

Math 002 – Term 052
Recitation hour (6.1–6.2)

Q1) Verify the following identities:

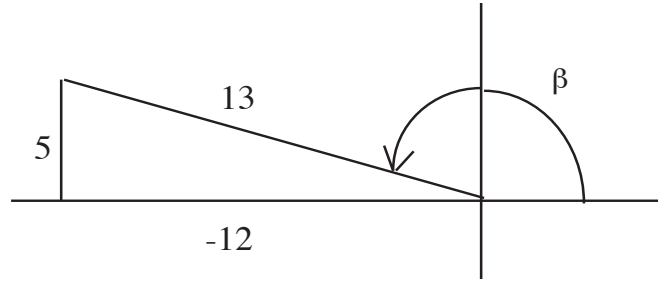
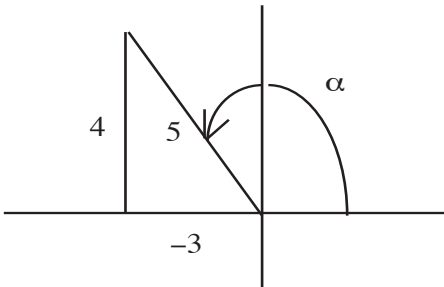
a) $\frac{\sin^2(-x) - \cos^2(-x)}{\sin(-x) - \cos(-x)} = \cos x - \sin x$ b) $\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \csc x - \cot x, 0 < x < \frac{\pi}{2}$

Solution: a) L.H.S. = $\frac{(-\sin x)^2 - \cos^2 x}{-\sin x - \cos x} = \frac{\sin^2 x - \cos^2 x}{-(\sin x + \cos x)} = \frac{(\sin x - \cos x)(\sin x + \cos x)}{-(\sin x + \cos x)}$
= $-(\sin x - \cos x) = \cos x - \sin x = \text{R.H.S.}$

b) L.H.S. = $\sqrt{\frac{1 - \cos x}{1 + \cos x}} \cdot \frac{1 - \cos x}{1 - \cos x} = \sqrt{\frac{(1 - \cos x)^2}{1 - \cos^2 x}} \sqrt{\frac{(1 - \cos x)^2}{\sin^2 x}} = \frac{|1 - \cos x|}{|\sin x|} = \frac{1 - \cos x}{\sin x}$
= $\frac{1}{\sin x} - \frac{\cos x}{\sin x} = \csc x - \cot x = \text{R.H.S.}$

Q2) Given $\csc \alpha = \frac{5}{4}$, α in quadrant II, and $\cos(\frac{\pi}{2} - \beta) = \frac{5}{13}$, β in quadrant II, find $\csc(\alpha + \beta)$.

Solution: $\cos(\frac{\pi}{2} - \beta) = \frac{5}{13} \implies \sin \beta = \frac{5}{13}$. Now $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
= $(\frac{4}{5})(-\frac{12}{13}) + (-\frac{3}{5})(\frac{5}{13}) = -\frac{48}{65} - \frac{15}{65} = -\frac{63}{65} \implies \csc(\alpha + \beta) = \frac{1}{\sin(\alpha + \beta)} = -\frac{65}{63}$



Q3) Find the exact values of the following expressions:

a) $\cos 165^\circ$ b) $\sin 13^\circ \sin 73^\circ + \sin 77^\circ \sin 17^\circ$ c) $\frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$

Solution: a) $\cos 165^\circ = \cos(45^\circ + 120^\circ) = \cos 45^\circ \cos 120^\circ - \sin 45^\circ \sin 120^\circ = \cos 45^\circ (-\cos 60^\circ) - \sin 45^\circ \sin 60^\circ$
= $(\frac{\sqrt{2}}{2})(-\frac{1}{2}) - (\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2}) = \frac{-\sqrt{2} - \sqrt{6}}{4} = -\frac{\sqrt{2} + \sqrt{6}}{4}$

b) = $\sin 13^\circ \cos(90^\circ - 73^\circ) + \cos(90^\circ - 77^\circ) \sin 17^\circ = \sin 13^\circ \cos 17^\circ + \cos 13^\circ \sin 17^\circ$
= $\sin(13^\circ + 17^\circ) = \sin 30^\circ = \frac{1}{2}$

c) = $\tan(69^\circ + 66^\circ) = \tan 135^\circ = -\tan 45^\circ = -1$

Q4) Prove that $\sin \alpha + \sin(120^\circ + \alpha) + \sin(240^\circ + \alpha) = 0$ for any angle α .

Proof: L.S. = $\sin \alpha + (\sin 120^\circ \cos \alpha + \cos 120^\circ \sin \alpha) + (\sin 240^\circ \cos \alpha + \cos 240^\circ \sin \alpha) = \sin \alpha + (\sin 60^\circ) \cos \alpha + (-\cos 60^\circ) \sin \alpha + (-\sin 60^\circ) \cos \alpha + (-\cos 60^\circ) \sin \alpha = \sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha - \frac{1}{2} \sin \alpha - \frac{\sqrt{3}}{2} \cos \alpha - \frac{1}{2} \sin \alpha = \sin \alpha - \sin \alpha = 0 = \text{R.S.}$