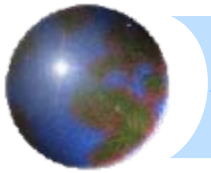


9.3 Nonlinear Systems of Equations



Ways to Solve Systems

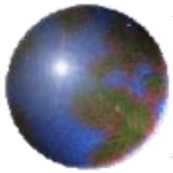
1. Graphing

2. Algebraically

- ❏ Substitution

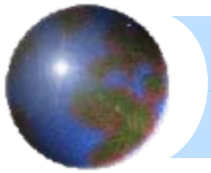
- ❏ Elimination can be used, but substitution is typically easier

- ❏ Factoring



Guideline for Solving Nonlinear Systems Algebraically:

1. Use Substitution.
2. Solve the easiest equation for one variable.
3. Plug it into the second equation to get the value(s) of one variable.
4. Plug those value(s) back into the first equation to get the value(s) of the other variable.
5. Use factoring, if needed.
6. Use elimination, if needed.

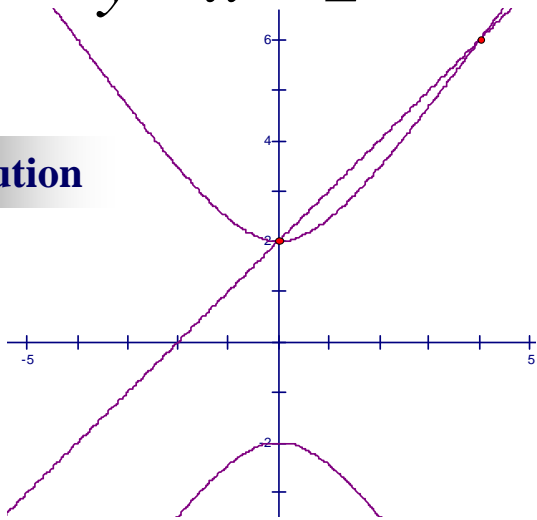


Ex: Solve each of the following systems

a. $\frac{y^2}{4} - \frac{x^2}{2} = 1$

$y - x = 2$

Solution

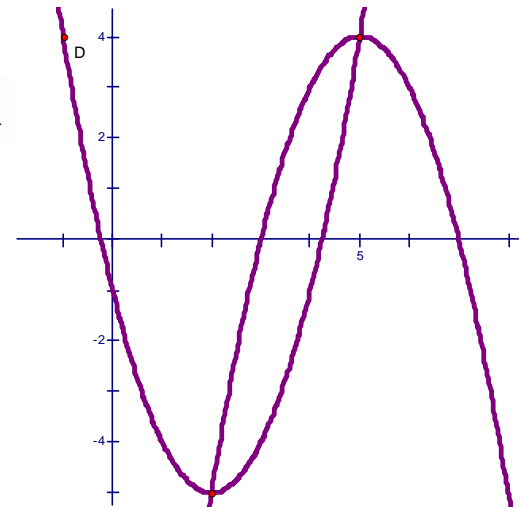


solutions: (0,2) and (4,6)

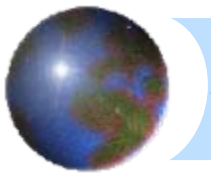
b. $y = (x - 2)^2 - 5$

$y = -(x - 5)^2 + 4$

Solution



solutions: (5,4) and (2,-4)



$$C. \quad (x-4)^2 + (y-2)^2 = 9$$

$$-x + y = 1$$

Solution

$$-x + y = 1 \Rightarrow y = x + 1$$

Rewrite 2nd equation for y

Substitute into 1st equation for y

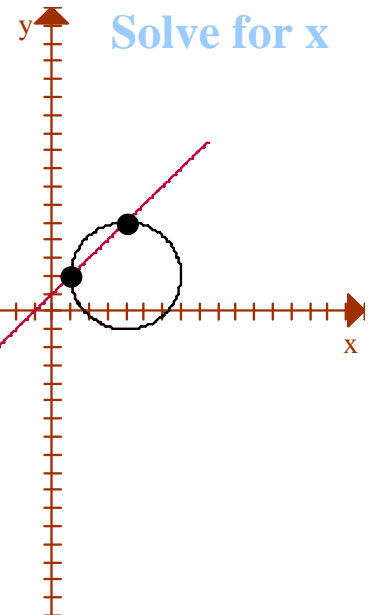
$$(x-4)^2 + ((x+1)-2)^2 = 9$$

$$x^2 - 8x + 16 + x^2 - 2x + 1 = 9$$

$$2x^2 - 10x + 8 = 0 \Rightarrow 2(x^2 - 5x + 4) = 0$$

$$2(x-4)(x-1) = 0$$

Solve for x



$$x = 4$$

Plug into 2nd equation

$$y = (4) + 1$$

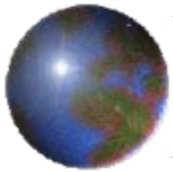
$$y = 5$$

$$x = 1$$

$$y = (1) + 1$$

$$y = 2$$

solutions are : (4, 5) (1, 2)



$$d. 5x^2 + y^2 = 30$$

$$y^2 - 16 = 9x^2$$

Solution

$$y^2 = 30 - 5x^2$$

$$(30 - 5x^2) - 16 = 9x^2$$

$$14 = 14x^2$$

$$x^2 = 1 \longrightarrow x = \pm 1$$

$$5(1) + y^2 = 30$$

$$5 + y^2 = 30$$

<-SAME->

$$5(-1) + y^2 = 30$$

$$5 + y^2 = 30$$

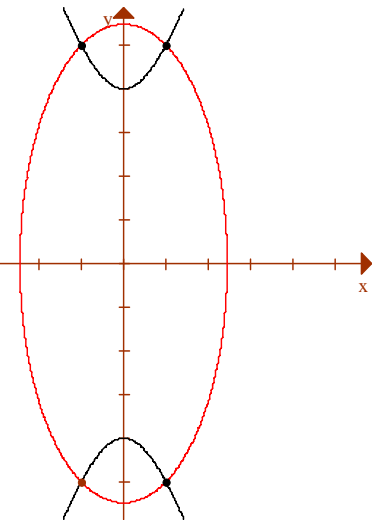
$$y^2 = 25$$

$$y = \pm 5$$

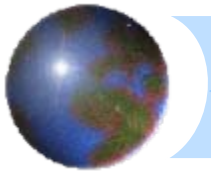
Rewrite first equation for y^2

Substitute y^2 in the 2nd equation

Substitute x back in the 1st equation



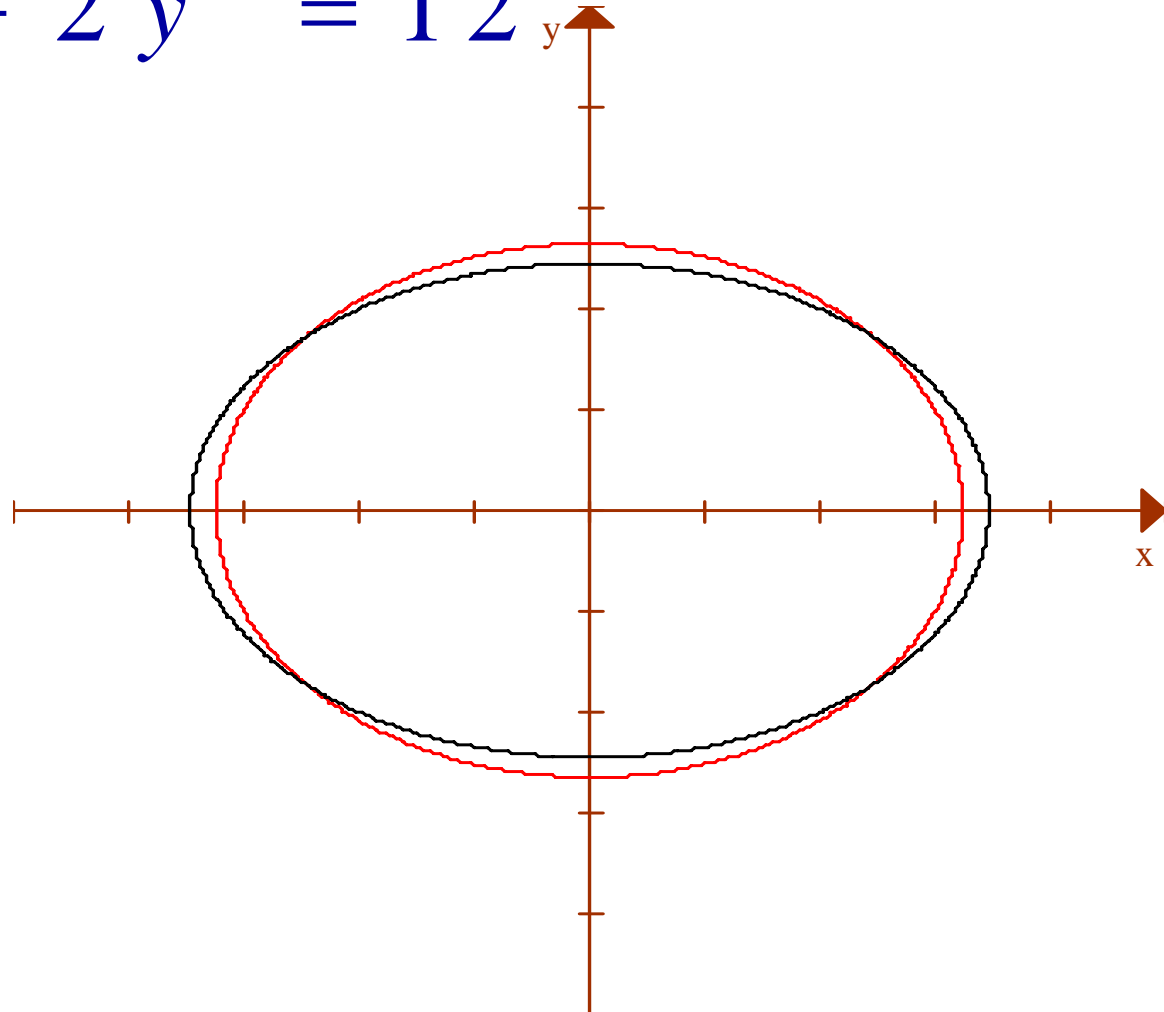
solutions are $(1, 5); (1, -5); (-1, 5); (-1, -5)$

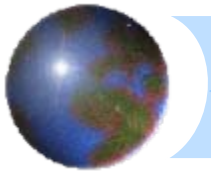


e. $2x^2 + 3y^2 = 21$

$$x^2 + 2y^2 = 12$$

Solution



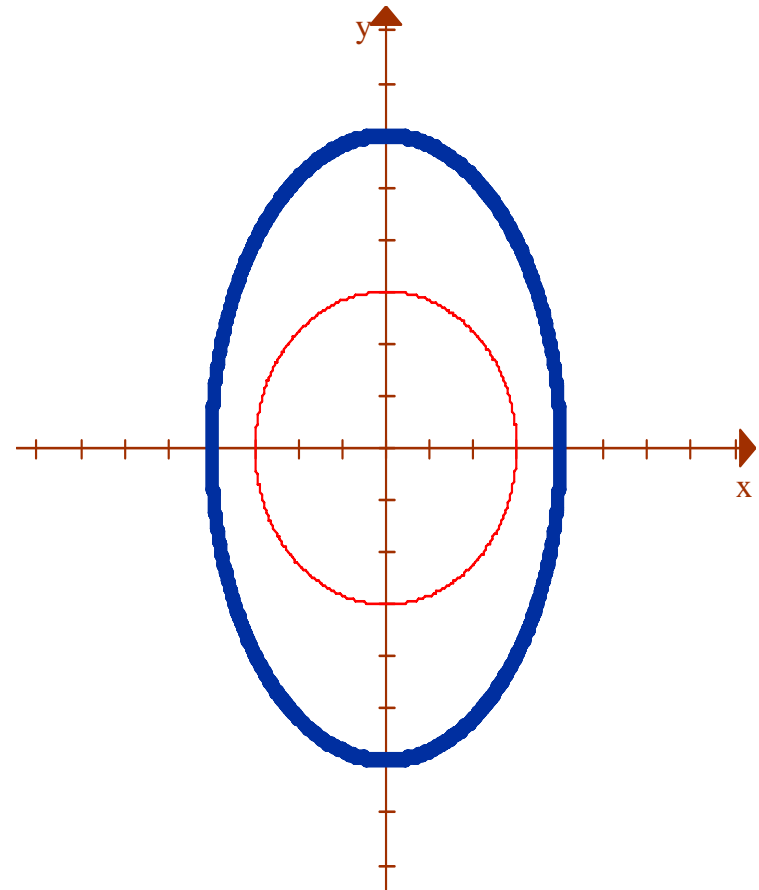


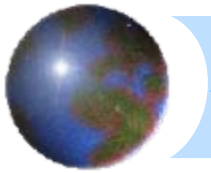
$$f. \quad 9x^2 + 4y^2 = 144$$

$$x^2 + y^2 = 9$$

Solution

no real solutions





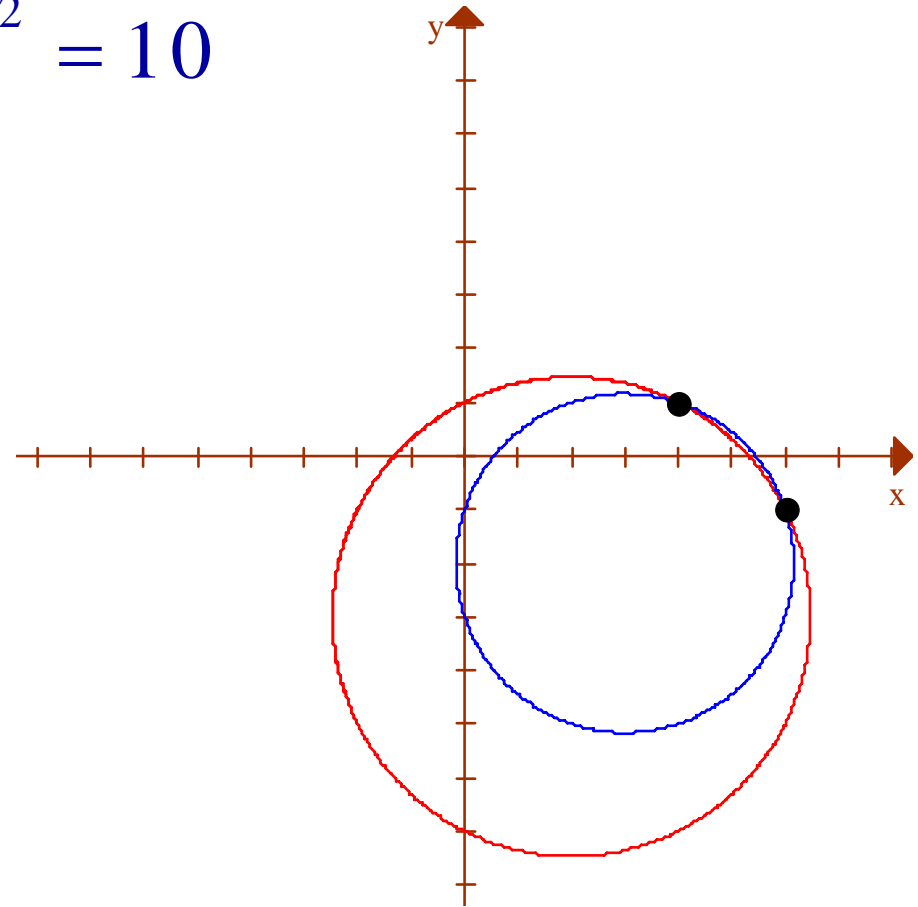
$$g. \quad (x - 2)^2 + (y + 3)^2 = 20$$

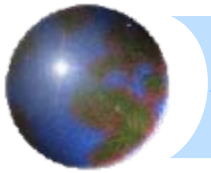
$$(x - 3)^2 + (y + 2)^2 = 10$$

Solution

The solutions are:

$(4, 1)$ and $(6, -1)$





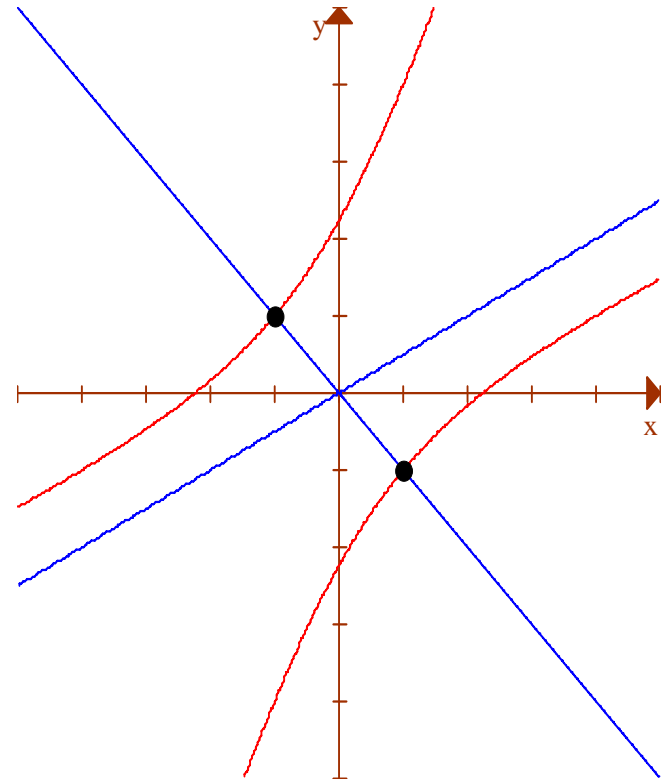
$$h. \quad x^2 - 3xy + y^2 = 5$$

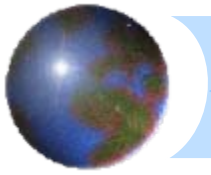
$$x^2 - xy - 2y^2 = 0$$

Solution

The solutions are:

$(-1, 1)$ and $(1, -1)$





Do exercises 33, 35, and 36