of $\tan(-22.5°)$ equals

(a) $\sqrt{2}-1$

(b) $1-\sqrt{2}$

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

(e) 1

(8)

16. The exact value of the expression

$$\sin\frac{13\pi}{12}\cos\frac{\pi}{12}$$

is equal to

(a) $-\frac{1}{4}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

(d) $-\frac{1}{8}$

(e) $\frac{1}{2}$

17. If $\csc x = \frac{-5}{3}$ and x is in quadrant III, then $\sin 2x + \cot\frac{x}{2}$ is equal to

(a) $\frac{47}{75}$

(b) $-\frac{51}{75}$

(c) $-\frac{63}{25}$

(d) $\frac{11}{75}$

(e) $-\frac{11}{75}$

18. The range of the function $f(x)=1-4\sin 2x\cos 2x\sin 4x$ is

A) $[-1,1]$

B) $[-1,3]$

C) $[-3,5]$

D) $[-2,2]$

E) $[-4,4]$

19. The expression $\cos 110^\circ$ is equal to

A) $-\sqrt{\dfrac{1-\cos 40^\circ}{2}}$

B) $\sqrt{\dfrac{1-\cos 40^\circ}{2}}$

C) $-\sqrt{\dfrac{1-\cos 220^\circ}{2}}$

D) $\sqrt{\dfrac{1+\sin 50^\circ}{2}}$

E) $-\sqrt{\dfrac{1+\sin 220^\circ}{2}}$

20. The value of the expression $\sin\frac{\pi}{8}\cos\frac{\pi}{8}$ is

A) $-\frac{\sqrt{2}}{2}$

B) $\frac{1}{4}$

C) $\frac{\sqrt{2}}{4}$

D) $\frac{\sqrt{2}}{2}$

E) $-\frac{\sqrt{2}}{4}$

21. $\sqrt{\frac{1+\cos 300^\circ}{2}} =$

A) $-\cos 150^\circ$

B) $\cos 150^\circ$

C) $\tan 150^\circ$

D) $-\sin 30^\circ$

E) $\sin 30^\circ$

(11)

22. If $\cos\theta = -\frac{12}{13}$ and $\csc\theta = -\frac{13}{5}$, then $\tan\frac{\theta}{2} =$

A) $\frac{1}{7}$

B) $\frac{3}{5}$

C) -5

D) $-\frac{4}{5}$

E) $\frac{1}{5}$

23. If $\frac{\tan 4x}{1-\tan^2 4x} = k\tan bx$, then $2k+b =$

A) 10

B) 8

C) 4

D) 9

E) 5

12

24. If $\frac{5\pi}{2} < \theta < 3\pi$, and θ' is the reference angle of θ, then $\sin\frac{\theta}{2}$ is equal to

A) $-\sqrt{\frac{1+\cos\theta'}{2}}$

B) $\frac{\sqrt{1+\cos\theta'}}{2}$

C) $-\sqrt{\frac{1-\cos\theta'}{2}}$

D) $\sqrt{\frac{1+\cos\theta'}{2}}$

E) $\sqrt{\frac{1-\cos\theta'}{2}}$

25. If $\sin\theta=\frac{4}{5}$, θ in Quadrant I, and $\cos\beta=-\frac{2}{\sqrt{5}}$, β in Quadrant III, then $\cot(\theta+2\beta)=$

A) $-\frac{7}{24}$

B) $\frac{7}{12}$

C) $\frac{-1}{\sqrt{5}}$

D) $-\frac{14}{3}$

E) $\frac{5}{24}$

26. If $\sec\theta = -\frac{13}{5}$, where $\frac{5\pi}{2} < \theta < 3\pi$, then the exact value of $\sin\left(\frac{\theta}{2}\right)$ is _____

(a) $\frac{-3\sqrt{13}}{13}$

(b) $\frac{-2\sqrt{13}}{13}$

(c) $\frac{6}{13}$

(d) $\frac{3\sqrt{13}}{13}$

(e) $\frac{-1}{13}$

27. If $\csc\theta = -\frac{5}{3}$, $\pi < \theta < \frac{3\pi}{2}$ then $\tan\frac{\theta}{2}$ is

A) -3

B) $\sqrt{3}$

C) 1

D) $-\sqrt{3}$

E) 3

28. $\sin 2x - \tan x =$

(a) $\tan x \cos 2x$

(b) $\cot x \cos 2x$

(c) $-\tan 2x \cos x$

(d) $\sin x \cos 2x$

(e) $\cos x \sin 2x$

29. $\sin(202.5°) =$

A) $-\dfrac{\sqrt{2-\sqrt{2}}}{2}$

B) $\dfrac{2-\sqrt{2}}{2}$

C) $\dfrac{\sqrt{2+\sqrt{2}}}{2}$

D) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

E) $\dfrac{\sqrt{\sqrt{2}-2}}{2}$

(15)

30. $\sin(-1057.5°) =$

✓A) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

B) $-\dfrac{\sqrt{2-\sqrt{2}}}{2}$

C) $-\dfrac{\sqrt{2+\sqrt{3}}}{2}$

D) $\dfrac{\sqrt{2+\sqrt{2}}}{2}$

E) $-\dfrac{\sqrt{2+\sqrt{2}}}{2}$

---

31. The expression $4\sin x\cos^3 x - 4\cos x\sin^3 x$ simplifies to

(a) $\sin 4x$

(b) $2\sin 4x$

(c) $2\cos 4x$

(d) $\cos 4x$

(e) $4\sin^2 4x$

---

32. If $\tan\alpha = -\frac{4}{3}$, $\frac{3\pi}{2} < \alpha < 2\pi$, then $\sec\frac{\alpha}{2} =$

(a) $-\frac{\sqrt{5}}{2}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{5}}$

(d) $-\frac{\sqrt{3}}{2}$

(e) $-\sqrt{5}$

33. $\cos^2 112.5^\circ =$

a) $\frac{2-\sqrt{2}}{4}$

b) $\frac{2+\sqrt{2}}{4}$

c) $\frac{-2-\sqrt{2}}{4}$

d) $\frac{-2+\sqrt{2}}{4}$

e) 1

34. The expression $\dfrac{\sin 2x - \sin x}{2\cos^2 x + \cos x - 1}$ simplifies to

(a) $\tan \dfrac{x}{2}$

(b) $\cot \dfrac{x}{2}$

(c) $\cos \dfrac{x}{2}$

(d) $\sin \dfrac{x}{2}$

(e) $\sec \dfrac{x}{2}$

35. $\dfrac{\sin 5° + \cos 5°}{\sqrt{2}} =$

(a) $\sin 50°$

(b) $\sin 40°$

(c) $\sin 10°$

(d) $\dfrac{\sqrt{2}}{2} \sin 10°$

(e) $\dfrac{\sqrt{2}}{2} \sin 50°$

(1)

Section 6.4 Functions Of The Form Of f(x) = a sin x + b cos x

1. The number of zeros of the function $f(x) = \sin x + \cos 2x$ in the interval $[0, 2\pi)$ is

(a) 3

(b) 2

(c) 4

(d) 6

(e) 5

2. If the function $y = -3\sin 2x - 3\cos 2x$ is written in the form $y = k\sin(2x + \beta)$, $0 < \beta < 2\pi$, then the values of k and β are

(a) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{4}$

(b) $k = -6,\ \beta = \frac{5\pi}{4}$

(c) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{8}$

(d) $k = -6,\ \beta = \frac{3\pi}{4}$

(e) $k = 3\sqrt{2},\ \beta = \frac{7\pi}{4}$

(2)

3. The range of $f(x) = \sqrt{3}\sin x - \cos x + 2$ is:

(a) $[0, 4]$

(b) $[-2, 2]$

(c) $[1, 3]$

(d) $[-1, 1]$

(e) $\left[-1, \sqrt{3}\right]$

4. If the function $f(x) = -\sin 2x + \sqrt{3}\cos 2x$ is written in the form $f(x) = k\sin(bx + \alpha)$ then the phase shift of $f(x)$ is:

A) $-\frac{\pi}{6}$

B) $\frac{2\pi}{3}$

C) $\frac{\pi}{6}$

D) $-\frac{\pi}{3}$

E) $\frac{\pi}{3}$

(3)

5. The expression $-\sqrt{2}\sin\frac{\pi}{5}+\sqrt{2}\cos\frac{\pi}{5}$ can be written as

A) $2\sin\frac{\pi}{20}$

B) $2\sin\frac{9\pi}{20}$

C) $-2\sin\frac{9\pi}{20}$

D) $-2\cos\frac{9\pi}{20}$

E) $2\sin\frac{7\pi}{20}$

6. The exact value of $2\sin^2 5°+2\sin^2 85°+5\sin 217°+5\cos 307°$ is equal to

(a) 2

(b) 1

(c) -3

(d) 7

(e) 0

(4)

7. If $\sin 40^\circ + \cos 40^\circ = k \sin \beta$, then

A) $k = \sqrt{2}, \ \beta = 85^\circ$

B) $k = \sqrt{2}, \ \beta = -45^\circ$

C) $k = 2, \ \beta = 40^\circ$

D) $k = \sqrt{2}, \ \beta = 80^\circ$

E) $k = 2, \ \beta = -80^\circ$

8. The minimum value of $f(x) = 5\sqrt{2}\sin\left(\frac{x}{2}\right) - 5\sqrt{2}\cos\left(\frac{x}{2}\right) - 2$ is

A) -12

B) -8

C) -10

D) -2

E) -3

9. The expression $-\sqrt{2}\sin 5^\circ - \sqrt{2}\cos 5^\circ$ can be written in the form

A) $2\sin 140^\circ$

B) $\frac{\sqrt{2}}{2}\sin 135^\circ$

C) $-2\sin 230^\circ$

D) $2\sin 230^\circ$

E) $2\sin 225^\circ$

5

10. The maximum value of $f(x) = \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$ is

A) $\frac{1}{2}$

B) $\sqrt{3}$

C) 1

D) $\frac{\sqrt{3}}{4}$

E) $\frac{\sqrt{3}-1}{2}$

11. The range of the function $f(x) = -3\sin x + 4\cos x + 3$ is

A) [0,4]

B) [−2,8]

C) [−2,4]

D) [−5,5]

E) [−4,4]

(6)

12. If $\sqrt{3}\cos x - \sin x = 2\sin(x+\alpha)$, then α is equal to

(a) $\frac{2\pi}{3}$

(b) $-\frac{\pi}{3}$

(c) $-\frac{\pi}{6}$

(d) $\frac{5\pi}{6}$

(e) $\frac{3\pi}{4}$

---

13. By using the **Reduction identity**, the range of the function $f(x) = 3\cos(x) - 4\sin(x) + 1$, in interval notation, is

(a) $[-4, 6]$

(b) $[0, 1]$

(c) $[-4, 4]$

(d) $[-4, 3]$

(e) $[-5, 5]$

---

14. If $y = -3\sin x + 3\sqrt{3}\cos x$ is rewritten as $y = k\sin(x+\alpha)$, $0 \leq \alpha < 360°$, then the values of k and α are given by

(a) $k = 6$, $\alpha = 120°$

(b) $k = 18$, $\alpha = 210°$

(c) $k = 6$, $\alpha = 240°$

(d) $k = 9$, $\alpha = 300°$

(e) $k = 6$, $\alpha = 330°$

7

15. If the amplitude of the function

$$f(x) = \sqrt{3}\sin(2x) + 2\cos(2x)$$

is A and the period is B, then

$$\frac{A^2}{\pi}B$$

is equal to

(a) 5
(b) $2\sqrt{7}$
(c) $\sqrt{7}$
(d) 10
(e) 7

16. When writing $-\sin x - \sqrt{3}\cos x$ in the form $A\sin(x+\alpha)$, $0 \leq \alpha \leq 2\pi$ then:

(a) $A = 2,\ \alpha = \frac{7\pi}{6}$

(b) $A = 2,\ \alpha = \frac{4\pi}{3}$

(c) $A = 2,\ \alpha = \frac{2\pi}{3}$

(d) $A = 2,\ \alpha = \frac{5\pi}{3}$

(e) $A = 2,\ \alpha = \frac{11\pi}{6}$

17. The maximum value of the function $f(x) = 3\sin x - 4\cos x$ is equal to:

a) 4
b) 7
c) 1
d) −1
● 5

Section 6.5 Inverse Trigonometric Functions

1. $\sin^{-1}\left(\sin\frac{4\pi}{3}\right) =$

(a) $\frac{4\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $-\frac{\pi}{3}$

(d) $\frac{\pi}{3}$

(e) $-\frac{2\pi}{3}$

2. $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)+\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$

a) $\frac{7\pi}{12}$

● $\frac{5\pi}{12}$

c) $\frac{-7\pi}{12}$

d) $\frac{\pi}{4}$

e) $\frac{\pi}{12}$

(2)

3. The value of $\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)^2 + \left(\cos^{-1}\left(-\frac{1}{2}\right)\right)^2$ is equal to

(a) $\frac{17\pi^2}{36}$

(b) $\frac{13\pi^2}{36}$

(c) $\frac{41\pi^2}{36}$

(d) $\frac{5\pi^2}{36}$

(e) $\frac{\pi^2}{36}$

4. The exact value of $\tan\left(\frac{\pi}{4} + \cos^{-1}\left(-\frac{12}{13}\right)\right)$ is equal to:

a) $\frac{12}{13}$

(b) $\frac{7}{17}$

c) $-\frac{5}{13}$

d) $\frac{13}{17}$

e) $\frac{17}{7}$

3

5. **Which one of the following is UNDEFINED?**

a) $\cos\left(\cos^{-1}\frac{5}{3}\right)$

b) $\tan\left(\tan^{-1}\frac{5}{3}\right)$

c) $\cot\left(\cot^{-1}\frac{5}{3}\right)$

d) $\sec\left(\sec^{-1}\frac{5}{3}\right)$

6. The exact value of $\tan\left[\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(-\frac{5}{13}\right)\right]$ is equal to

(a) $-\frac{33}{56}$

(b) $-\frac{17}{65}$

(c) $\frac{56}{33}$

(d) $-\frac{15}{56}$

(e) $\frac{41}{65}$

7. The value of $\cos\left(2\tan^{-1}\left(-\frac{4}{5}\right)\right)$

A) $\frac{9}{41}$

B) $-\frac{9}{41}$

C) $\frac{5}{\sqrt{41}}$

D) $-\frac{40}{41}$

E) $-\frac{4}{\sqrt{41}}$

8. The exact value of the expression $\sin^{-1}\left\{\cos\left(-\frac{2\pi}{3}\right)\right\}$ is equal to

(a) $-\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $-\frac{\pi}{3}$

(d) $\frac{2\pi}{3}$

(e) $\frac{\pi}{6}$

(5)

9. The exact value of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)+\tan\left(\cos^{-1}-\frac{1}{2}\right)$ is

(a) $-\frac{\pi}{6}-\sqrt{3}$

(b) $\frac{7\pi}{6}+\frac{\sqrt{3}}{3}$

(c) $\frac{\pi}{6}+\sqrt{3}$

(d) $\frac{5\pi}{6}-1$

(e) $-\frac{\pi}{6}+\sqrt{3}$

10. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\sin^{-1}\left(\sin\frac{5\pi}{7}\right)=$

(a) $-\frac{\pi}{21}$

(b) $\frac{7\pi}{6}$

(c) $\frac{2\pi}{21}$

(d) $-\frac{3\pi}{14}$

(e) $-\frac{5\pi}{21}$

11. The expression $\sin(\tan^{-1}\frac{1}{2} - \cos^{-1}\frac{4}{5})$ is:

(6)

A) $\frac{4\sqrt{5}}{25}$

B) $-\frac{2\sqrt{5}}{25}$

C) $-\frac{4\sqrt{5}}{5}$

D) $\frac{2\sqrt{5}}{25}$

E) $\frac{1}{2}$

12. The expression $\sin^{-1}(\sin\frac{5\pi}{3})$ is:

A) $-\frac{5\pi}{3}$

B) $\frac{5\pi}{3}$

C) $\frac{\pi}{3}$

D) $-\frac{\pi}{3}$

E) undefined

13. The exact value of $\csc(\sin^{-1}(-\frac{4}{5}) - \tan^{-1}(-\frac{5}{12}))$ is equal to:

A) $-\frac{48}{65}$

B) $-\frac{65}{48}$

C) $\frac{65}{15}$

D) $\frac{65}{33}$

E) $-\frac{65}{33}$

14. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) =$

A) $\frac{7\pi}{6}$

B) $\frac{11\pi}{6}$

C) $\frac{5\pi}{6}$

D) $\frac{\pi}{3}$

E) $\frac{\pi}{6}$

8

15. The exact value of the expression $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-1)$ is

A) $\frac{7\pi}{12}$

B) $\frac{5\pi}{12}$

C) $-\frac{7\pi}{12}$

D) $\frac{\pi}{2}$

E) $-\frac{5\pi}{12}$

16. $\text{Sin}\left(2\cos^{-1}\left(\frac{-4}{5}\right)\right) =$

A) $-\frac{24}{25}$

B) $\frac{-12}{25}$

C) $\frac{24}{25}$

D) $\frac{-25}{24}$

E) $\frac{12}{25}$

17. $\cot\left[2\cos^{-1}\left(-\frac{4}{5}\right)\right] =$ (9)

A) $-\frac{7}{24}$

B) $-\frac{24}{25}$

C) $-\frac{5}{12}$

D) $\frac{24}{7}$

E) $\frac{7}{24}$

18. $\sin^{-1}[\sin\frac{3\pi}{5}]-\cos^{-1}[\cos\frac{3\pi}{5}]=$

A) $\frac{6\pi}{5}$

B) $-\frac{6\pi}{5}$

C) $\frac{\pi}{5}$

D) π

E) $-\frac{\pi}{5}$

(10)

19. If $\sec^{-1}2+\cos^{-1}x=\frac{\pi}{2}$, then $x=$

A) $\frac{1}{2}$

B) $-\frac{1}{2}$

C) 1

D) $\frac{\sqrt{3}}{2}$

E) $-\frac{\sqrt{3}}{2}$

20. The Domain and the Range of the function $f(x)=-\frac{\pi}{3}-2\sin^{-1}\left(\frac{x}{2}-3\right)$ are respectively

A) $[0,4],\left[-\frac{4\pi}{3},\frac{2\pi}{3}\right]$

B) $[4,8],\left[-\frac{4\pi}{3},\frac{2\pi}{3}\right]$

C) $[-1,1],\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

D) $[4,8],\left[-\frac{\pi}{3},\frac{2\pi}{3}\right]$

E) $[4,6],\left[-\frac{\pi}{3},\frac{\pi}{3}\right]$

21. If $\cos^{-1} x + 2\sin^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$, then $x =$

(a) $\frac{24}{25}$

(b) $\frac{18}{25}$

(c) $\frac{4}{5}$

(d) $-\frac{21}{25}$

(e) $\frac{6}{5}$

22. Which one of the following statements is **FALSE** about the graph of $y = \tan^{-1}(x+1) - \frac{\pi}{2}$?

(a) the graph has only one x-intercept

(b) the graph has only one y-intercept

(c) the graph has two asymptotes $y = -\pi$ and $y = 0$

(d) the graph increases for all real numbers x

(e) the graph lies completely under the x-axis

23. The range of the function $y = -\cos^{-1}(2-x) - \frac{\pi}{2}$

A) $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$

B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C) $\left[-\frac{3\pi}{2}, +\frac{\pi}{2}\right]$

D) $\left[-1-\frac{\pi}{2}, 1-\frac{\pi}{2}\right]$

E) $[1,3]$

24. If $\cos^{-1}\frac{x}{2} + \sin^{-1}\left(-\frac{3}{5}\right) = \frac{\pi}{3}$, then $x =$

(a) $\frac{4-3\sqrt{3}}{5}$

(b) $\frac{3-4\sqrt{3}}{5}$

(c) $\frac{8-6\sqrt{3}}{5}$

(d) $\frac{4+3\sqrt{3}}{10}$

(e) $\frac{4-3\sqrt{3}}{20}$

25. The range of $y = \frac{\pi}{2} - 2\sin^{-1}(x-4)$ is

(a) $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) $[-\pi, \pi]$

(d) $\left[-\frac{\pi}{2}, \pi\right]$

(e) $\left[-\pi, \frac{3\pi}{2}\right]$

26. The solution set of the inverse trigonometric equation $\sin^{-1}\frac{-3}{5} + \tan^{-1}x = \frac{\pi}{2}$ is:

a) $-\frac{4}{3}$

b) $\frac{4}{3}$

c) $\frac{4}{5}$

d) $-\frac{4}{5}$

e) 1

27. The **solution** of the equation $\sin^{-1}\frac{3}{5}+\cos^{-1}x=\pi$ is given by $x=$

(a) $\frac{-3}{5}$

(b) $\frac{4}{5}$

(c) $\frac{-4}{5}$

(d) $\frac{3}{5}$

(e) $\frac{-2}{5}$

28. The domain D and the range R of the function $f(x)=-3\sin^{-1}(2x-1)$ are given by:

a) $D=[0,\ 4]$; $R=\left[\frac{-3\pi}{2},\ \frac{3\pi}{2}\right]$

b) $D=[0,\ 1]$; $R=[-\pi,\ \pi]$

c) $D=[0,\ 1]$; $R=\left[\frac{-\pi}{3},\ \frac{\pi}{3}\right]$

[redacted] $D=[-2,\ 2]$; $R=\left[\frac{-3\pi}{2},\ \frac{3\pi}{2}\right]$

e) $D=[0,\ 1]$; $R=\left[\frac{-3\pi}{2},\ \frac{3\pi}{2}\right]$

(9)

Section 6.6 Trigonometric Equations

1. .The sum of all solutions of the equation $\sin 3x \cos x - \cos 3x \sin x - \frac{1}{2} = 0$, where $0 \le x < 2\pi$, is

A) $\frac{4\pi}{3}$

B) 0

C) $\frac{5\pi}{12}$

D) π

E) 3π

2. The sum of the solutions of the equation $\sin(\frac{6x-\pi}{3}) + \frac{\sqrt{3}}{2} = 0$ in the interval $[0, 2\pi)$ is:

A) $\frac{7\pi}{3}$

B) $-\frac{2\pi}{3}$

C) $\frac{11\pi}{3}$

D) $\frac{9\pi}{4}$

E) $-\frac{\pi}{4}$

(2)

3. The sum of the solutions of the equation $\cos 4\theta + 3 = 5\cos 2\theta$ for $0 \le \theta < \frac{3\pi}{2}$

A) $\frac{13\pi}{6}$

B) $\frac{13\pi}{3}$

C) 2π

D) $\frac{11\pi}{6}$

E) π

4. The number of solutions for the equation $2\sin^2 x \cos x - \cos x = 0$ over the interval $[0, 2\pi)$ is

A) 3

B) 4

C) 6

D) 5

E) 2

5. The sum of the solutions of the equation $\sin x = \cos\frac{x}{2}$ in the interval $[0, 2\pi)$ is

A) 2π

B) π

C) 3π

D) $\frac{8\pi}{3}$

E) $\frac{10\pi}{3}$

6. The sum of the zeros of $\cos 2x = \cos x$ in $[0, 2\pi)$ is

(a) 2π

(b) π

(c) $\frac{5\pi}{6}$

(d) $\frac{3\pi}{2}$

(e) $\frac{8\pi}{9}$

7. If $0 \leq x < 2\pi$, then the number of all solutions of the equation $2\sin\left(2x + \frac{\pi}{6}\right) - 1 = 0$ is

(a) 4

(b) 6

(c) 8

(d) 2

(e) 10

8. The solution set of the equation $\sin^{-1} x + \cos^{-1} \frac{4}{5} = \frac{\pi}{4}$ is equal to

A) $\left\{\frac{\sqrt{2}}{10}\right\}$

B) $\left\{\frac{3}{5}\right\}$

C) $\left\{\frac{7\sqrt{2}}{10}\right\}$

D) $\left\{\frac{3\sqrt{3}-4}{10}\right\}$

E) $\left\{-\frac{\sqrt{2}}{10}\right\}$

9. The sum of solutions in $[0, 2\pi)$ of the equation $\sin x + \cos x = 1$ is

(a) $\frac{\pi}{2}$

(b) $\frac{5\pi}{2}$

(c) π

(d) $\frac{3\pi}{2}$

(e) 2π

10. The equation

$$\cos^{-1}(x) + \sin^{-1}(3 + 4x) = \frac{\pi}{2}$$

has:

(a) Exactly one solution which is a negative integer

(b) Two real solutions

(c) A solution $x \in (-3, -2)$

(d) A solution $x \in (0, 1]$

(e) no solution at all

11. The sum of the solutions of the trigonometric equation $\sqrt{3}\sin x + \cos x = 1$, where $\pi < x < 3\pi$, is:

a) $\frac{14\pi}{3}$

b) 4π

c) $\frac{10\pi}{3}$

d) $\frac{8\pi}{3}$

e) $\frac{18\pi}{3}$

12. If $0 \leq x < 2\pi$ and $\sin x \tan^2 x = \sin^2 x - \sin x$ then the sum of all possible values of x equals

(a) 0

(b) $\frac{\pi}{2}$

(c) $-\frac{\pi}{2}$

(d) The sum is undefined because there is no solution.

(e) π

13. The sum of all solutions of the equation

$$\cos 2x + \cos x = 0$$

in the interval $0 \leq x \leq 2\pi$ is equal to:

(a) 2π

(b) 3π

(c) 4π

(d) 5π

(e) 6π

7

14. The number of solutions in $[0, 2\pi)$ of the equation $2\sin^3 x = \sin x$ is

(a) 6

(b) 3

(c) 7

(d) 4

(e) 5

15. The sum of all solutions of the equation

$$2\sin x \cos x - \sqrt{3}\sin x - 2\sqrt{2}\cos x + \sqrt{6} = 0, \quad \text{where } 0 \le x < 2\pi$$

is equal to

(a) 2π

(b) $\frac{9\pi}{4}$

(c) $\frac{5\pi}{6}$

(d) $\frac{11\pi}{4}$

(e) $\frac{17\pi}{6}$

16. **The sum of the solutions of the equation $2\sin x - \cos 2x = \frac{1}{2}$, $0 \le x < \pi$, is equal to**

A) $\frac{5\pi}{6}$

B) $\frac{2\pi}{3}$

C) 0

D) $\frac{\pi}{6}$

E) π

17. If $0 \leq x < 2\pi$, then the sum of all solutions of

$$2\sin x - 1 - \csc x = 0$$

is equal to

(a) $\frac{\pi}{2}$

(b) 3π

(c) $\frac{5\pi}{3}$

(d) 0

(e) $\frac{7\pi}{2}$

18. If $0 \leq x < 2\pi$ and $\sin x \tan^2 x = \sin^2 x - \sin x$ then the sum of all possible values of x equals

(a) 0

(b) $\frac{\pi}{2}$

(c) $-\frac{\pi}{2}$

(d) The sum is undefined because there is no solution.

(e) π

19. The sum of the solutions of

$2\sin x \cos x + \sqrt{3}\cos x + 2\sin x + \sqrt{3} = 0$ in the interval $\left[0, \frac{3\pi}{2}\right)$ is

A) $\frac{7\pi}{3}$

B) 3π

C) 4π

D) $\frac{11\pi}{6}$

E) $\frac{7\pi}{6}$

20. If $0 \le x \le 2\pi$, then the sum of all solutions of the equation

$$\sin 4x \cos x - \cos 4x \sin x = 1$$

is equal to :

a) $-\frac{\pi}{3}$

b) $\frac{2\pi}{3}$

c) 2π

d) 5π

e) $\frac{5\pi}{2}$

21. The number of solutions of the equation $\sqrt{2}\sin 2x = 1$ in the interval $[0, 2\pi)$ is equal to:

a) 2
b) 4
c) 8
d) 6
e) 1

22. The number of solutions of the equation $\cos\left(2x + \frac{2\pi}{3}\right) - \frac{\sqrt{3}}{2} = 0$ in $[0, 2\pi]$

A) 4
B) 3
C) 2
D) 1
E) 5