(9)

## Section 10.1 Gaussian Elimination Method

1. If $(a, b, c)$ is the solusion of the system $\begin{cases} x-3y+z=8 \\ 2x-5y-3z=2 \\ x+4y+z=1 \end{cases}$, then $a+b+c=$

A) $-4$

B) $-6$

C) 6

D) $-1$

E) 4

2. If $(a, b, c)$ is the solution of the system $\begin{cases} x-y+3z = 10 \\ 2x-y+7z = 24, \\ 3x-6y+7z = 21 \end{cases}$

then $a+b+c=$

A) 8

B) 13

C) 6

D) 10

E) 12

(2)

3. If the system $\left[\begin{array}{ccc|c} -1 & 0 & 1 & -4 \\ 2 & 1 & 1 & 5 \\ -1 & 1 & 1 & -1 \end{array}\right]$ is written as $\left[\begin{array}{ccc|c} 1 & a & b & 4 \\ 0 & 1 & c & -3 \\ 0 & 0 & 1 & -2 \end{array}\right]$, then the sum of $a$, $b$ and $c$ is

a) 2

b) 0

c) 1

d) −1

e) −2

4. If $(a,b,c)$ is the solution of the system of equations

$$\begin{cases} x+3y-z=1 \\ 2x+5y-3z=1 \\ x+4y+z=0 \end{cases}$$

then $a+b+c=$

a) −9

b) −2

c) −7

d) 0

e) 12

5. The system of linear equation $\begin{cases} x + 2y = 1 \\ x + 3y + z = 4 \\ 2y + 2z = 6 \end{cases}$

has

A) three solutions only

B) no solution

C) a unique solution

D) infinitely many solusions

E) two solutions only

6. If the echelon form of the augmented matrix for the linear system

$$\begin{cases} x - 3y + z = 8 \\ 2x - 5y - 3z = 6 \\ x - 6y + 7z = -7 \end{cases}$$

is $\left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 0 & 1 & m & n \\ 0 & 0 & 1 & p \end{array}\right]$, then

(a) $m = -5$, $n = -10$, and $p = 5$

(b) $m = 3$, $n = -6$, and $p = -3$

(c) $m = -5$, $n = 10$, and $p = -3$

(d) $m = -2$, $n = 7$, and $p = -1$

(e) $m = -3$, $n = 6$, and $p = -2$

(4)

**7.** The echelon form of the system $\begin{cases} 4x-5y-z=2 \\ 3x-4y+z=-2 \\ x-2y-z=3 \end{cases}$ is:

a) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 1 & -\frac{13}{6} \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -11 \\ 0 & 0 & 1 & -\frac{13}{6} \end{bmatrix}$

c) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 0 & -\frac{13}{6} \end{bmatrix}$

d) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 1 & -\frac{5}{6} \end{bmatrix}$

e) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5

8. When the augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & 2 & 4 & 5 \\ 2 & 1 & 1 & 6 \end{array}\right]$ of a given system of linear equations is written in the echelon form

$$\left[\begin{array}{ccc|c} 1 & m & n & 2 \\ 0 & 1 & k & 1 \\ 0 & 0 & 1 & -3/2 \end{array}\right],$$

then the product $mnk$ is equal to

(a) $-1$

(b) 0

(c) 1

(d) 2

(e) $-2$

9. If the **echelon** form of the augmented matrix of the system

$$\begin{cases} x-3y+z=1 \\ 2x-5y-3z=2 \\ x-4y+z=11 \end{cases} \quad \text{is equal to} \quad \left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & p & q \\ 0 & 0 & r & t \end{array}\right],$$

then the sum $p+q+r+t$ is equal to:

*a*) $-9$

*b*) $-12$

*c*) 7

*d*) $-6$

*e*) 8

**10.** The echelon form of the matrix $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 2 & 0 & -2 & 4 \\ 0 & 4 & 2 & 11 \end{bmatrix}$ is given by:

a) $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 3/2 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 2 & 8 & 6 \\ 0 & 0 & -18 & -9 \end{bmatrix}$

e) $\begin{bmatrix} 1 & -3 & 2 & 4 \\ 0 & 1 & 8 & 6 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$

**11.** If $(x, y, z) = (a, b, c)$ is the solution of the following system of equations, then what is the value of $a + b + c$?

$$\begin{aligned} 4x + 8y + 6z &= 1 \\ x + 2y + z &= 0 \\ x + y - 2z &= -1 \end{aligned}$$

(a) $-\frac{1}{2}$

(b) $1\frac{1}{2}$

(c) $-1\frac{1}{2}$

(d) 0

(e) $\frac{1}{2}$

**12.** The solution of the following system is $(x, y, z) = (a, b, c)$:

$$x - \frac{1}{y} + \frac{2}{z} = 1$$
$$3x + \frac{2}{y} + \frac{4}{z} = 4$$
$$\frac{1}{y} + \frac{2}{z} = 5$$

What is the value of $abc$?

(a) 1

(b) $-1$

(c) $-4$

(d) 4

(e) 2

**13.** If $(x, y, z) = (a, b, c)$ is the solution of the following system of equations, then what is the value of $a + b + c$?

$$4x + 8y + 6z = 1$$
$$x + 2y + z = 0$$
$$x + y - 2z = -1$$

(a) $-\frac{1}{2}$

(b) $1\frac{1}{2}$

(c) $-1\frac{1}{2}$

(d) 0

(e) $\frac{1}{2}$

8

14. The system of linear equations $\begin{cases} x-3y+z &= 5 \\ -7y+2z &= 12-3x \\ 2x-6y+2z &= 10 \end{cases}$

(A) is dependent

B) has three solutions only

C) is independent

D) is inconsistent

E) has two solutions only

15. If the augumented matrix of a system of linear equations is

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -x^2 & -4 & -3 \\ 0 & 0 & 4x^2-1 & x-\frac{1}{2} \end{array}\right)$$

then the system is inconsistent if $x =$

(A) $-\frac{1}{2}$

B) $\frac{1}{2}$

C) 2

D) $\frac{1}{4}$

E) $-\frac{1}{4}$

16. If the system $\begin{cases} x-2y+z=1 \\ y+3z=m^2 \\ y+kz=4 \end{cases}$ has a unique solution, then

a) $k \neq 3$

b) $k \neq 3$ and $m \neq \pm 2$

c) $k = 3$ and $m \neq \pm 2$

d) $k = 1$

e) $k \neq 3$ and $m = \pm 2$

17. The system of equations $\begin{cases} x+2y-2z=3 \\ 5x+8y-6z=14 \\ 3x+4y-2z=8 \end{cases}$

a) is dependent

b) is independent

c) is inconsistent

d) has the unique solution $\left\{\left(2, \frac{1}{2}, 0\right)\right\}$

e) has the unique solution $\left\{\left(0, \frac{5}{2}, 1\right)\right\}$

(10)

18. The system $\begin{cases} x+y=1 \\ y+z=1 \\ x+kz=1 \end{cases}$ has no solution if $k$ is equal to:

a) $-1$

b) 1

c) 0

d) $-2$

e) 2

19. The system

$$\begin{cases} x+y=k \\ y+z=2 \\ x-z=1 \end{cases}$$

is :

a) independent for $k \neq 3$

b) dependent for $k \neq 3$

c) inconsistent for $k \neq 3$

d) dependent for $k=4$

e) independent for $k=5$

20. The value of the constant $k$ for which the system of equations:

$$x + kz = 1$$
$$y + z = 2$$
$$2x + y = 5$$

has **no solution**, is equal to:

a) $-3/2$

b) $-5/2$

c) $3/2$

d) $-1/2$

e) $-1$

21. Consider the **augmented matrix** of a **linear system**

$$\begin{bmatrix} 1 & -2 & -2 & M & -1 \\ 1 & 1 & 1 & M & 2 \\ 1 & 2 & 2 & M & 1 \end{bmatrix}$$

Which one of the following statements is **TRUE** ?

(*a*) The system is independent.

(*b*) The system is dependent

(*c*) The system has the solution set $\left\{\left(2, 1, \frac{1}{2}\right)\right\}$

(*d*) The system has the solution set $\{(5, -1, -1)\}$

(*e*) The system has no solution.

**22.** If the augmented matrix of a system of equations is $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & C^2 & 1 \\ 0 & 2 & 8 & C \end{array}\right]$, which one of the following is FALSE?

A) The system is inconsistent for all $C \neq -2$

B) The system is inconsistent for $C = -2$

C) The system has unique solution for all $C \neq \pm 2$.

D) The system has infinitely many solutions for $C = 2$.

E) The system can be made consistent or inconsistent for a suitable value of $C$.

**23.** Which one of the following statements is TRUE about the linear system of equations which has the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 0 \\ 1 & 2 & (c^2-1) & c+1 \end{array}\right).$$

(a) The system is consistent if $c = 0$, with infinitely many solutions.

(b) The system is consistent for all $c \neq 0$, with exactly one solution.

(c) The system can be made consistent for a suitable choice of $c$.

(d) The system is inconsistent for all values of $c$.

(e) The system is consistent for $c > 0$.

## Section 10.2 The Algebra of Matrices

1. If $A=\begin{bmatrix}2 & -1 & -2 & 5\\ 3 & 0 & 1 & 4\end{bmatrix}$, $B=\begin{bmatrix}-6 & 3 & 1 & 0\\ 3 & 2 & 7 & 4\end{bmatrix}$ and $3A+5X=3X+B$, then $X=$

A) $\begin{bmatrix}-6 & 3 & \frac{7}{2} & -\frac{15}{2}\\ -3 & 1 & 2 & -4\end{bmatrix}$

B) $\begin{bmatrix}-4 & 2 & -1 & 5\\ 6 & 2 & 8 & 8\end{bmatrix}$

C) $\begin{bmatrix}0 & 0 & -5 & 15\\ 12 & 2 & 10 & 16\end{bmatrix}$

D) $\begin{bmatrix}0 & 0 & -\frac{5}{2} & \frac{15}{2}\\ 6 & 1 & 5 & 8\end{bmatrix}$

E) $\begin{bmatrix}-12 & 6 & 7 & -15\\ -6 & 2 & 4 & -8\end{bmatrix}$

2. If $\begin{bmatrix}x+2 & 8 & -3\\ 1 & 2y & 2x+1\\ 7 & -2 & y+2\end{bmatrix}=\begin{bmatrix}2x+6 & 8 & -3\\ 1 & 18 & -7\\ 7 & -2 & 11\end{bmatrix}$, then $x+y=$

A) $-13$

B) $-5$

C) $5$

D) $4$

E) $13$

(2)

3. Suppose that $A = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 22 \\ 10 \end{bmatrix}$, and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. If $AX = B$, then the **matrix** $X$ **is equal to**

(a) $\begin{bmatrix} 2 & 2 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 22 \\ 10 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 22 \\ 10 \end{bmatrix}$

(c) $\begin{bmatrix} 11 \\ 5 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 11 \\ 5 \end{bmatrix}$

(e) $\begin{bmatrix} 22 \\ 10 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & \frac{3}{2} \end{bmatrix}$

4. Given the matrices:

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 3 & 5 & 1 \end{bmatrix},\ B = \begin{bmatrix} 5 & 0 \\ -3 & 1 \\ 0 & -1 \end{bmatrix},\ \text{and}\ C = \begin{bmatrix} \frac{3}{2} & 1 \\ 0 & \frac{3}{2} \end{bmatrix}.$$

Then the matrix $AB - 2C$ is equal to:

a) $\begin{bmatrix} 6 & -1 \\ 3 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 6 & 1 \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 6 & 2 \\ 1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 6 \\ 1 & 0 \end{bmatrix}$

e) $\begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

5. If $A=\begin{bmatrix}0 & -2 & 7\\ 5 & 4 & 3\end{bmatrix}$, $B=\begin{bmatrix}3 & 1\\ -1 & 5\\ 6 & 0\end{bmatrix}$, $C=\begin{bmatrix}40 & -10\\ 28 & 23\end{bmatrix}$, and $D = AB - C$, then the **element** in the **second row** and **second column** of the matrix $D$ is equal to

(*a*) 2
(*b*) 48
(*c*) 0
(*d*) 28
(*e*) −10

6. If $A=\begin{bmatrix}-1 & 2 & 3\\ 3 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1 & 4\\ -2 & 3 & -3\end{bmatrix}$, then the matrix $X$ for which $3X+2B=X-2A$ is equal to :

*a*) $\begin{bmatrix}1 & -6 & -7\\ -1 & -2 & 4\end{bmatrix}$

(*b*) $\begin{bmatrix}1 & -3 & -7\\ -1 & -2 & 2\end{bmatrix}$

*c*) $\begin{bmatrix}1 & -3 & -14\\ -1 & -4 & 2\end{bmatrix}$

*d*) $\begin{bmatrix}1 & -3 & -7\\ -2 & -2 & 4\end{bmatrix}$

*e*) $\begin{bmatrix}2 & -3 & -14\\ -1 & -2 & 2\end{bmatrix}$

7. A linear system written in a matrix form

$$\begin{bmatrix} -2 & 6 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

is

(a) dependent

(b) inconsistent

(c) independent

(d) consistent with only two solutions

(e) consistent with only three solutions

8. Let $A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and $I$ be the $3 \times 3$ identity matrix. Then the element in the second row and third column of the matrix $3A^2 - 2I$ is

(a) 6

(b) 4

(c) 7

(d) 5

(e) 10

9. Given $A = \begin{bmatrix} 0 & -2 & 7 \\ 5 & 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -1 & 5 \\ 6 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 40 & -10 \\ 28 & 23 \end{bmatrix}$. If $X = AB - C$, then the element in the second row and second column of matrix $X$ is equal to

(a) 2

(b) 48

(c) 0

(d) 28

(e) $-10$

10. If $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + 5\begin{bmatrix} 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$, then $a - 2b$ is equal to:

a) 4

b) 3

c) $-3$

d) $-1$

e) $-1$

11. If $A=\begin{bmatrix}-2 & 2\\ 3 & 4\\ 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}4 & 1\\ 1 & -2\\ 3 & -4\end{bmatrix}$, then the matrix $X$ for which $3X-4B=X+2A$ is equal to:

a) $\begin{bmatrix}6 & 4\\ 5 & 0\\ 7 & -8\end{bmatrix}$

b) $\begin{bmatrix}2 & 3\\ 4 & 2\\ 4 & -4\end{bmatrix}$

c) $\begin{bmatrix}-10 & 0\\ 1 & 0\\ -5 & -6\end{bmatrix}$

d) $\begin{bmatrix}2 & 1\\ 2 & -1\\ 0 & 5\end{bmatrix}$

e) $\begin{bmatrix}2 & 8\\ -1 & 7\\ -2 & 3\end{bmatrix}$

12. If $A=\begin{bmatrix}1 & x\\ 2 & 3\end{bmatrix}$ and $A^2-4A=I$, then $x$ is equal to:

a) 2

b) 1

c) 0

d) $-1$

e) $-2$

7

13. If $A=\begin{pmatrix}2 & 0 & -2\\ 3 & -1 & 0\end{pmatrix}$ and $B=\begin{pmatrix}5 & 2 & -1\\ 0 & -3 & 1\\ -2 & 6 & 0\end{pmatrix}$ then the element in the second row and third column of the matrix $AB$ is

A) 0

B) 2

C) $-4$

D) $-2$

E) 10

14. Given that $A=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}, B=\begin{pmatrix}2 & 0\\ x & 1\end{pmatrix}, C=\begin{pmatrix}0 & 0\\ 6 & 1\end{pmatrix}$. If $AB=2A^2-C$, then $x=$

A) $-4$

B) 4

C) 0

D) 2

E) $-2$

15. If $A=\begin{bmatrix}2 & 3 & -1\\ 0 & -5 & 2\\ 7 & 1 & -2\end{bmatrix}$ then the element in the third row and second column of $(2A)^2+A$ is

a) 57

b) $-14$

c) 29

d) $-6$

e) 17

8

16. If $C = AB$ where

$$A = \begin{pmatrix} 1 & -1 & 0 & 1 \\ 2 & 3 & -1 & 4 \\ -1 & 2 & 1 & 3 \\ 0 & 1 & -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 1 & -1 & 2 & -1 \\ 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 2 \end{pmatrix}$$

then $c_{32}$, the third row and second column of $C$ equals

(a) 5

(b) −5

(c) 6

(d) −6

(e) 0

17. If $C = BA$ where $A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & -2 \\ -3 & 4 & -5 \\ -6 & 7 & -8 \\ -9 & 10 & -11 \end{bmatrix}$

then $c_{23}$, the element in the second row and third column of $C$ is equal to

A) 32

B) −32

C) 158

D) −158

E) 116

18. Suppose that A, B and C are nonsingular matrices of order $n$, then which one of the following statements is always TRUE?

A) $A(B+C)=AB+CA$

B) $A^2-B^2=(A+B)(A-B)$

C) $AC=BC$, implies that $A=B$

D) $AB=0$, implies that $A=0$ or $B=0$

E) $(A+B)^2=A^2+2AB+B^2$

19. If $A, B$, and $C$ are matrices each of order $n \times n$, then which one of the following is TRUE?

(a) $(AB)C = A(BC)$

(b) $(A+B)^2 = A^2+2AB+B^2$

(c) $(A+B)\cdot C = A+(B\cdot C)$

(d) $C(AB)+C(BA) = 2C(AB)$

(e) $(A-B)(A+B) = A^2-B^2$

20. Let $A, B$, and $C$ be any $n \times n$ matrices and let $O$ be the $n \times n$ zero matrix. Which one of the following statements is **FALSE**?

(a) $AB - BA = O$

(b) $A(BC) = (AB)C$

(c) $OA = AO = O$

(d) $A(B+C) = AB+AC$

(e) $A^2+B^2$ is an $n \times n$ matrix

(9)

## Section 10.3 The Inverse of a Matrix

1. If

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

then what is the element in row 2 column 3 of $A^{-1}$?

(a) 0

(b) $\frac{1}{3}$

(c) $-\frac{1}{3}$

(d) 3

(e) 1

2. If $A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 6 \\ -1 & -1 & 2 \end{bmatrix}$, then the sum of the elements in the second row of $A^{-1}$ is equal to:

a) $-\frac{2}{3}$

b) $\frac{5}{3}$

● $-\frac{1}{3}$

d) $-4$

e) 2

3. Let $A^{-1} = \begin{bmatrix} 1 & 0 & 7 \\ 2 & 1 & -1 \\ 7 & 3 & 1 \end{bmatrix}$, then the sum of the elements in the 2nd row of the matrix $A$ is:

a) 14

b) 10

c) 9

d) 16

e) 2

(2)

**4.** Let $A=\begin{bmatrix}1 & 0 & 7\\ 0 & 1 & 0\\ 0 & 3 & 1\end{bmatrix}$, then the elements of the first row of the **inverse matrix** $A^{-1}$ are:

a) $a_{11}=-3$ , $a_{12}=-21$ , $a_{13}=7$

b) $a_{11}=1$ , $a_{12}=1$ , $a_{13}=-7$

c) $a_{11}=1$ , $a_{12}=-21$ , $a_{13}=-11$

d) $a_{11}=3$ , $a_{12}=21$ , $a_{13}=-7$

(e) $a_{11}=1$ , $a_{12}=21$ , $a_{13}=-7$

**5.** Let $A=\begin{bmatrix}1 & 0 & 1\\ 2 & 2 & 0\\ 0 & 0 & -1\end{bmatrix}$. Then the sum of the elements of the second row of $A^{-1}$ is equal to

(a) $-\frac{3}{2}$

(b) $-4$

(c) $\frac{5}{2}$

(d) $\frac{7}{2}$

(e) $-\frac{1}{2}$

**6.** If $A=\begin{bmatrix}1 & 0 & 4\\ 2 & 1 & 0\\ 1 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0 & 2\\ m & 1 & -4\\ \frac{1}{2} & 0 & k-\frac{1}{2}\end{bmatrix}$ are inverses of each other, then

(a) $k=0$ and $m=2$

(b) $k=-1$ and $m=0$

(c) $k=1$ and $m=2$

(d) $k=2$ and $m=0$

(e) $k=3$ and $m=-1$

7. If $A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{-1}{2} & 1 \end{bmatrix}$, then the matrix $A$ is equal to

(a) $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 4 \\ 4 & 0 \end{bmatrix}$

8. The matrix $M$ and its inverse are given by

$$M = \begin{pmatrix} 2 & 2 & -1 \\ 0 & 3 & -1 \\ -1 & -2 & 1 \end{pmatrix}, \quad M^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ x & y & z \end{pmatrix}$$

The sum $x + y + z$ equals

(a) 11

(b) 9

(c) 13

(d) 7

(e) 5

(4)

**9.** The solution of the matrix equation:

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

is given by:

a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2/7 & 3/7 \\ 1/7 & -2/7 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} -2/7 & 3/7 \\ 1/7 & -2/7 \end{bmatrix}$

e) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 & 1 \\ 1/3 & 1/2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

**10.** The system of equations $\begin{cases} 3x-5y=-18 \\ 2x-3y=-9 \end{cases}$, has the solution in the form:

(a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -18 \\ -9 \end{bmatrix}$

b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -18 \\ -9 \end{bmatrix}$

c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -18 \\ -9 \end{bmatrix}$

d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -18 \\ -9 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ -2 & 3 \end{bmatrix}$

e) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -18 \\ -9 \end{bmatrix}$

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, then the element in the second row, third column of $A^{-1}$ is equal to

(a) $-1$

(b) 0

(c) 1

(d) $-2$

(e) 2

12. If $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$, then the sum of the elements of the first row of the inverse matrix is equal to

A) $\frac{3}{2}$

B) $-1$

C) $-\frac{2}{3}$

D) $-\frac{3}{2}$

E) $\frac{2}{3}$

13. Let $A = \begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. If the solution of the system $AX = B$ is $X = CB$, then the sum of all elements of the matrix $C$ is

A) $-1$

B) 5

C) $-6$

D) 12

E) $-11$

14. If $A=\begin{bmatrix}1 & 2 & -1\\ 2 & 5 & 1\\ 3 & 6 & -2\end{bmatrix}$ then the product of all elements in the main diagonal of $A^{-1}$ is

A) 1

B) −16

C) 6

D) 8

E) −9

15. Let $A=\begin{bmatrix}1 & 1 & 0\\ -1 & 0 & 1\\ 3 & 3 & -1\end{bmatrix}$, then the sum of all elements of the matrix $A^{-1}$ is equal to:

a) 3

b) −2

c) 1

d) 0

e) −4

16. If the inverse of the matrix $A=\begin{bmatrix}1 & 2 & -1\\ 2 & 3 & -1\\ 3 & 6 & -n\end{bmatrix}$ is $A^{-1}=\begin{bmatrix}0 & 2 & m\\ -1 & -1 & 1\\ -3 & 0 & 1\end{bmatrix}$, then the sum of $m$ and $n$ is

A) −4

B) −3

C) 1

D) 2

E) −1

(7)

17. Let $A=\begin{bmatrix} 2 & -3 \\ -1 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $X=\begin{bmatrix} x \\ y \end{bmatrix}$. If the solution of the system $AX=B$ is $X=CB$, then the matrix $C$ is equal to :

a) $\begin{bmatrix} \frac{1}{5} & -\frac{3}{5} \\ -\frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

b) $\begin{bmatrix} -\frac{1}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$

c) $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{bmatrix}$

d) $\begin{bmatrix} \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

e) $\begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

18. The system $\begin{cases} 3x-5y=-18 \\ 2x-3y=-9 \end{cases}$, has the solution in the form

a) $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 9 & -15 \\ 6 & -9 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -8 \\ -9 \end{bmatrix}$

c) $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -5 & 3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -18 \\ -9 \end{bmatrix}$

d) $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -15 & 9 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \end{bmatrix}$

e) $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -18 \\ -9 \end{bmatrix}$

19. If $A$ is a matrix of order $n$ such that $A^3 - \frac{1}{2}I_n = 3A^2 - A$, then

A) $A^{-1} = 2(A^2 - 3A + 1)$

B) $A^{-1} = 2(A^3 - 6A + 2I_n)$

C) $A$ has no inverse

D) $A^{-1} = A$

✓E) $A^{-1} = 2A^2 - 6A + 2I_n$

20. If the matrix equation $A^4 = I$ is true and $A^{-1}$ exists, then $A^{-1}$ is equal

(a) $A^3$

(b) $A$

(c) $A^4$

(d) $I$

(e) $A^8$

21. Which of the following is TRUE for square matrices $A$ and $B$ which are the same size?

(a) If $AB = 0$ then $A = 0$ or $B = 0$

(b) $(A + B)^2 = A^2 + 2AB + B^2$

(c) $(A - B)(A + B) = A^2 - B^2$

(d) $(AB)^{-1} = B^{-1}A^{-1}$

(e) $A(BC) = (BA)C$

(1)

## Section 10.4 Determinants

1. What is the value of the determinant

$$\begin{vmatrix} 4 & -1 & 3 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix} ?$$

(a) $-1$
(b) 1
(c) 2
(d) $-2$
(e) $-15$

2. What is the value of the following determinant?

$$\begin{vmatrix} -1 & 2 & 2 & 3 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & 6 & 6 \\ 2 & -4 & -4 & -2 \end{vmatrix}$$

(a) 24

(b) $-36$

(c) $-24$

(d) 36

(e) 12

3. The value of the determinant $\begin{vmatrix} 2 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & 2 & 1 & 2 \end{vmatrix}$ is equal to

(a) $-6$

(b) 8

(c) 2

(d) $-1$

(e) 10

4. **The value of the determinant $\begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 1 \\ 1 & 2 & 1 & -2 \end{vmatrix}$ is equal to:**

a) 4

b) $-6$

c) 3

d) $-3$

e) 0

5. The value of the determinant $\begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix}$ is equal to:

a) 6

b) 12

c) $-12$

d) 0

e) 1

6. The determinant of the matrix $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 3 & 7 & -3 & 11 \\ 2 & 3 & -5 & 11 \\ 2 & 6 & 1 & 8 \end{bmatrix}$ is

A) 1

B) 3

C) 2

D) $\frac{3}{2}$

E) $\frac{2}{3}$

7. $\begin{vmatrix} 1 & 2 & 1 & 4 \\ -1 & 1 & 0 & 1 \\ 2 & 4 & 3 & 3 \\ 3 & 6 & 0 & 2 \end{vmatrix} =$

A) 0

B) −25

C) 25

D) 15

E) −75

(4)

8. If $A=\begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 3 & 6 & -9 \\ 0 & 1 & 8 & -6 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$, then $|A|=$

a) $-6|B|$

b) $6|B|$

c) $-3|B|$

(d) $3|B|$

e) $2|B|$

9. If $\begin{vmatrix} 2 & 2 & 2 \\ x-1 & y-2 & z-3 \\ 1 & 2 & 3 \end{vmatrix}=3$, then $\begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ 4 & 4 & 4 \end{vmatrix}=$

(a) $-6$

b) $-8$

c) 6

d) $-12$

e) 0

**10.** If $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ 2 & 3 & 4 \end{vmatrix} = -3$, then the value of $\begin{vmatrix} 2 & 3 & 4 \\ x-4 & y-6 & z-8 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \end{vmatrix}$ is equal to

(a) $-1$

(b) $\frac{1}{9}$

(c) $-3$

(d) 3

(e) 1

**11.** If $\begin{vmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \\ a & 2b & 3c \end{vmatrix} = 3$, then $\begin{vmatrix} 2a & 6c+10 & 4b+7 \\ a & 3c & 2b \\ 2 & -6 & -4 \end{vmatrix}$

A) $-6$

B) $-12$

C) 18

D) 10

E) $-8$

6

12. Let $x$ be a nonzero real number and $A = \begin{bmatrix} x & x^4 & x \\ 0 & x^3 & 1 \\ x^5 & 2x^8 & -3x^5 \end{bmatrix}$. Then the determinant of $A$ is equal to

(a) $-5x^9$
(b) $-3x^5$
(c) $-3x^9$
(d) $6(x^5 - x^3)$
(e) $4x^5$

13. The **minor** and the **cofactor** of the **element** 0 in the **matrix** $\begin{bmatrix} -3 & 2 & 1 \\ -5 & 6 & 0 \\ -2 & -1 & 3 \end{bmatrix}$ are respectively

(*a*) 7 and -7
(*b*) $-7$ and 7
(*c*) 7 and 7
(*d*) $-7$ and $-7$
(*e*) 0 and $-7$

14. The sum of the minor $M_{13}$ and the cofactor $C_{32}$ of the matrix $\begin{bmatrix} 4 & -3 & 3 \\ 2 & 1 & -4 \\ 6 & -2 & -1 \end{bmatrix}$ is

A) –32
B) 34
C) 24
D) 12
E) –10

7

15. If $A=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$, then the **cofactor** of the element in the **second row** and the **third column** of the matrix $AB$ is equal to:

a) 2

b) −1

c) −2

d) 0

e) 4

16. If $A=\begin{bmatrix}5 & -2 & -3\\ 2 & 4 & -1\\ 4 & -5 & 6\end{bmatrix}$, then $M_{21}+C_{23}=$

a) −10

b) −18

c) −8

d) −9

e) 0

17. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 3 \\ 1 & 5 & 1 \end{bmatrix}$, then the sum of the cofactors of the elements of the **third row** of the matrix $A$ is

(a) $-4$
(b) 11
(c) 6
(d) $-7$
(e) 8

18. If $A = \begin{bmatrix} -3 & 0 & -1 & 0 \\ 2 & 4 & 6 & 0 \\ 0 & 0 & -2 & 5 \\ 1 & 0 & 3 & 0 \end{bmatrix}$, **then the cofactor $C_{34}$ is equal to:**

**(a) 32**
**(b) 12**
**(c) 10**
**(d) 48**
**(e) 4**

19. If $A$ is a matrix of order $3\times3$ with $|A| = 4$ and $B$ is a matrix of order $4\times4$ with $|B| = 3$, then $\left|2B^{-1}\right| - \frac{1}{6}|2A| =$

A) 0
B) 24
C) $-24$
D) 2
E) $-2$

20. If $A = \begin{bmatrix} 0 & -3 & -1 \\ x & 1 & 0 \\ 2 & -2 & 4 \end{bmatrix}$ is a nonsingular matrix, then $|A^{-1}| =$

A) $\frac{4x}{6-5x}$

B) $\frac{1}{12x+5}$

C) $\frac{1}{14x+2}$

D) $\frac{3}{5x+6}$

E) $\frac{1}{10x+2}$

21. If $A$ and $B$ are $4\times4$ matrices such that $|A|=3$ and $|B|=4$, then $4|3(AB)^{-1}|-|2B^{-1}|=$

a) 23

b) $\frac{27}{4}$

c) 28

d) 36

e) $\frac{81}{4}$

22. If $A$ and $B$ are $2 \times 2$ matrices with $|A| = 5$, $|B| = 2$, then $|3AB| + 2|B^{-1}|$ is equal to

(a) 91
(b) 63
(c) 84
(d) 103
(e) 37

23. The matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & k & 2 \end{bmatrix}$ is a **singular matrix** if $k$ is equal to:

a) 6
b) $-6$
c) $-2$
d) $-3$
e) 3

24. Let $A, B$ be two invertible matrices such that $|A| = 2$ and $|B| = 4$, then $|2A| + |A^{-1}B| =$

a) 18
b) 10
c) 20
d) 22
e) 16

(11)

**25.** If $A$ and $B$ are two matrices of order 4 such that $|A|=4$ and $|B|=5$, then the value of $|AB|-5|B^{-1}|$ is equal to:

*a*) 5
*b*) −5
*c*) 19
*d*) 21
*e*) 45

**26.** If $A$ and $B$ are two matrices of **order** $3\times 3$ and $|A|=4$ and $|B|=5$, then the **value** of $2|A|-|2B^{-1}|=$

(*a*) $-72$
(*b*) $-2$
(*c*) $\frac{32}{5}$
(*d*) 7
(*e*) $\frac{38}{5}$

**27.** If $Z$ is a $5\times 5$ matrix and $|Z|=3$, then $|2Z^{-1}|$ is equal to

(a) $\frac{25}{3}$
(b) 50
(c) 10
(d) $\frac{64}{9}$
(e) $\frac{32}{3}$

(12)

28. Let $A$ and $B$ be $3 \times 3$ matrices. Which one of the following statements is FALSE?

(a) $|A^{-1}| = |A|$

(b) $(AB)^{-1} = B^{-1}A^{-1}$

(c) $(|A| + 1)^2 = |A|^2 + 2|A| + 1$

(d) $AA^{-1} = B^{-1}B$

(e) $|3A| = 27|A|$

29. If

$$M = \begin{pmatrix} 5 & 6 \\ 4 & 0 \end{pmatrix}$$

and $I$ is the $2 \times 2$ identity matrix, then the sum of the values of $x$ which satisfy

$$\det(M - xI) = 0$$

is

(a) 5
(b) $-5$
(c) 10
(d) $-3$
(e) $-10$