King Fahd University of Petroleum and Minerals
Mathematical Sciences Department
Prep-Year Math I
Class Test # I
Term(041)

Instructor: Bassam Al-Absi **Oct. 21, 2004**

Student's Name: **Sample Key Solution...** **ID: ....SEC:**

(1) **If the sum of the reciprocals of two positive consecutive integers is one more their product, what are the integers?**

Sol:

Let the first integer be $x \Rightarrow$ the consecutive integer is $x+1$

$$\frac{1}{x}+\frac{1}{x+1}=\frac{1}{x(x+1)}+1 \Rightarrow x+1+x=x+x^2+1$$

$$\Rightarrow x^2-x=0 \Rightarrow x=\cancel{0}\ (\text{rejected}) \text{ or } \boxed{x=1} \Rightarrow \boxed{x+1=2}$$

....

(2) **If** $\frac{\sqrt[3]{-8}-\left(-i^7\right)^{15}}{i+\sqrt{-4}\sqrt{-1}}=2x-3yi$, where $i=\sqrt{-1}$, find the values of x and y.

Sol:

$$\frac{\sqrt[3]{-8}-\left(-i^7\right)^{15}}{-i+\sqrt{-4}\sqrt{-1}}=\frac{-2+i^{105}}{-i+(2i)i}=\frac{-2+i}{-2-i}=\frac{-2+i}{-2-i}\bullet\frac{-2+i}{-2+i}=\frac{4-4i+i^2}{(-2)^2-i^2}$$

$$=\frac{3-4i}{5}=\frac{3}{5}-\frac{4}{5}i \underset{\text{Given}}{=} 2x-3yi$$

$$\Rightarrow 2x=\frac{3}{5} \Rightarrow x=\frac{3}{10}$$

$$\text{and } -3y=-\frac{4}{5} \Rightarrow y=\frac{4}{15}$$

....

(3) **Factor** $a^2+2ab+b^2-x^2-2xy-y^2$

Sol:

$$a^2+2ab+b^2-x^2-2xy-y^2=\left(a^2+2ab+b^2\right)-\left(x^2+2xy+y^2\right)$$

$$=(a+b)^2-(x+y)^2=(a+b+x+y)(a+b-x-y)$$

(4) **If** $-2 < x < -1$, simplify $\dfrac{\sqrt{(x-2)^2 + 2(x-2)+1}}{x-1}$

Sol:

$$\frac{\sqrt{(x-2)^2+2(x-2)+1}}{x-1} = \frac{\sqrt{x^2-4x+4+2x-4+1}}{x-1}$$

$$= \frac{\sqrt{x^2-2x+1}}{x-1} = \frac{\sqrt{(x-1)^2}}{x-1} = \frac{|x-1|}{x-1} = -1$$

....

(5) **Find the product** $\left(\sqrt[3]{5}+4\right)\left(\sqrt[3]{25}-4\sqrt[3]{5}+16\right)$

Sol:

$$\left(\sqrt[3]{5}+4\right)\left(\sqrt[3]{25}-4\sqrt[3]{5}+16\right) = \left(\sqrt[3]{5}\right)^3 + (4)^3 = 5+64 = 69$$

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(6) **Simplify** $\dfrac{2+6(x-2)^{-1}}{1-3(x-4x^{-1})^{-1}}$

Sol:

$$\frac{2+6(x-2)^{-1}}{1-3(x-4x^{-1})^{-1}} = \frac{2+\dfrac{6}{x-2}}{1-\dfrac{3}{x-\dfrac{4}{x}}} = \frac{2x-4+6}{x-2} \div \left(1-\frac{3}{\dfrac{x^2-4}{x}}\right)$$

$$= \frac{2(x+1)}{x-2} \div \left(1-\frac{3x}{x^2-4}\right) = \frac{2(x+1)}{x-2} \div \left(\frac{x^2-3x-4}{x^2-4}\right)$$

$$= \frac{2\cancel{(x+1)}}{\cancel{x-2}} \bullet \frac{\cancel{(x-2)}(x+2)}{(x-4)\cancel{(x+1)}} = \frac{2(x+2)}{(x-4)}, x \neq -1, 2$$

....

(7) **Find all real values of** N **for which the equation** $x^2 + N^2 = 2(N+1)x$ **will have one real root that is a double root.**

Sol:

$x^2 + N^2 = 2(N+1)x$ rewrite as $x^2 - 2(N+1)x + N^2 = 0$

One double solution $\Rightarrow b^2 - 4ac = 0$, where $a = 1, b = -2(N+1), c = N^2$

$\Rightarrow \left(-2(N+1)\right)^2 - 4N^2 = 0 \Rightarrow N = -\dfrac{1}{2}$ (simplify and solve)

<u>(8)</u> **If the coefficient of** x^2y in the product $\left(x+x^2y+Mxy\right)\left(x-2xy+3\right)$ is equal to 3, **find the value of** M **.**

<u>Sol:</u>

$$\left(x+x^2y+Mxy\right)\left(x-2xy+3\right)$$

$$=x^2\underline{-2x^2y}+3x+x^3y-2x^3y^2+\underline{3x^2y}+\underline{Mx^2y}-2Mx^2y^2+3Mxy$$

Adding the underlined terms gives $(M+1)x^2y \Rightarrow M+1=3 \Rightarrow M=2$

....

<u>(9)</u> **Simplify each of the following expressions by rationalizing the denominator:**

(a) $\sqrt[4]{\dfrac{m^3n^5}{4r^6}}$ **, where** m, n and r are positive real numbers.

<u>Sol:</u>

$$\sqrt[4]{\frac{m^3n^5}{4r^6}}=\frac{n\sqrt[4]{m^3n}}{r\sqrt[4]{2^2r^2}}\bullet\frac{\sqrt[4]{2^2r^2}}{\sqrt[4]{2^2r^2}}=\frac{n\sqrt[4]{m^3n}\sqrt[4]{4r^2}}{r\cdot 2\cdot r}=\frac{n\sqrt[4]{m^3n}\sqrt[4]{4r^2}}{2r^2}$$

(b) $$\frac{\sqrt{x}-\sqrt{x-2}}{\sqrt{x}+\sqrt{x-2}}=\frac{\sqrt{x}-\sqrt{x-2}}{\sqrt{x}+\sqrt{x-2}}\bullet\frac{\sqrt{x}-\sqrt{x-2}}{\sqrt{x}-\sqrt{x-2}}=\frac{\left(\sqrt{x}-\sqrt{x-2}\right)^2}{x-(x-2)}$$

$$=\frac{2x-2-2\sqrt{x}\sqrt{x-2}}{2}=x-1-\sqrt{x}\sqrt{x-2}$$

....

<u>(10)</u> **If** x and y are positive integers, simplify $\left(\dfrac{\left(\frac{1}{2}x^2y^{-3}\right)^{-1}\left(2x^{-1}y^2\right)^3}{-2\left(x^{-3}y\right)^2\left(-2x^2y^{-1}\right)^3}\right)^{\frac{-1}{5}}$

<u>Sol:</u>

$$=\left(\frac{\left(2x^{-2}y^3\right)\left(2^3x^{-3}y^6\right)}{-2\left(x^{-6}y^2\right)\left(-8x^6y^{-3}\right)}\right)^{\frac{-1}{5}}=\left(x^{-2-3+6-6}\bullet y^{3+6-2+3}\right)^{-\frac{1}{5}}$$

$$=\sqrt[5]{\left(\frac{1}{x^5y^{10}}\right)}=\frac{1}{xy^2}$$

(11) **Fill in the following blanks with the correct answers:**

(a) The multiplicative inverse of (-0.75) **is** $-\frac{4}{3}$

(b) The least common denominator of the fractions $\frac{4}{2b^2-6b+4}, \frac{2}{b^2-b-2}$

is... $2(b-2)(b+1)(b-1)$ **.**

(c) If $x=-2$ is a solution for the equation $3x^2+kx=2$, then the

other solution is ... $\frac{-1}{3}$ **and** $k=7$

using the two properties $x_1 x_2=\frac{c}{a}$ and $x_1+x_2=\frac{-b}{a}$

(d) The equation $|x-1|=-3$ **has ...no. solution(s)**

(e) For a nonzero real number k, the types of the solutions for the equation $x^2+2kx-3k^2=0$ are **two distinct real solutions.**

(Using the discriminant that is equal to $16k^2$ **which is positive for any nonzero real number** k **)**