**KING FAHD UNIVERSITY OF PETROLUEM AND MINERALS**
**Faculty of Science, Prep-Year Math Program**
**Math 001 - Term 041**

## CLASS TEST 2

**Code 1**

**Name:** **ID#:** **Sr. #:** **Section:**

**[Provide neat and complete solution. Show all necessary steps for full credit.]**

**Question1**

If $f(x)=2x^2+5$ and $g(x)=3x+a$, find $a$ so that the graph of $fog$ crosses the $y-axis$ at 23.

**4 points**

$(f\circ g)(x)=f(g(x))=f(3x+a)=2(3x+a)^2+5$

$f\circ g$ crosses the $y-axis$ when $x=0\Rightarrow 2a^2+5=23\Rightarrow 2a^2=18\Rightarrow a^2=9\Rightarrow a=\pm 3$

**Question2** Given the equation of a circle $2x^2+2y^2+8x-4y+2=0$.

a) Write this equation in the standard form.

**3 points**

$x^2+y^2+4x-2y=-1$

$(x^2+4x+4)+(y^2-2y+1)=-1+4+1$

$(x+2)^2+(y-1)^2=4$

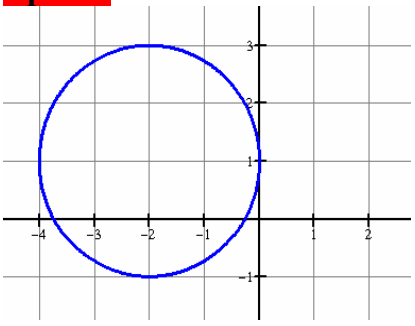
b) Find the center and the radius of this circle.

**1 point**

$C:(-2,1), r=2$

c) Determine whether the circle is tangent to the $x-$axis, $y-$axis or both.

**2 points**




so, $y-$axis only

## Question3

Given the function $f(x) = 4x - x^2,\ x \le 2..$

d) Write $f^{-1}(x)$.

**4 points**

let $f(y) = x \Rightarrow 4y - y^2 = x,$ complete the square for $y$

$\Rightarrow y^2 - 4y = -x,$ add 4 to both sides

$\Rightarrow (y-2)^2 = -x + 4$

$\Rightarrow y - 2 = \pm\sqrt{-x+4}$

$\Rightarrow y = 2 \pm \sqrt{-x+4}, y \le 2$

$\Rightarrow y = 2 - \sqrt{-x+4}$

$\Rightarrow f^{-1}(x) = 2 - \sqrt{-x+4}$

e) State the **domain** and **range** of $f^{-1}$

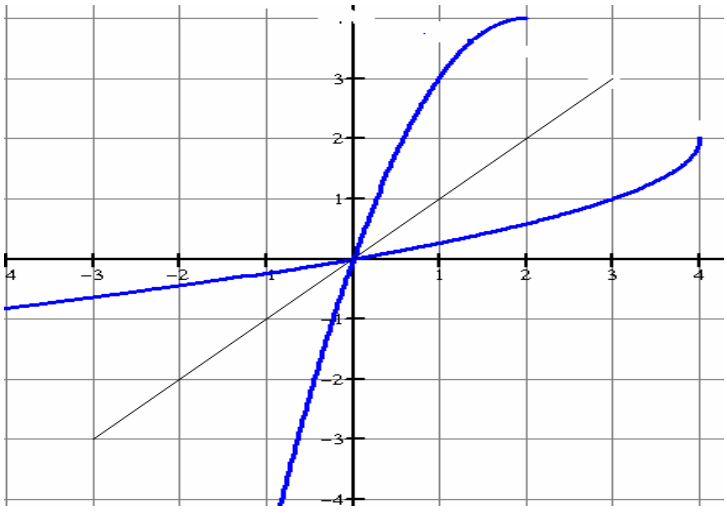
**2 points**

$D_{f^{-1}} : -x + 4 \ge 0 \Rightarrow x \le 4 \Rightarrow D_{f^{-1}} = (-\infty.4]$

$R_{f^{-1}} = D_f = (-\infty,2]$

f) Sketch the graphs of $f$ and $f^{-1}$ in the same coordinate system.

**Bonus(+3)**



**Question 4** If the line $L$ passes through the vertex of the quadratic function $f(x) = -2x^2 + 4x + 1$ and is perpendicular to the line $3y - 2x + 1 = 0$. Write the equation of the line $L$.

**5 points**

$* h = \frac{-b}{2a} = 1, k = f(h) = f(1) = 3 \Rightarrow$ The vertex is (h, k) = (1,3)

$* 3y - 2x + 1 = 0 \Rightarrow 3y = 2x - 1 \Rightarrow y = \frac{2}{3}x - \frac{1}{3} \Rightarrow m = \frac{2}{3}$

$*$ The slope of the required line is $-\frac{3}{2}$

$*$ Eq.: $y - 3 = -\frac{3}{2}(x - 1)$

**Question 5** The graph of the equation $y = |x - 2| + 1$ is reflected across the $y - axis$, then shifted 2 units left, then shifted 1 unit down. The equation of the new graph is $y = |ax + b| + c$. Find the value of $a + b + c$.

**5 points**

$y_1 = |-x - 2| + 1$

$y_2 = |-(x + 2) - 2| + 1 = |-x - 4| + 1 =$

$y_{new} = |-x - 4| + 1 - 1 = |-x - 4|$

Hence,

$a = -1,\ b = -3, c = 0 \Rightarrow a + b + c = -5$

*Note*:

$y_{new} = |-x - 4| = |-(x + 4)| = |x + 4|$

Hence,

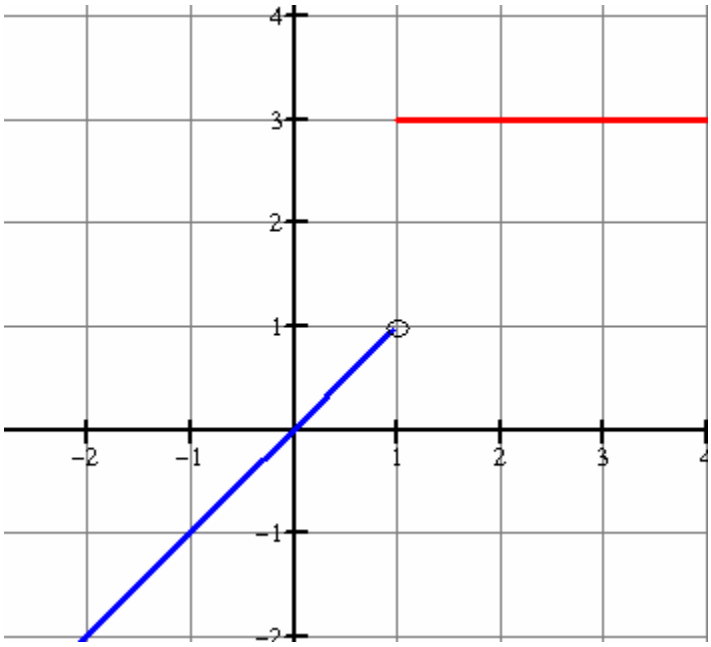
$a = 1,\ b = 3, c = 0 \Rightarrow a + b + c = 5$

**One solution is enough.**

**Question 6**

a) If $g(x)=\begin{cases} x & if \quad x<1 \\ 3 & if \quad x\geq 1 \end{cases}$, then find the range of $g(x)$.

**3 points**




$R=(-\infty,1)\cup\{3\}$

b) If $h(x)=[x]$, where $[x]$ is the greatest integer of less than or equal to $x$, then find the value of $h(-0.5)+h^2(\sqrt{8})$.

**4 points**

$[-0.5]+([2.?])^2=-1+(2)^2=-1+4=3$

**Question7** If $f(x)=x^2+1$ and $g(x)=|x|-4$, find:

$a)$ $\dfrac{f(1+h)-f(1)}{h}$

**3 points**

$$\frac{f(1+h)-f(1)}{h}=\frac{(1+h)^2+1-2}{h}=\frac{1+2h+h^2-1}{h}\equiv\frac{2h+h^2}{h}=\frac{h(2+h)}{h}=2+h$$

$b)$ $(f\,o\,g)(2)$ + $(g\,o\,g)(-1)$

**3 points**

$$f(g(2))+g(g(-1))=f(-2)+g(-3)=5-1=4$$

$c)$ The Domain of $\left(\dfrac{f}{\sqrt{g}}\right)(x)$

**3 points**

$$\frac{f}{\sqrt{g}}=\frac{x^2+1}{\sqrt{|x|-4}}$$

$$|x|-4>0\Rightarrow|x|>0\Rightarrow x<-4 \;\; or \;\; x>4$$

$$\therefore D=(-\infty,4)\cup(4,\infty)$$

**Question 8** If the quadratic function $f(x)=2x^2+bx+c$ decreases on $(-\infty,2]$ and has $x$-intercept $(3,0)$ find the value of $b+c$

**5 points**

$$*\,h=2=\frac{-b}{2a}=\frac{-b}{2(2)}=\frac{-b}{4}\Rightarrow b=-8$$

$$*\,f(3)=0\Rightarrow 2(9)-24+c\Rightarrow c=6$$

$$*\,b+c=-8+6=-2$$

**Question 9**

Identify the equation or the set of ordered pair that define $y$ as a function of $x$.

**3 points**

a) $\{(2,5),,(-4,3),(-2,5)\}$

function

b) $|x|+y^3=1$

function