King Fahd University of Petroleum and Minerals
University Diploma Programs
Prep.-Year Math II (032)
Class Test I

Student's Name: KEY **ID:** ________ **Sec:** ______

Part I: MULTIPLE-CHOICE **(2 points each)**

1) If the terminal side of an angle θ in standard position is passing through the point $P\left(2\sqrt{5},-4\right)$, then $\sin\theta$ is equal to

$r = \sqrt{20+16} = 6$

$\sin\theta = \frac{-4}{6} = \boxed{\frac{-2}{3}}$

(a) $-\frac{2}{3}$

(b) $\frac{\sqrt{5}}{3}$

(c) $\frac{-2\sqrt{5}}{5}$

(d) $-\frac{\sqrt{5}}{2}$

2) The exact value of $3^{4\log_3\sqrt{5}+\frac{1}{3}\log_3 64}$ is

$= 3^{\log_3 25\cdot 4} = 3^{\log_3 100} = \boxed{100}$

(a) 10

(b) 20

(c) 100

(d) 5

3) The **supplementary** angle of a $23^0\ 15'\ 33''$ angle is equal to

(a) $57^0\ 44'\ 27''$

(b) $157'\ 45'\ 27''$

(c) $236'\ 44'\ 27''$

(d) $156^0\ 44'\ 27''$

$179^o\ 59'\ 60''$
$23^o\ 15'\ 33''$
$156^o\ 44'\ 27''$

4) Which one of the following statements is TRUE for any **nonzero** real number x ?

(a) $\log(x^2+4) = \log x^2 + \log 4$

(b) $\log x^2 = 2\log x$

(c) $\log x^4 = 2\log x^2$

(d) $\log\left(\frac{x^2}{4}\right) = 2\log x - 2\log 2$

(e) $\frac{\log x}{\log y} = \log(x-y)$

5) If S is the interval in which the graph of the function $f(x) = -2^{-x} + 2$ lies above the x-axis and y is the its horizontal asymptote, then:

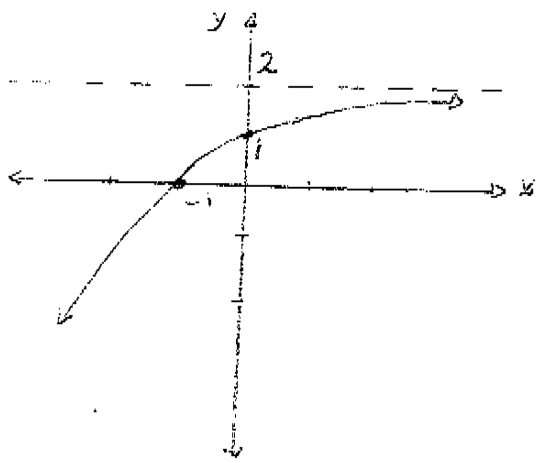
(a) $S = (-\infty, \infty)$, y=0

(b) $S = (-1, \infty)$, y = 1

(c) $S = (-1, \infty)$, $y = 2$

(d) $S = (1, \infty)$, $y = 0$

$y = -\left(\frac{1}{2}\right)^x + 2$



6) The exact value of the expression $\sec 0^0 \tan 30^0 + \csc 630^0 \cot 60^0$ is equal to

(a) $\frac{2\sqrt{3}}{3}$

$= 1 \cdot \frac{\sqrt{3}}{3} + \csc 270^0 \cdot \frac{\sqrt{3}}{3}$

(b) 0

$= \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3} = 0$

(c) $\frac{\sqrt{3}}{3}$

(d) $-\frac{\sqrt{3}}{3}$

Part II: Written

1) Given the function $f(x) = |\log_3(3-x)|$. (6 points)

(a) Find the domain of $f(x)$ is

$3 - x > 0 \Rightarrow x < 3 \Rightarrow D = (-\infty, 3)$

(b) Find the range of $f(x)$ is , $[0, \infty)$

(c) Find the x-intercept of the graph of $f(x)$

$|\log_3(3-x)| = 0 \Rightarrow \log_3(3-x) = 0$

$3 - x = 3^0 \Rightarrow 3 - x = 1 \Rightarrow \boxed{x = 2}$

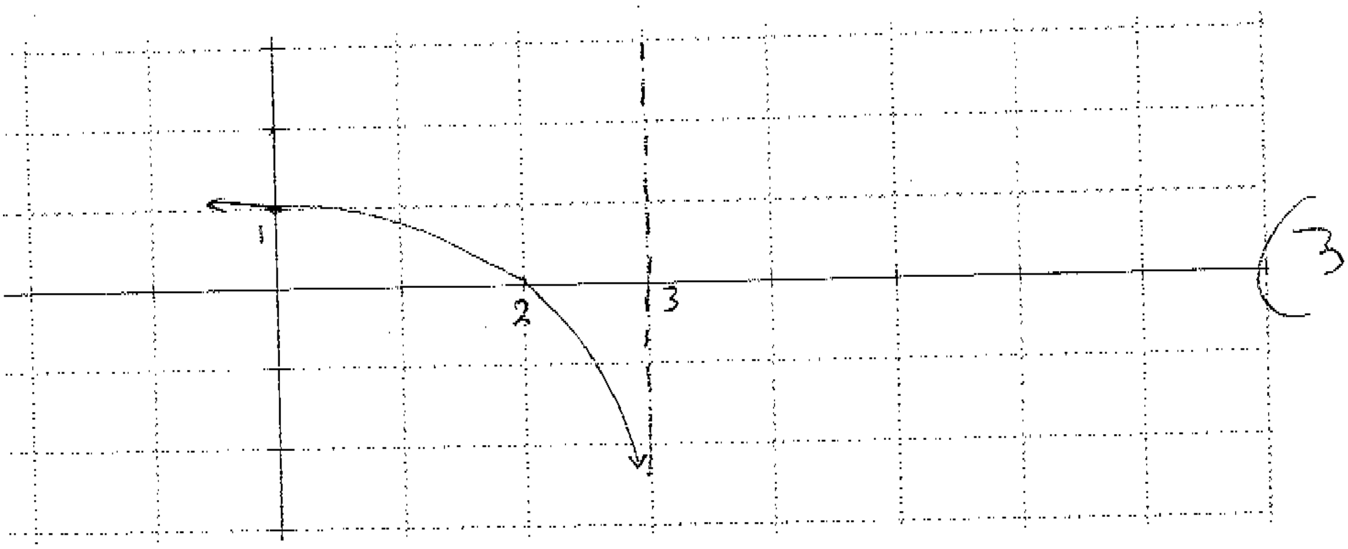
(d) Find the y-intercept of the graph of $f(x)$

$y = |\log_3(3-0)| = |\log_3 3| = |1| = \boxed{1}$

(e) Find the asymptote to the graph of $f(x)$

$3 - x = 0 \Rightarrow x = 3$

(f) Sketch the graph of $f(x)$



2) Find the exact measure in <u>**degrees**</u> of the central angle subtended by the arc length $\frac{25\pi}{6}$ feet in a circle of radius 5 feet. **(3 points)**

$S = r\theta$

$\Rightarrow \theta = \frac{S}{r} = \frac{5\pi}{6} \cdot \frac{1}{5} = \frac{\pi}{6}$ radian

$\Rightarrow \boxed{\theta = 30^{\circ}}$

3) Write $2\log(x+3) - \log(x^2-9) + \log(x-3)$ as a single logarithm. **(3 points)**

$= \text{Log}(x+3)^2 - \text{Log}(x^2-9) + \text{Log}(x-3)$

$= \text{Log}\,\frac{(x+3)^2 \cdot (x-3)}{(x-3)(x+3)}$

$= \boxed{\text{Log}(x+3)}$

4) If $\cos\theta < 0$ and $\csc\theta = \dfrac{13}{5}$, find the exact value of $\sec\theta$ **(4 points)**

$\csc\theta = \dfrac{13}{5} = \dfrac{r}{y}$, $\theta \in Q\ \mathrm{II}$

$x = -\sqrt{(13)^2 - (5)^2} = -\sqrt{169-25} = -12$

$\Rightarrow \sec\theta = \dfrac{r}{x} = \boxed{\dfrac{13}{-12}}$ 2

5) Solve $e^x - 2e^{-x} = 1$ for x **(4 points)**

multiply by e^x

$\Rightarrow e^{2x} - 2 = e^x$

$e^{2x} - e^x - 2 = 0$ 2

$\Rightarrow (e^x - 2)(e^x + 1) = 0$

$\Rightarrow e^x = 2$ or $e^x = -1$ which is impossible as $e^x > 0\ \forall x \in \mathbb{R}$

$\Rightarrow \ln e^x = \ln 2$

$\Rightarrow \boxed{x = \ln 2}$

6) Express $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$ in terms of $\csc x$. **(3 points)**

$= \dfrac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$

$= \dfrac{2 + 2\cos x}{\sin x(1+\cos x)}$

$= \dfrac{2\cancel{(1+\cos x)}}{\sin x\cancel{(1+\cos x)}}$

$= 2 \cdot \dfrac{1}{} = \boxed{2\csc x}$

7) Solve $2\log_2(2x-1)-2\log_4(x+1)=3$ for x. **(4 points)**

$$\log_2(2x-1)^2-2\frac{\log_2(x+1)}{\log_2 4}=3 \Rightarrow \log_2(4x^2-4x+1)-\log_2(x+1)=3$$

$$\Rightarrow \log_2\frac{4x^2-4x+1}{x+1}=3 \Rightarrow \frac{4x^2-4x+1}{x+1}=8 \Rightarrow 4x^2-4x+1=8x+8$$

$$\Rightarrow 4x^2-12x-7=0 \Rightarrow (2x+1)(2x-7)=0 \Rightarrow x=-\frac{1}{2} \text{ or } x=\frac{7}{2}$$

$x=-\frac{1}{2}$ is rejected (?)

$\Rightarrow x=\boxed{\frac{7}{2}}$ is the only solution

8) Let W be the wrapping function. Find the value of x and y such that **(3 points)**

$$W\left(\frac{-20\pi}{3}\right)=P(x,y) \rightarrow x=\cos\left(\frac{-20\pi}{3}\right)=\cos\left(\frac{20\pi}{3}\right)=\cos\left(6\pi+\frac{2\pi}{3}\right)$$

$$=\cos\frac{\pi}{3}=\boxed{\frac{1}{2}}$$

1.5

$$y=\sin\left(-\frac{20\pi}{3}\right)=-\sin\left(\frac{20\pi}{3}\right)$$

$$=-\sin\left(6\pi+\frac{2\pi}{3}\right)$$

$$=-\sin\left(\frac{2\pi}{3}\right)=-\sin\frac{\pi}{3}=\boxed{-\frac{\sqrt{3}}{2}}$$

1.5

9) Give the trigonometric function $g(x) = -2\sec(\pi x + \frac{\pi}{2}) + 1$ over the interval $\left[-\frac{1}{2}, 3\right]$

(6 points)

(a) Find the general equation of the asymptotes to the graph of g

$$\pi x + \frac{\pi}{2} = \frac{(2n+1)\pi}{2} \Rightarrow \pi x = \frac{2n\pi}{2} + \cancel{\frac{\pi}{2}} - \cancel{\frac{\pi}{2}}$$

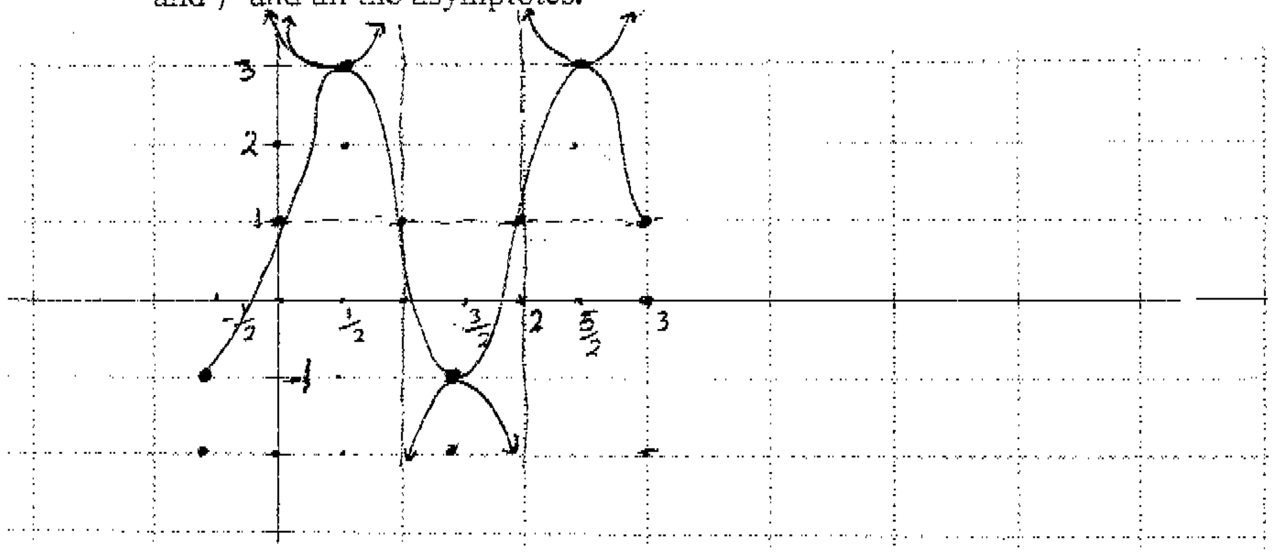
$\Rightarrow x = \boxed{n}$, $n \in$ Integers. 3

(b) Find the period of the graph g $\quad P = \frac{2\pi}{\pi} = \boxed{2}$

(c) Find the phase shift of the graph g $\quad = \frac{-c}{b} = \frac{-\frac{\pi}{2}}{\pi} = \boxed{-\frac{1}{2}}$

(d) Find the range of the graph g $\quad (-\infty, -|a|+d] \cup [|a|+d, \infty) = (-\infty, -1] \cup [3, \infty)$

(e) Sketch the graph of $g(x) = -2\sec(\pi x + \frac{\pi}{2}) + 1$ and $f(x) = -2\cos(\pi x + \frac{\pi}{2}) + 1$ over the above interval showing all x-intercepts , maximum, minimum values of both g and f and all the asymptotes.



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