

King Fahd University of Petroleum and Minerals
University Diploma Programs
Prep.-Year Math II (032)
Class Test I

Student's Name: KEY ID: _____ Sec: _____

Part I: MULTIPLE-CHOICE

(2 points each)

1) If the terminal side of an angle θ in standard position is passing through the point

$P(2\sqrt{5}, -4)$, then $\sin\theta$ is equal to

(a) $-\frac{2}{3}$

(b) $\frac{\sqrt{5}}{3}$

(c) $\frac{-2\sqrt{5}}{5}$

(d) $-\frac{\sqrt{5}}{2}$

$$r = \sqrt{20 + 16} = 6$$

$$\sin\theta = \frac{-4}{6} = \boxed{-\frac{2}{3}}$$

2) The exact value of $3^{4\log_3\sqrt{5} + \frac{1}{3}\log_3 64}$ is

(a) 10

(b) 20

(c) 100

(d) 5

$$= 3^{\log_3 25.4} = 3^{\log_3 100} = \boxed{100}$$

3) The supplementary angle of a $23^{\circ} 15' 33''$ angle is equal to

(a) $57^{\circ} 44' 27''$

(b) $157^{\circ} 45' 27''$

(c) $236^{\circ} 44' 27''$

✓ (d) $156^{\circ} 44' 27''$

$$\begin{array}{r} 179^{\circ} 59' 60'' \\ 23^{\circ} 15' 33'' \\ \hline 156^{\circ} 44' 27'' \end{array}$$

4) Which one of the following statements is TRUE for any nonzero real number x ?

(a) $\log(x^2 + 4) = \log x^2 + \log 4$

(b) $\log x^2 = 2 \log x$

✓ (c) $\log x^4 = 2 \log x^2$

(d) $\log\left(\frac{x^2}{4}\right) = 2 \log x - 2 \log 2$

(e) $\frac{\log x}{\log y} = \log(x - y)$

5) If S is the interval in which the graph of the function $f(x) = -2^{-x} + 2$ lies above the x -axis and y is the its horizontal asymptote, then:

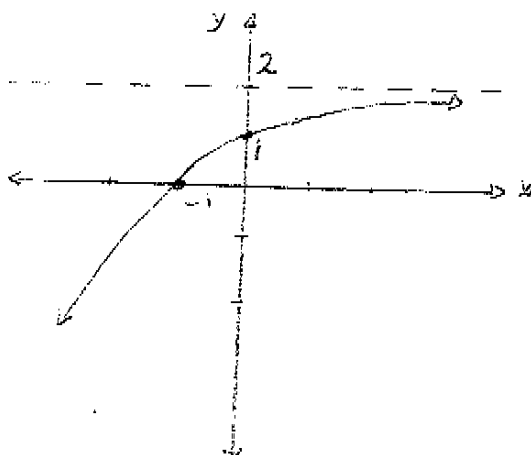
(a) $S = (-\infty, \infty)$, $y=0$

$$y = -\left(\frac{1}{2}\right)^x + 2$$

(b) $S = (-1, \infty)$, $y = 1$

✓ (c) $S = (-1, \infty)$, $y = 2$

(d) $S = (1, \infty)$, $y = 0$



6) The exact value of the expression $\sec 0^\circ \tan 30^\circ + \csc 630^\circ \cot 60^\circ$ is equal to

(a) $\frac{2\sqrt{3}}{3}$

$$= 1 \cdot \frac{\sqrt{3}}{3} + \csc 270^\circ \cdot \frac{\sqrt{3}}{3}$$

$$= \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3} = 0$$

(b) 0

(c) $\frac{\sqrt{3}}{3}$

(d) $-\frac{\sqrt{3}}{3}$

Part II: Written

1) Given the function $f(x) = |\log_3(3-x)|$.

(6 points)

(a) Find the domain of $f(x)$ is

$$3-x > 0 \Rightarrow x < 3 \Rightarrow D = (-\infty, 3)$$

(b) Find the range of $f(x)$ is , $[0, \infty)$

(c) Find the x-intercept of the graph of $f(x)$

$$|\log_3(3-x)| = 0 \Rightarrow \log_3(3-x) = 0$$

$$3-x = 3^0 \Rightarrow 3-x = 1 \Rightarrow \boxed{x=2}$$

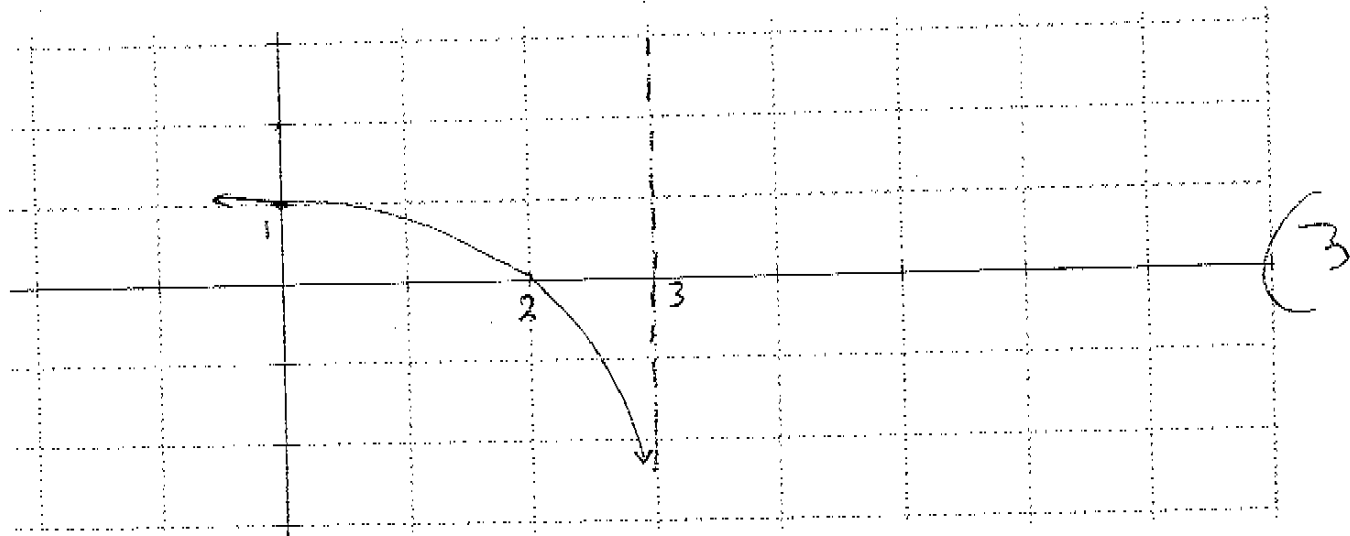
(d) Find the y-intercept of the graph of $f(x)$

$$y = |\log_3(3-0)| = |\log_3 3| = |1| = \boxed{1}$$

(e) Find the asymptote to the graph of $f(x)$

$$3 - x = 0 \Rightarrow x = 3$$

(f) Sketch the graph of $f(x)$



2) Find the exact measure in degrees of the central angle subtended by the arc length $\frac{25\pi}{6}$

feet in a circle of radius 5 feet.

$$s = r\theta$$

(3 points)

$$\Rightarrow \theta = \frac{s}{r} = \frac{25\pi}{6} \cdot \frac{1}{5} = \frac{5\pi}{6} \text{ radian}$$

$$\Rightarrow \boxed{\theta = 30^\circ}$$

3) Write $2\log(x+3) - \log(x^2-9) + \log(x-3)$ as a single logarithm.

(3 points)

$$= \log(x+3)^2 - \log(x^2-9) + \log(x-3)$$

$$= \log \frac{(x+3)^2 \cdot (x-3)}{(x-3)(x+3)}$$

$$= \boxed{\log(x+3)}$$

4) If $\cos\theta < 0$ and $\csc\theta = \frac{13}{5}$, find the exact value of $\sec\theta$

(4 points)

$$\csc\theta = \frac{13}{5} = \frac{r}{y}, \quad \theta \in \mathcal{Q} \text{ II}$$

$$x = -\sqrt{(13)^2 - (5)^2} = -\sqrt{169 - 25} = -12$$

$$\Rightarrow \sec\theta = \frac{r}{x} = \frac{13}{-12}$$

5) Solve $e^x - 2e^{-x} = 1$ for x

(4 points)

multiply by e^x

$$\Rightarrow e^{2x} - 2 = e^x$$

$$e^{2x} - e^x - 2 = 0$$

$$\Rightarrow (e^x - 2)(e^x + 1) = 0$$

$$\Rightarrow e^x = 2 \quad \text{or} \quad e^x = -1 \text{ which is impossible as } e^x > 0 \forall x \in \mathbb{R}$$

$$\Rightarrow \ln e^x = \ln 2$$

$$\Rightarrow \boxed{x = \ln 2}$$

6) Express $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x}$ in terms of $\csc x$.

(3 points)

$$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1 + \cos x)}$$

$$= \frac{2 + 2\cos x}{\sin x(1 + \cos x)}$$

$$= \frac{2(1 + \cancel{\cos x})}{\sin x(1 + \cancel{\cos x})}$$

$$= 2 \cdot \frac{1}{\sin x} = \boxed{2\csc x}$$

7) Solve $2\log_2(2x-1) - 2\log_4(x+1) = 3$ for x .

(4 points)

$$\log_2(2x-1) - 2 \frac{\log_2(x+1)}{\log_2 4} = 3 \Rightarrow \log_2(4x^2 - 4x + 1) - \log_2(x+1) = 3$$

$$\Rightarrow \log_2 \frac{4x^2 - 4x + 1}{x+1} = 3 \Rightarrow \frac{4x^2 - 4x + 1}{x+1} = 8 \Rightarrow 4x^2 - 4x + 1 = 8x + 8$$

$$\Rightarrow 4x^2 - 12x - 7 = 0 \Rightarrow (2x+1)(2x-7) = 0 \Rightarrow x = -\frac{1}{2} \text{ or } x = \frac{7}{2}$$

$x = -\frac{1}{2}$ is rejected (?)

$\Rightarrow x = \boxed{\frac{7}{2}}$ is the only solution

8) Let W be the wrapping function. Find the value of x and y such that

(3 points)

$$W\left(\frac{-20\pi}{3}\right) = P(x, y) \Rightarrow x = \cos\left(\frac{-20\pi}{3}\right) = \cos\left(\frac{20\pi}{3}\right) = \cos\left(6\pi + \frac{2\pi}{3}\right) \\ = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}} \quad (1.5)$$

$$y = \sin\left(\frac{-20\pi}{3}\right) = -\sin\left(\frac{20\pi}{3}\right)$$

$$= -\sin\left(6\pi + \frac{2\pi}{3}\right)$$

$$= -\sin\left(\frac{2\pi}{3}\right) = -\sin\frac{\pi}{3} = \boxed{-\frac{\sqrt{3}}{2}}$$

(1.5)

9) Give the trigonometric function $g(x) = -2\sec(\pi x + \frac{\pi}{2}) + 1$ over the interval $[-\frac{1}{2}, 3]$

(6 points)

(a) Find the general equation of the asymptotes to the graph of g

$$\pi x + \frac{\pi}{2} = \frac{(2n+1)\pi}{2} \Rightarrow \pi x = \frac{2n\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2}$$

$$\Rightarrow x = \boxed{n}, n \in \text{Integers.} \quad (3)$$

(b) Find the period of the graph g $P = \frac{2\pi}{\pi} = \boxed{2}$

(c) Find the phase shift of the graph $g = \frac{-c}{b} = \frac{-\frac{\pi}{2}}{\pi} = \boxed{-\frac{1}{2}}$

(d) Find the range of the graph g $(-\infty, -|a|+d] \cup [|a|+d, \infty) = (-\infty, -1] \cup [3, \infty)$

(e) Sketch the graph of $g(x) = -2\sec(\pi x + \frac{\pi}{2}) + 1$ and $f(x) = -2\cos(\pi x + \frac{\pi}{2}) + 1$ over

the above interval showing all x-intercepts, maximum, minimum values of both g

and f and all the asymptotes.

