

P.1 THE REAL NUMBER SYSTEM (الأعداد الحقيقية)

Numbers

1) **Natural numbers** = {1, 2, 3, ...} الأعداد الطبيعية

2) **Whole numbers** = { 0, 1, 2, 3, ...} الأعداد الكلية

3) **Integers** = { ..., -3, -2, -1, 0, 1, 2, 3, ... } الأعداد الصحيحة

4) **Rational numbers** = { all terminating or repeating decimals }
الأعداد النسبية

Ex1: All of the following numbers are rational numbers

1) $-1.121212\dots = -1.\overline{12}$

5) -3

8) $\frac{22}{7}$

2) 0.121212

6) $\sqrt{4}$

3) 0.45

7) $\frac{11}{12}$

4) 0



5) **Irrational numbers** = { all nonterminating and nonrepeating
الأعداد غير النسبية decimals }

Ex2: All of the following numbers are irrational numbers

1) 1.2350174...

2) 0.272772777...

3) $\sqrt{2}, \sqrt{3}, \sqrt[3]{7}$

4) $\pi, -\pi, \frac{1}{\pi}$

5) $e = 2.71828183...$



6) **Real numbers** = { all rational or irrational numbers }

الأعداد الحقيقية

Prime and composite numbers

1) A **prime Number** is a positive integer other than one whose only factors are 1 and itself. الأعداد الأولية

2) A **composite number** is a positive integer other than one that is not prime(has more than two factors). الأعداد غير الأولية

Ex3:

- The first 10 prime numbers are { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
- The first 10 composite numbers are {4, 6, 8, 9 , 10, 12, 14, 15, 16, 18}

Even and Odd Numbers

1) Even Numbers: Any number that can be divided by 2

الأعداد الزوجية

Ex: ..., -8, -4, -2, 0, 2, 4, 6, 8, ...

2) Odd Numbers : Any number that cannot be divided by 2

الأعداد الفردية

Ex: ..., -9, -3, -1, 1, 3, 5, 7, 9, ...



Ex4:

Determine which of the following numbers are

- a) Integers
- b) Rational Numbers
- c) Irrational Numbers
- d) Real Numbers
- e) Prime Numbers
- f) Composite Numbers

$$\{0, 1, -1, 0.\bar{2}, 0.222, 0.222\dots, \sqrt{2}, \sqrt{3}, \sqrt{4}, \pi^2, 3.14, \frac{22}{7}, 1.121221222, 1.121221222\dots, 45, 51, 57, 52, -205\}$$

Solution

a) **Integers:** $\{0, 1, -1, \sqrt{4}, 45, 51, 57, 52, -205\}$

b) **Rational numbers:** $\{0, 1, -1, 0.\bar{2}, 0.222, 0.222\dots, \sqrt{4}, 3.14, \frac{22}{7}, 1.121221222, 45, 51, 57, 52, -205\}$

c) **Irrational numbers:** $\{\sqrt{2}, \sqrt{3}, \pi^2, 1.121221222\dots\}$

d) **Real numbers:** all the given numbers

e) **Prime numbers:** $\{\sqrt{4}\}$

f) **composite numbers:** $\{45, 51, 57, 52\}$



Ex5: TURE OR FALSE

1) Every even number is composite

False. Take 2.

2) The sum of two prime numbers is a prime number

False. $7(\text{prime}) + 5(\text{prime}) = 12(\text{composite})$

3) The only positive integer that is niether prime nor composite is 1.

True

Sets (المجموعات)

Def.:

A set is a collection of distinct objects. The objects in the set are called elements.

Ex6: Consider the following sets $A = \{1, 2, 3\}$


$$B = \{-1, 1, 0, 4\}$$

$$C = \{2, 3\}$$

★ 1, 2, 3 are the elements of A

-1, 1, 0, 4 are the elements of B

2, 3 are the elements of C




★ $2 \in A$ “is an element of”

$-10 \notin A$ “is **not** an element of”

★ $\{2\} \subseteq A$ “is subset of”

$\{-10\} \not\subseteq A$ “is **not** subset of”



★ The **union** (اتحاد) of two sets, $A \cup B$, is the set of all elements that belong to either A or B or both.

The **intersection** (تقاطع) of two sets, $A \cap B$, is the set of all elements that are common to both A and B .

$$A \cap C = \{2, 3\}$$

$A \cap B = \{ \} = \phi$ Empty set A and C are called **disjoint sets**.

$$A \cup B = \{-1, 0, 1, 2, 3, 4\}$$



Notes:

1) Negative Integers = $\{-1, -2, -3, \dots\}$

2) Positive Integers = $\{1, 2, 3, \dots\}$

3) Naturals \subseteq Wholes \subseteq Integers \subseteq Rationals \subseteq Reals

4) Rational \cap Irrational = ϕ

Rational \cup Irrational = Real numbers

5) Finite set: all the elements of the set can be listed

Ex: $\{1, 2, 3, 4\}$

5) Infinite set: not all the elements of the set can be listed

Ex: set of all integers

Set Builder Notation

$$\{ x \mid x \text{ has some property} \}$$

The set of $\underbrace{\quad}$ \uparrow \uparrow \uparrow such that \uparrow x has the given property
all elements x

Ex7: List the elements of the following sets

1) $A = \{ x \mid x \text{ is a prime number} < 10 \}$

Sol.: $\{2, 3, 5, 7\}$

2) $B = \{ x \mid x \text{ is the smallest positive odd number} \}$

Sol.: $\{ 1 \}$

3) $C = \{ y \mid y = 2x + 1, x \text{ is a natural number} \}$

Sol.: $\{3, 5, 7, 9, 11, \dots\}$

4) $D = \{ x^2 \mid x \text{ is integer} \}$

Sol.: $\{0, 1, 4, 9, 16, 25, \dots\} \equiv$ The set of perfect square numbers

Ex8:

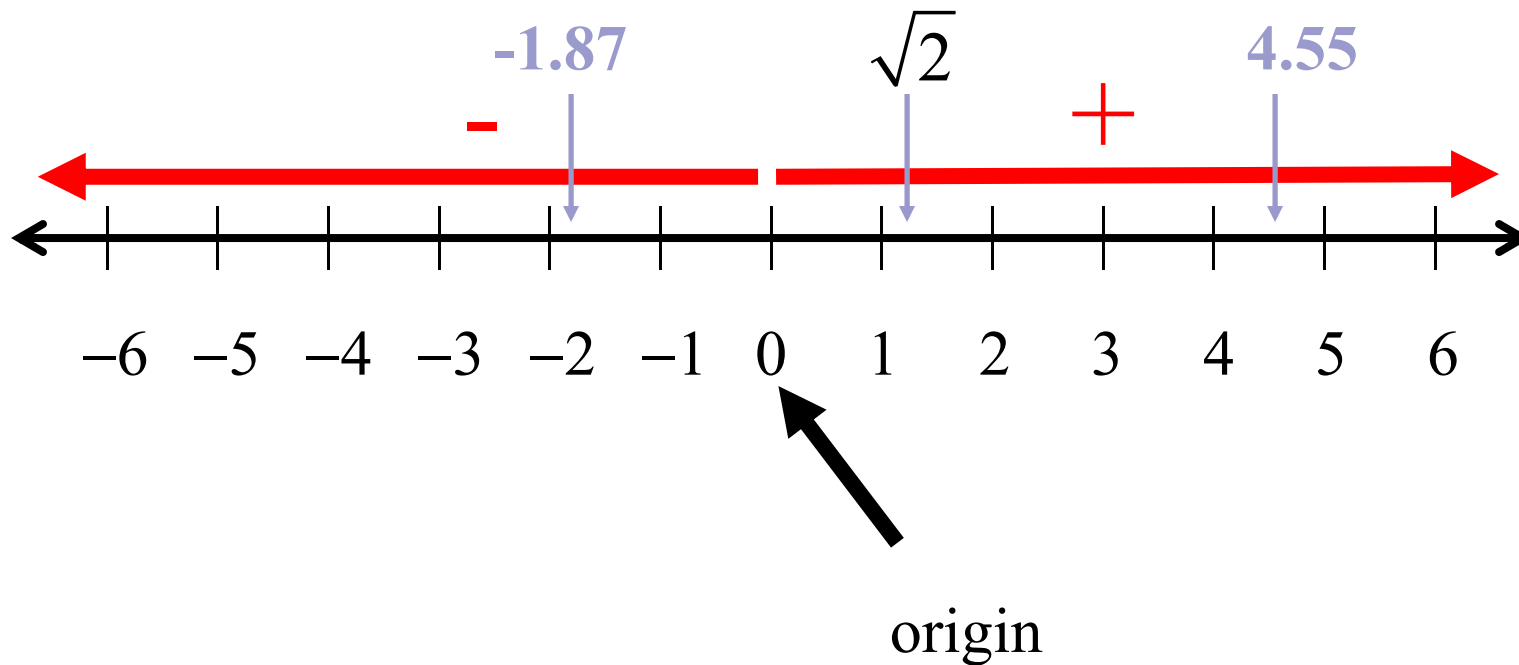
Let $A = \{-6, -4, -2, 0\}$

$B = \{ y \mid y = 2x - 2, x \text{ is an integer with } -3 \leq x < 1 \}$

Find $A \cap B$

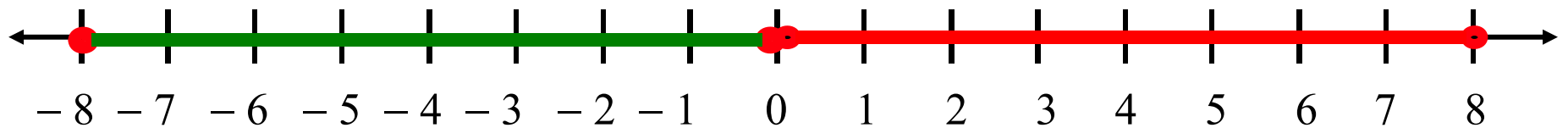
The Real Number Line

خط الأعداد الحقيقية



Absolute Value and Distance (القيمة المطلقة والمسافة)

For a real number x , $|x|$ is the distance between x and 0.




$$|-8| = 8$$

$$|8| = 8$$

Def.: Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$



Ex9: Write without the absolute value notation

1) $|-2|$


2) $|2|$

3) $|0|$

4) $|2 - \pi|$

5) $|\pi - 2|$

6) $|\pi - 2|$


$$7) |\sqrt{5} - 2| - |\sqrt{5} - 3|$$

$$8) |-x|, x < 0$$

$$9) |x + 3| + |x - 2|, -1 < x < 1$$

$$10) \left| \frac{x + 7}{|x| + |x - 1|} \right|, 0 < x < 1$$

Absolute Value Properties:

1) $|a| \geq 0$ **nonnegative property**

2) $|ab| = |a||b|$

3) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}, b \neq 0$

4) $|a + b| \leq |a| + |b|$ **triangle inequality**

5) $|a - b| = |b - a|$

6) $|-a| = |a|$

Def.:


If P and Q are two points on a real number line with coordinates a and b , respectively, the **distance between P and Q** , denoted by $d(P, Q)$, is

$$d(P, Q) = |b - a|$$

Ex10: Let P and Q be points on a real number line with coordinates -3 and 8 , respectively. Find the distance between P and Q .

Solution

$$d(P, Q) = |8 - (-3)| = 11$$



Ex11: Use the absolute value symbol to describe the following statements:

1) The distance between a and 3 is less than 5

Solution $|a - 3| < 5$

2) x is more than 2 units from 4 but less than 7 units from 4

Solution $2 < |a - 4| < 7$

3) The distance between x and -3 is not more than 8

Solution $|a + 3| \leq 8$



Inequality Symbols رموز المتراجحة

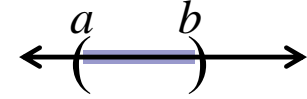

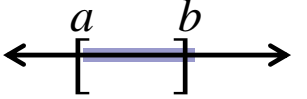

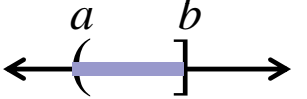

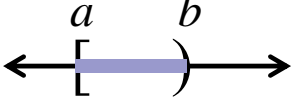




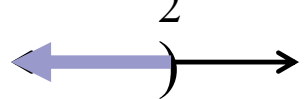

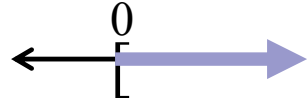
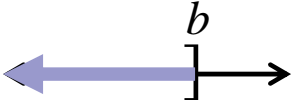
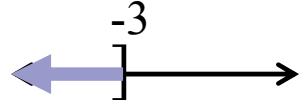
$>$ **Greater than**

$<$ **Less than**

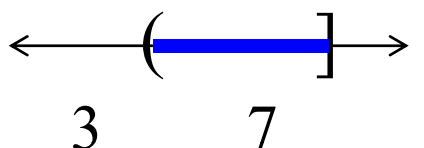
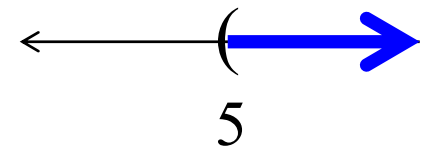
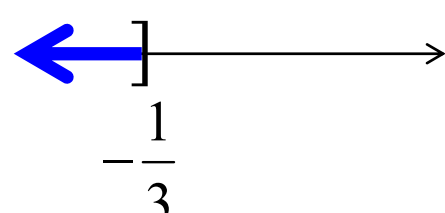
\geq **Greater than or equal**

\leq **Less than or equal**

Interval Notation الفترة

Interval	Graph	Example
(a, b)		$(3, 5)$ 
$[a, b]$		$[4, 7]$ 
$(a, b]$		$(-1, 3]$ 
$[a, b)$		$[-2, 0)$ 
(a, ∞)		$(1, \infty)$ 
$(-\infty, b)$		$(-\infty, 2)$ 
$[a, \infty)$		$[0, \infty)$ 
$(-\infty, b]$		$(-\infty, -3]$ 

Ex12: Complete the following table

Set Notation	The Graph	Interval Notation
$\{x \mid 3 < x \leq 7\}$		$(3, 7]$
$\{x \mid x > 5\}$		$(5, \infty)$
$\{x \mid x \leq -\frac{1}{3}\}$		$\left(-\infty, -\frac{1}{3}\right]$

Notes

★) or (means not included in the solution

★] or [means included in the solution

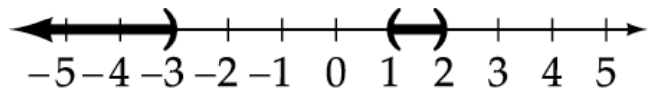
★ “AND” as a connector means intersection \cap

★ “OR” as a connector means union \cup

Ex13: Graph the following. Write a. and b. using interval notation. Write c. and d. using set-builder notation

a) $\{x|x < -3\} \cup \{x|1 < x < 2\}$

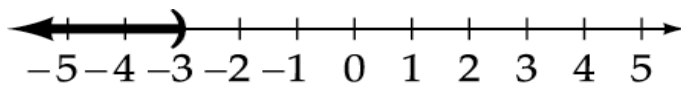
Sol.



$(-\infty, -3) \cup (1, 2)$

b) $\{x|x < -3\} \cap \{x|x < 2\}$

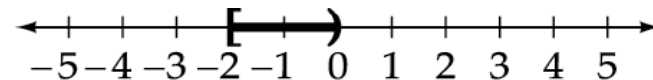
Sol.



$(-\infty, -3)$

c) $(-4, 0) \cap [-2, 5]$

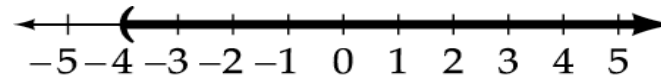
Sol.



$\{x|-2 \leq x < 0\}$

d) $(-4, \infty) \cup (0, \infty)$

Sol.



$\{x|x > -4\}$

Basic Terminologies: (مصطلحات اساسية)

Addition: الجمع + plus
Subtraction: الطرح - minus
Multiplication: الضرب \times times
Division: القسمة \div Divided by

Arithmetic Operations

العمليات الحسابية

If $a + b = c$, then c is called the **sum** ناتج الجمع

If $a - b = c$, then c is called the **difference** ناتج الطرح

If $a \times b = c$, then c is called the **product** ناتج الضرب

If $a \div b = c$, then c is called the **quotient** ناتج القسمة

$-a$ is called the **additive inverse** of a النظير الجمعي

$\frac{1}{a}$ is called the **multiplicative inverse (reciprocal)** of a
النظير الضربي المقلوب

$\frac{a}{b}$, where a and b are real number $b \neq 0$ is called **a fraction**.
كسر

a is called the **numerator** and b is called the **denominator**
بسط مقام

Ex14: Find the additive and multiplicative inverse for the following numbers

1) 2

2) $-\frac{\pi}{3}$

3) $-2\frac{3}{4}$



The **addends** of a variable expression are called **terms**

Ex15: Consider the following expression

$$3x^2 - 4xy + 5x - y - 7$$

variable terms : $3x^2, -4xy, 5x, -y$

Note: the sign of the term is sign that immediately precedes it

constant term: -7

coefficients: $3, -4, 5, -1$

The **multipliers** of a variable expression are called **factors**

in the expression $4xy$ $4, x,$ and y are called factors

Order of Operations ترتيب العمليات

- 1) Brackets الأقواس
- 2) Exponents الأسس
- 3) Multiplications and division as they occur from left to right
- 4) Addition and subtraction as they occur from left to right

Ex16: Simplify the expression $\frac{2 + 2 \times 3 \div 2^2 + 1}{2 - 2 \div 2}$

Ex17: Evaluate the expression $\frac{x^3 - y^3}{x^2 + xy + y^2}$ when $x = 2$ and $y = -3$

Properties of Real Numbers

Let a , b and c be real numbers.

	Addition	Multiplication
الإغلاق	Closure $a+b$ is a unique real number	ab is a unique real number
التبديل	Commutative $a + b = b + a$	$ab = ba$
التجميع	Associative $(a+b)+c=a+(b+c)$	$(ab)c=a(bc)$
المحايد	Identity $a + 0 = 0 + a = a$	$a \cdot 1 = 1 \cdot a = a$
النظير	Inverse $-a + a = a + (-a) = 0$	$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$
التوزيع	Distributive $a(b+c)=ab+ac$	

Ex18: Identify the property of real illustrated in each statement.

1) $(2a)b = 2(ab)$ Associative property of multiplication

2) $3(x + 2) = 3x + 6$ Distributive property

3) $1 + \sqrt{3}$ is a real number Closure property

4) $(3 \cdot \frac{1}{3})x = 1 \cdot x$ Inverse property of multiplication

5) $3(x + y) = 3(y + x)$ Commutative property of addition

6) $x + (y + z) = (y + z) + x$ Commutative property of addition

7) $x + (y + z) = (x + y) + z$ associative property of addition

Property of Equality

An equation is a statement of equality between two expressions

$x^2 + 3x = 5$ is an equation “ **which can be solved**”

$x^2 + 3x$ is an expression “ **which can not be solved**”

Let a , b and c be real numbers.

Reflexive

$$a = a$$

إنعكاس

Symmetric

if $a = b$, then $b = a$

تماثل

Transitive

if $a = b$ and $b = c$, then $a = c$

التعدي

Substitution

if $a = b$, then a may be replaced by b in any expression that involves a

التعويض



Ex19: Identify the property of real illustrated in each statement.

1) $x + 2y = 7$, then $7 = x + 2y$

Symmetric

2) $x + 2y = z$ and $y = 3$, then $x + 2(3) = z$

Substitution

3) $4a - 1 = 7b$ and $7b = 5c + 2$, then $4a - 1 = 5c + 2$

Transitive

4) $x + 2y = x + 2y$

Reflexive