P.6 COMPLEX NUMBERS

(الاعداد المركبة)

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Objectives:

- Introduction to Complex Numbers
- Addition and Subtraction of Complex Numbers
- Multiplication Complex Numbers
- Division Complex Numbers

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Introduction:

Remember that $\sqrt{-4}$ is impossible in the set of real numbers since there is no real number b for which $b^2 = -4$

Mathematician invented an expanded number system called the complex number system.

First they defined the new number $\sqrt{-1}$

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The Imaginary Unit *i* (الوحدة التخيلية)

$$i = \sqrt{-1}, \quad \text{where} \quad i^2 = -1.$$

Principal Square Root of a Negative Number

If b is a positive real number, then

$$\sqrt{-b} = i\sqrt{b}.$$

The number $i\sqrt{b}$ is called an imaginary number.

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Ex: Simplify: a. $\sqrt{-90} = \sqrt{90}\, i = \sqrt{9\cdot 10}\, i = 3\sqrt{10}\, i$

b. $\sqrt{-64} = i\sqrt{64} = 8i$

c. $\sqrt{16} + \sqrt{-50}$

$= \sqrt{16} + i\sqrt{50}$ — $a + bi$ form

$= \sqrt{16} + \sqrt{25\cdot 2}\, i$ — Simplify using the product property of radicals.

$= 4 + 5\sqrt{2}\,i$

Equality of Complex Numbers

$a + bi = c + di$ **if and only if** $a = c$ and $b = d$.

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Complex Numbers

A complex number is an expression of the the form

a + bi

- where ***a*** and ***b*** are real numbers
- The real number ***a*** is called the **real part**, and the real number ***b*** is called the **imaginary part**, of the complex number $a + bi$.

Ex:

	Real part	Imaginary part
$2 + 7i$	2	7
$20 - 3i$	20	-3
$4 + 0i$ (Real Number)	4	0
$0 + 6i$ (pure Imaginary Number)	0	6

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Adding and Subtracting Complex Numbers

1. Write each complex number in the **standard form** ***a* + *bi***.
(صورة قياسية)

2. Add or subtract the real parts of the complex numbers.

3. Add or subtract the imaginary parts of the complex numbers.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i.$$

Ex: Simplify each of the following

a. $(11 + 5i) + (8 - 2i)$

$= (11 + 8) + (5i - 2i)$ Group real and imaginary terms.

$= 19 + 3i$ $a + bi$ form

b. $(11 + \sqrt{-16}) - (6 + \sqrt{-9})$

$= (11 + i\sqrt{16}) - (6 + i\sqrt{9})$ Group real and imaginary parts.

$= (11 - 6) + [\ 4 - 3\ ]i$

$= 5 + i$ $a + bi$ form

The product of two complex numbers is defined as:

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

Note: $\sqrt{-6}.\sqrt{-24} \neq \sqrt{(-6)(-24)}$

Ex: *a.* $\sqrt{-25} \cdot \sqrt{-5} = i\sqrt{25} \cdot i\sqrt{5}$

$= 5i \cdot i\sqrt{5}$

$= 5\sqrt{5} \quad i^2$

$= 5\sqrt{5}\,(-1)$

$= -5\sqrt{5}$

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b. $7i\,(11 - 5i) = 77i - 35i^2$

$= 77i - 35\,(-1)$

$= 35 + 77i$

c. $(2 + 3i)(6 - 7i) = 12 - 14i + 18i - 21i^2$

$= 12 + 4i - 21i^2$

$= 12 + 4i - 21(-1)$

$= 12 + 4i + 21$

$= 33 + 4i$

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Conjugate (مرافق) of a Complex Number

The **complex conjugate** of the number $a+bi$ is $a-bi$, and the complex conjugate of $a-bi$ is $a+bi$. The multiplication of complex conjugates gives **a real number**.

Ex: $(5+2i)(5-2i)$

$(a+bi)(a-bi) = a^2 - b^2i^2 = a^2 - b^2(-1) = a^2 + b^2$

$= (5^2 - 4i^2)$

$= 25 - 4(-1)$

$= 29$

Number	Conjugate	Multiplication
$3+2i$	$3-2i$	13
$-5i+9 = 9-5i$	$9+5i$	106
7	7	49
$-4i$	$4i$	16

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Dividing Complex Numbers

A rational expression, containing one or more complex numbers, is in **simplest form** when there are **no imaginary numbers** in the denominator.

Ex: Simplify $\dfrac{7-9i}{6i}$

Sol:

$$= \frac{7-9i}{6i} \cdot \frac{i}{i}$$ Multiply the expression by $\frac{i}{i}$

$$= \frac{7i - 9i^2}{6i^2}$$

$$= \frac{7i - 9(-1)}{6(-1)}$$ Replace i^2 by -1 and simplify.

$$= \frac{9+7i}{-6} = -\frac{3}{2} - \frac{7}{6}i$$ Write the answer in the form $a + bi$.

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Ex: Find the conjugate $\bar{z}$ of the complex number $z = \dfrac{5+3i}{2+i}$

Sol:

$$z = \frac{5+3i}{2+i} \cdot \frac{2-i}{2-i}$$

Multiply the numerator and denominator by the conjugate of 2 + *i*.

$$= \frac{10-5i+6i-3i^2}{2^2+1^2}$$

$$= \frac{10+i-3(-1)}{4+1}$$

Replace i^2 by –1 and simplify.

$$= \frac{13+i}{5}$$

$$= \frac{13}{5} + \frac{1}{5}i$$

Write the answer in the form *a* + *bi*.

The conjugate is $\bar{z} = \dfrac{13}{5} - \dfrac{1}{5}i$ (watch)

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Ex: Write the complex number in standard form $\left(-1+\sqrt{-5}\right)^2$

Sol:

$$\left(-1+\sqrt{-5}\right)^2$$

$$= \left(-1+\sqrt{5}\,i\right)^2$$

You may use the perfect square formula

$$= (1)^2 - 2(1)\sqrt{5}\,i + \left(\sqrt{5}\,i\right)^2$$

$$= 1 - 2\sqrt{5}\,i + 5i^2$$

$$= -4 - 2\sqrt{5}\,i$$

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Powers of i:

$$i = i$$
$$i^2 = -1$$
$$i^3 = i^2 . i = -i$$
$$i^4 = \left(i^2\right)^2 = \left(-1\right)^2 = 1$$

Notice the pattern: $i, -1, -i, 1$

Powers of *i*
If n is a positive integer, then $i^n = i^r$,
where r is the remainder of the division of n by 4.

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Ex:

1. $i^{14} = i^2$ — 2 is the remainder of dividing 14 by 4
2. $i^{153} = i^1 = i$ — 1 is the remainder of dividing 153 by 4

Ex: Write the following complex number i^{-35} in standard from:

Sol:

$$i^{-35} = \frac{1}{i^{35}} = \frac{1}{i^3} = \frac{1}{-i}$$

$$= \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} \cdot = \frac{i}{-(-1)} = i$$

$$= 0 + i$$

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Ex: If $\dfrac{\sqrt[3]{-8}+\sqrt{-9}}{\sqrt{-1}+\sqrt[3]{-8}} = x + yi$, find x and y.

Sol:

$$= \frac{-2+3i}{i+-2} \qquad = \frac{-2+3i}{-2+i} \qquad = \frac{-2+3i}{-2+i}\cdot\frac{-2-i}{-2-i}$$

Multiply the numerator and denominator by the conjugate of 2 + i.

$$= \frac{4+2i-6i-3i^2}{(-2)^2+1^2} \qquad = \frac{7-4i}{5}$$

Replace i^2 by –1 and simplify.

$$= \frac{7}{5} - \frac{4}{5}i$$

Write the answer in the form $a + bi$.

Thus $x = \dfrac{7}{5}$ $y = -\dfrac{4}{5}$

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Ex: Find the sum of the real part and imaginary part of the complex number $z = 3(2+5i) - 2i(2-3i) + \sqrt{-12}\cdot\sqrt{-27}$

Sol:

$$z = 6 + 15i - 4i + 6i^2 + \sqrt{12}\ i\sqrt{27}\ i$$

$\sqrt{-12}\ \sqrt{-27} \neq \sqrt{12\cdot 27}$

$$= 6 + 15i - 4i + 6(-1) + \sqrt{12\cdot 27}\ \ i^2$$

$$= 6 + 15i - 4i + 6(-1) + 18\ (-1)$$

$$= 6 + 15i - 4i + 6(-1) + 18\ (-1)$$

$$= -18 + 11i$$

the real part is -18 the real part is 11

the sum is -7

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