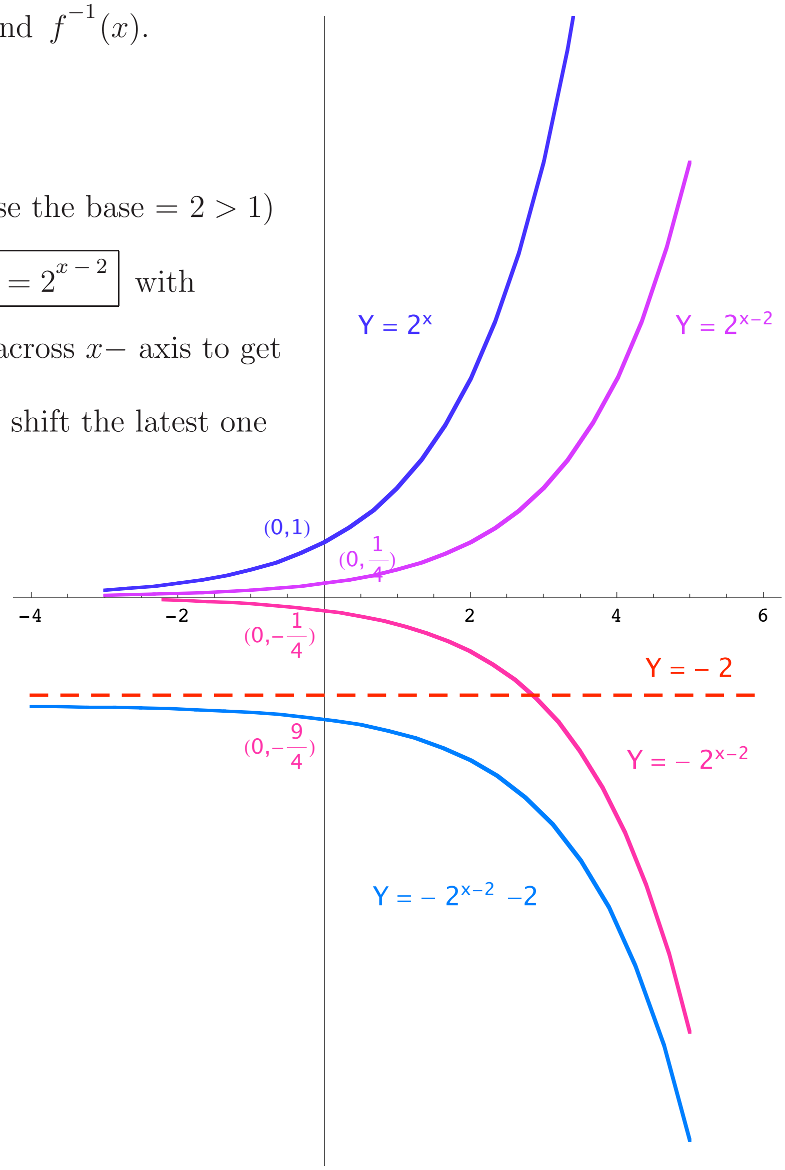
Math 002 – Term 052
Recitation hour (4.2 & 4.3)

Q1) For the function $f(x) = -\left(\frac{1}{2}\right)^{-x+2} - 2$, sketch the graph of $f(x)$, find the $x-$ and $y-$ intercepts, the range, asymptote(s), and then find $f^{-1}(x)$.



<u>**Solution**</u>: $f(x) = -\left(\frac{1}{2}\right)^{-x+2} - 2 = -\,2^{x-2} - 2$

First, we draw $\boxed{y = 2^x}$ (which is increasing because the base $= 2 > 1$)

Next, shift this graph 2 units to the right to get $\boxed{y = 2^{x-2}}$ with $y-$ intercept $= (0, \frac{1}{4})$. Then reflect the last graph across $x-$ axis to get $\boxed{y = -\,2^{x-2}}$ with $y-$ intercept $= (0, -\frac{1}{4})$. Finally shift the latest one 2 units downward to get $\boxed{f(x) = -\,2^{x-2} - 2}$

From the graph of $f(x)$: $y-$ intercept $= (0, -\frac{9}{4})$, the range $= (-\infty, -2)$, and the horizental asymptote is $y = -2$. To find $f^{-1}(x)$:

interchange x with y, so $x = -2^{y-2} - 2 \implies$

$2^{y-2} = -x - 2 \implies y - 2 = \log_2(-x-2) \implies$

$y = f^{-1}(x) = 2 + \log_2(-x-2)$.

Notice that $D_{f^{-1}} = (-\infty, -2) = R_f$

Q2) For the function $f(x) = -\log_2(3-x)$, sketch the graph of $f(x)$, find the $x-$ and $y-$ intercepts, the domain, asymptote(s), and then find $f^{-1}(x)$.

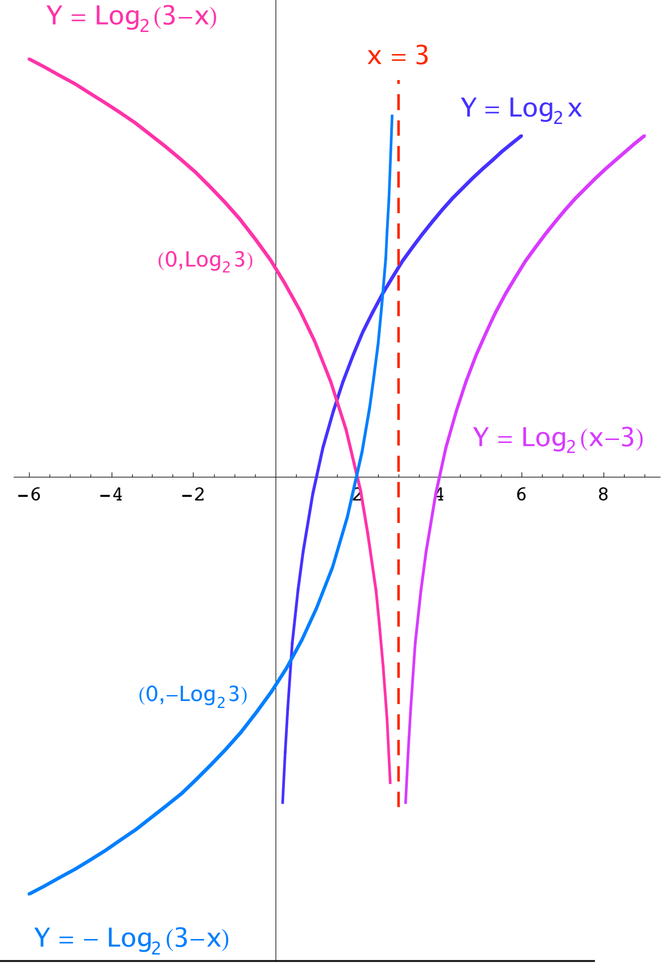
<u>**Solution**</u>: (a) First, we draw $\boxed{y = \log_2 x}$ (which is increasing because the base $= 2 > 1$) and has an x–intercept = (1,0)

(b) Next, shift graph (a) 3 units to the right to get $\boxed{y = \log_2(x-3)}$ with $x-$ intercept $= (4, 0)$, vertical asymptote $x = 3$.

(c) Then reflect graph (b) across the line $x = 3$ to get $\boxed{y = \log_2(3-x)}$ with $x-$ intercept $= (2, 0)$, and $y-$ intercept $= (0, \log_2 3)$.

(d) Finally reflect graph (c) across $x-$ axis to get $\boxed{f(x) = -\log_2(3-x)}$

From the graph of $f(x)$: $x-$ intercept $= (2,0)$,
$y-$ intercept $= (0, -\lg_2 3) = (0, \log_2 \frac{1}{3})$,
the domain $= (-\infty, 3)$, and the vertical asymptote is $x = 3$. To find $f^{-1}(x)$: interchange x with y, so $x = -\log_2(3-y) \implies -x = \log_2(3-y) \implies$
$3 - y = 2^{-x} \implies y = f^{-1}(x) = 3 - 2^{-x}$.
Notice that $\mathrm{R}_{f^{-1}} = (-\infty, 3) = \mathrm{D}_f$



Q3) If the graph of the logarithmic function $f(x) = \log_b x$ passes through the point $(\frac{1}{64}, -3)$, then find $f(2)$.

Solution: $-3 = \log_b \frac{1}{64} = \log_b \left(\frac{1}{2}\right)^6 = \log_b 2^{-6} = -6 \log_b 2 \implies \log_b 2 = \frac{-3}{-6} = \frac{1}{2}$
$\implies f(2) = \log_b 2 = \frac{1}{2}$

Q4) If a bacteria population starts with 1000 bacteria and doubles every three hours, then the number of bacteria after t hours is $N(t) = 1000 \cdot 2^{t/3}$. When will the population reach 128000?

Solution: $128000 = 1000 \cdot 2^{t/3} \implies 128 = 2^{t/3} \implies 2^7 = 2^{t/3} \implies 7 = \frac{t}{3} \implies t = 21$ hours.

Solved by: A. Al-Shallal