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Section 7.3 Vectors

1. If $\mathbf{v} = \langle a,b \rangle$ is a vector whose magnitude is 50 and direction angle $\theta = \frac{5\pi}{6}$, then the horizontal component a and the vertical component b of $\mathbf{v}$ are

A) $a = -25\sqrt{3}$ and $b = 25$

B) $a = 25\sqrt{3}$ and $b = 25$

C) $a = -25$ and $b = 25\sqrt{3}$

D) $a = -25\sqrt{3}$ and $b = -25$

E) $a = \frac{-50\sqrt{3}}{3}$ and $b = 50\sqrt{3}$

2. Given the vectors $\mathbf{u} = <3,-5>$, and $\mathbf{v} = <-3,-1>$. Then the magnitude M and the direction angle θ of the vector $\mathbf{u}+\mathbf{v}$ are:

a) $M = 6,\ \theta = \frac{3\pi}{2}$

b) $M = 4,\ \theta = \frac{3\pi}{4}$

c) $M = 6,\ \theta = \frac{\pi}{2}$

d) $M = 4,\ \theta = \pi$

e) $M = 6,\ \theta = \frac{\pi}{4}$

3. Suppose that the vector $\mathbf{u} = \overrightarrow{PQ}$, where the initial point is $P(5, 4)$ and terminal point is $Q\ (5,11)$. If $\mathbf{v} = \sqrt{3}\,\mathbf{i} - 8\mathbf{j}$, then the **magnitude** M and the **direction angle** α of the vector $\mathbf{u}+\mathbf{v}$ are

(a) $M = 4,\ \alpha = \frac{\pi}{6}$

(b) $M = 2,\ \alpha = \frac{11\pi}{6}$

(c) $M = 2,\ \alpha = \frac{7\pi}{6}$

(d) $M = 2,\ \alpha = \frac{5\pi}{6}$

(e) $M = 2,\ \alpha = \frac{5\pi}{36}$

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4. The magnitude M and the direction angle α of the vector $\mathbf{u} = 2\langle -1, 6\rangle - 3\langle 5, 4\rangle$ is

(a) $M = 17, \quad \alpha = 180°$

(b) $M = 17, \quad \alpha = 0°$

(c) $M = \sqrt{7}, \quad \alpha = 90°$

(d) $M = \sqrt{13}, \quad \alpha = 0°$

(e) $M = 17, \quad \alpha = 270°$

5. If $\mathbf{u} = \langle -2, 4\rangle$ and $\mathbf{v} = \langle -3, -2\rangle$, then $\|3\mathbf{u} - 4\mathbf{v}\|$ is equal to

(a) $2\sqrt{109}$

(b) $2\sqrt{85}$

(c) $\sqrt{85}$

(d) $2\sqrt{15}$

(e) $2\sqrt{119}$

6. **If**

$$\vec{u} = \langle 2\sqrt{3}, -3\rangle \quad \text{and} \quad \vec{v} = \langle -\sqrt{3}, 4\rangle$$

then the magnitude and direction angle of $\vec{u} + \vec{v}$ are equal to

(a) $2, \pi/3$

(b) $\sqrt{3}, \pi/3$

(c) $2, \pi/6$

(d) $\sqrt{3}, \pi$

(e) $4, \pi/6$

7\. What is the direction angle θ and the magnitude $\|v\|$ of the vector $v = \sqrt{3}i - 3j$?

(a) $\theta = \frac{11\pi}{6}$, $\|v\| = 2\sqrt{3}$

(b) $\theta = \frac{2\pi}{3}$, $\|v\| = 2\sqrt{3}$

(c) $\theta = \frac{5\pi}{3}$, $\|v\| = 2\sqrt{3}$

(d) $\theta = \frac{5\pi}{3}$, $\|v\| = \sqrt{3}$

(e) $\theta = \frac{2\pi}{3}$, $\|v\| = \sqrt{3}$

8\. The measure of the smallest positive angle between the vectors

$u = -i - 2j$ and $v = -i + 3j$

is equal to :

a) 135°

b) 45°

c) 120°

d) 150°

e) 225°

(4)

9. The smallest positive angle between the vectors $\mathbf{u} = \langle 2, 1 \rangle$ and $\mathbf{v} = \langle -1, -3 \rangle$ is

(a) $\frac{3\pi}{4}$

(b) $\frac{2\pi}{3}$

(c) $\frac{3\pi}{2}$

(d) $\frac{5\pi}{4}$

(e) $\frac{5\pi}{6}$

10. If $\mathbf{u} = \langle -2, 7 \rangle$, then a nonzero vector that is perpendicular to $\mathbf{u}$ is

(a) $\langle 14, 4 \rangle$

(b) $\langle -1, 1 \rangle$

(c) $\langle 2, -7 \rangle$

(d) $\langle 1, -1 \rangle$

(e) $\langle 7, -2 \rangle$

11. Let $\theta = \cos^{-1}(-\frac{3}{5})$ be the direction angle of a vector u. If $\|u\| = 20$, then **the vertical component of** u is equal to :

a) 16

b) 12

c) −16

d) −12

e) 4

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12. If $\mathbf{v} = 2i + j$ and $\mathbf{w} = 6i + 3j$, then $\text{Proj}_{\mathbf{w}}\,\mathbf{v}$ is equal to

(a) $\sqrt{5}$

(b) $\frac{\sqrt{5}}{5}$

(c) $3\sqrt{5}$

(d) $\frac{12\sqrt{5}}{5}$

(e) $\frac{1}{9}$

13. Given the vectors $\mathbf{v} = \langle 6, -7\rangle$ and $\mathbf{w} = \langle 3, 4\rangle$. the value of $\text{Proj}_{\mathbf{w}}\,\mathbf{v}$ is equal to

(a) -2

(b) $\frac{13}{5}$

(c) $-\frac{21}{5}$

(d) $\frac{41}{5}$

(e) -6

14. For the vectors $\vec{u} = <0, 5>$ and $\vec{v} = <-2, 2>$, the smallest positive angle between the vectors $\vec{u} + \vec{i}$ and $\vec{v} + \vec{j}$ is

A) $\cos^{-1}\frac{1}{3}$

B) $\cos^{-1}\left(-\frac{2}{\sqrt{13}}\right)$

C) 120°

D) 45°

E) 135°

15. The cosine of the smallest positive angle between the vectors $\mathbf{u} = \langle -1,1 \rangle$ and $\mathbf{v} = \langle 1,7 \rangle$ is equal to:

A) $\frac{7}{10}$

B) $-\frac{1}{5}$

C) $\frac{4}{5}$

D) $\frac{6}{5\sqrt{2}}$

E) $\frac{3}{5}$

16. The smallest positive angle between the vectors $V = -2i - 2\sqrt{3}j$ and $W = -5i + 5\sqrt{3}j$ is

A) $\frac{2\pi}{3}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{6}$

D) $\frac{5\pi}{6}$

E) $\frac{4\pi}{3}$

17. The vertical component of the vector with magnitude $\sqrt{3}$ and direction angle 300^0 is

A) $\frac{3}{2}$

B) $\frac{1}{2}$

C) $-\frac{\sqrt{3}}{2}$

D) $-\frac{3}{2}$

E) $\frac{\sqrt{3}}{2}$

18. If θ is the smallest positive angle between the vectors $u = 3\mathbf{i} - 4\mathbf{j}$ and $v = -2\mathbf{i} + \mathbf{j}$, then $\tan\theta$ is equal to

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{3}$

(e) $-\frac{1}{5}$

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19. If $v = 2i + j$ and $w = 6i + 3j$, then $\text{Proj}_w v$ is equal to

A) $\frac{3\sqrt{5}}{5}$

B) $\sqrt{5}$

C) $3\sqrt{5}$

D) $\frac{\sqrt{5}}{5}$

E) $\frac{\sqrt{5}}{15}$

20. If $\vec{u} = \langle -1, 2 \rangle$, $\vec{v} = \langle 3, -3 \rangle$, and $\vec{w} = \langle -3, 4 \rangle$ then $\text{proj}_{\vec{w}} (\vec{u} + \vec{v})$ is

A) -2

B) $\frac{3}{5}$

C) $\frac{\sqrt{5}}{5}$

D) 4

E) $-2\sqrt{5}$

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21. If α is the smallest positive angle between the two vectors $\mathbf{u} = 4\mathbf{i} - 3\mathbf{j}$ and $\mathbf{v} = \langle 4,1 \rangle$, then $\cos\alpha =$

A) $\frac{\sqrt{17}}{13}$

B) $\frac{17}{85}$

C) $\frac{13}{85}$

D) $\frac{13\sqrt{17}}{85}$

E) $\frac{13}{17}$

22. If $\mathbf{u} = \langle 3,4 \rangle$ has the same direction as $\mathbf{v} = 9\mathbf{i} + b\mathbf{j}$, then the value of b is equal to

A) 16

B) $-\frac{27}{4}$

C) $\frac{3}{4}$

D) -16

E) 12

23. If $\vec{v} = \langle 2, 5 \rangle$, $\vec{u} = \langle 5, -2 \rangle$ and if $\vec{w} = \langle a, b \rangle$ is a unit vector in the opposite direction of $\vec{v} - \vec{u}$, then

(a) $a = \frac{3\sqrt{58}}{58}$, $b = -\frac{7\sqrt{58}}{58}$

(b) $a = -\frac{3\sqrt{58}}{58}$, $b = -\frac{7\sqrt{58}}{58}$

(c) $a = -\frac{7\sqrt{58}}{58}$, $b = \frac{3\sqrt{58}}{58}$

(d) $a = -\frac{7\sqrt{58}}{58}$, $b = -\frac{3\sqrt{58}}{58}$

(e) $a = \frac{3\sqrt{58}}{58}$, $b = \frac{7\sqrt{58}}{58}$

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24. Which one of the following statements is **false** for nonzero vectors $\mathbf{u}$ and $\mathbf{v}$:

A) If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, then $proj_u \mathbf{v} = 0$

B) If $proj_u \mathbf{v} = 0$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal

C) If $proj_u \mathbf{v} = \|\mathbf{v}\|$, then $\mathbf{u}$ and $\mathbf{v}$ are parallel

D) $proj_v \mathbf{v} = \|\mathbf{v}\|$

E) $proj_u \mathbf{v} - proj_v \mathbf{u} = 0$

25. Given the vectors $\mathbf{u} = \langle -2,2 \rangle$, $\mathbf{v} = \langle 2,-4 \rangle$. If the vector $\mathbf{w} = \langle a,b \rangle$ is a unit vector in the opposite direction of $\mathbf{u} - \frac{1}{2}\mathbf{v}$, then $a - b =$

A) $\frac{3}{5}$

B) $\frac{7}{5}$

C) $-\frac{3}{5}$

D) $\frac{1}{5}$

E) $-\frac{7}{5}$

26. Given that a vector V with $\|V\| = 5$ and direction angle $\theta = 30^\circ$, the vector W of magnitude 2 and in the opposite direction of V is

A) $\langle -\sqrt{3}, -1 \rangle$

B) $\langle \sqrt{3}, 1 \rangle$

C) $\langle -2\sqrt{3}, -2 \rangle$

D) $\left\langle -\frac{1}{2}, \sqrt{3} \right\rangle$

E) $\langle -1, -\sqrt{3} \rangle$

27. Given the vectors $\mathbf{u} = \langle -4, 10 \rangle$ and $\mathbf{v} = \langle -5, 1 \rangle$. If the vector $\mathbf{w} = \langle a, b \rangle$ is a **unit** vector **in the opposite direction** of $\frac{1}{2}\mathbf{u} - \mathbf{v}$, then $a + b$ is equal to

(a) $-\frac{7}{5}$

(b) $-\frac{3}{5}$

(c) $-\frac{4}{5}$

(d) $-\frac{2}{5}$

(e) $-\frac{9}{5}$

28. If $u = \langle 3,3 \rangle$ and $v = 3j$, then a vector of length 3 in the opposite direction of $u + \frac{1}{3}v$ is:

a) $\left\langle \frac{-9}{5}, \frac{-12}{5} \right\rangle$

b) $\left\langle \frac{9}{5}, \frac{-12}{5} \right\rangle$

c) $\left\langle \frac{-9}{5}, \frac{12}{5} \right\rangle$

d) $\left\langle \frac{-3}{5}, \frac{-12}{5} \right\rangle$

e) $\left\langle \frac{-9}{5}, \frac{-4}{5} \right\rangle$

29. The sum of all values of the constant k for which the two vectors:

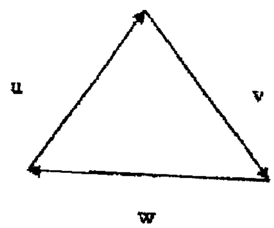
$$\mathbf{u}=(k-1)\mathbf{i}+\mathbf{j} \text{ and } \mathbf{v}=3\mathbf{i}+(k-1)^2\mathbf{j}$$

are perpendicular is equal to:

a) -3
b) 4
c) -2
d) 3
e) -1

30. For the vectors u, v and w shown in the figure. Which one of the following relations is TRUE?

a) $u+v+w=0$
b) $v+w-u=0$
c) $w+u-v=0$
d) $u+v-w=0$



31. Given the vectors $u=\langle 2,-2\rangle$ and $v=\langle -12,15\rangle$. If the vector $w=\langle m,n\rangle$ is a **unit** vector in the **opposite** direction of $\frac{1}{2}u+\frac{1}{3}v$, then $m+n=$

a) $-\frac{1}{5}$

b) $\frac{7}{5}$

c) $\frac{2}{5}$

d) $-\frac{2}{5}$

e) $-\frac{7}{5}$

32. Given the vectors $\mathbf{u} = 10\mathbf{i} - 8\mathbf{j}$ and $\mathbf{v} = 12\mathbf{i} - 6\mathbf{j}$. the direction angle of the vector $\frac{1}{2}\mathbf{u} - \frac{1}{6}\mathbf{v}$, in radians, is equal to

(a) $\frac{7\pi}{4}$

(b) $\frac{5\pi}{4}$

(c) $\frac{3\pi}{4}$

(d) $\frac{\pi}{4}$

(e) $\frac{5\pi}{3}$

33. Given the vectors $w = \langle y, x \rangle$ and $v = \langle x, y \rangle$, then $proj_w v + proj_v w$ is equal to:

a) $\frac{4xy}{\|w\|}$

b) $\frac{2xy}{\|v\|}$

c) $\frac{2xy}{\|v + w\|}$

d) $\|w\|$

e) $2(x^2 + y^2)$