P.3 POLYNOMIALS (كثيرات الحدود)

Def : A **monomial** is a constant, a variable, or a product between a constant and one or more variables. The variable(s) must have *positive integer exponents* (1, 2, 3, …). The constant is called the **coefficient** (معامل).

The **degree** (درجة) of a monomial is the sum of the exponents of the variables.

Ex: Complete the following table

The monomial	The coefficient	The degree
$2x^3y^2$	2	2+3=5
$-21x^3$	-21	3
$-3(2x^2y)^2 = -12x^4y^2$	-12	4+2=6

A **polynomial** is a finite sum of monomials. Each monomial is called a **term (حد)**.

Ex: $xy^2-2x-2y+3$, and x^2+x+1 are examples of polynomials

The **degree** of a polynomial is the largest degree of the terms in the polynomial.

Ex: Complete the following table

The Polynomials	The degree	The coefficients	The terms
$-2x^3y^2+10xy+1$	5	-2,10,1	$-2x^3y^2$,$10xy$, 1
$9x^3+x-1$	3	9, 1, -1	$9x^3$, x, -1
$-2x^3y^2+10(2x^2y)^2+1$	6 (why?)	-2, 40, 1	$-2x^3y^2$, $40x^4y^2$, 1

Like terms: same variable raised to the same power

Ex: 1) $2x^2, 5x^2$ 2) $7x^2y, -3x^2y$ (like terms)

3) $3x^2, 3x^3$ 4) $3x^2, 3xy^2$ (not like terms)

Simplified polynomial: all like terms are added/subtracted (combined)

Ex: $4x^2 + 3x + 5x - 10x^2 + 2 = -6x^2 + 8x + 2$

A polynomial of two terms is called a binomial **(ثنائية الحدود)**

Ex: $7xy^2 + 2y$

A polynomial of three terms is called a trinomial **(ثلاثية الحدود)**

Ex: $8x^2 + 12xy + 2y^2$

The standard form of a polynomial of **degree n** in the variable x is:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 ,$$

where $a_n \neq 0$ and n is a nonnegative integer.

a_n :is called the leading coefficient

(coefficient of the variable with the largest exponent)

a_0 :is called the constant term

(the term without a variable)

$a_n x^n$:is called the leading term

terms are arranged in decreased powers of x

Ex: The degree is 3.

$$5x^3 + 2x^2 - 5x + 12$$

The leading coefficient is 5. The constant term is 12.

Ex: Which of the following is a polynomial?

$a)\ x^2 + x + 1$ (polynomial)

$b)\ x^2 + x^{-1} + 1$ (*not* polynomial)

$c)\ \left(\frac{1}{x}\right)^2 + \frac{1}{x} + 1$ (*not* polynomial)

$d)\ \frac{x^2 - 1}{x - 1}$ (*not* polynomial)

$e)\ \sqrt{x}$ (*not* polynomial)

$f)\ \sqrt{2}$ (*not* polynomial)

$g)\ 5$ (*mono*mial)

$h)\ \sqrt{x^2 + x + 1}$ (*not* polynomial)

Operations on Polynomials

Addition and Subtraction: add (subtract) the coefficients of like terms.

Ex: Perform the indicated operation

a) $(3x^2 + 4x + 5) + (2x^2 + 7x - 2)$

Sol.

$5x^2 + 11x + 3$

b) $(u^3 - 3u^2 - 4u + 8) - (u^3 - 2u + 4)$

Sol.

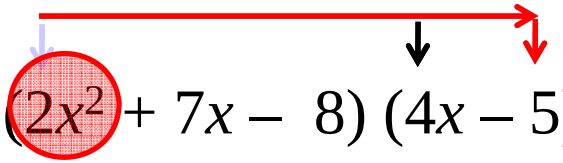
$u^3 - 3u^2 - 4u + 8 - u^3 + 2u - 4 = -3u^2 - 2u + 4$

Multiplication: use the distributive property

Ex: Perform the indicated operation

a) $(2x^2 + 7x - 8)(4x - 5)$

Sol.



Now apply the same property repeatedly

$$(2x^2 + 7x - 8)(4x - 5) = 8x^3 - 10x^2 + 28x^2 - 35x - 32x + 40$$

$$= 8x^3 + 18x^2 - 67x + 40$$

b) $(m - n + k)(m + 2n - 3k)$

$$= m^2 + 2mn - 3km - mn - 2n^2 + 3kn + mk - 2kn - 3k^2$$

$$= m^2 + mn - 2km + 5kn - 2n^2 - 3k^2$$

Special Products

Difference of Two Squares (فرق بين مربعين)

$$(x-a)(x+a) = x^2 - a^2$$

square of the first

square of the last

Squares of Binomials, or Perfect Squares (مربع كامل)

$$(x \pm a)^2 = x^2 \pm 2ax + a^2$$

square of the first

square of the last

twice the product

Cubes of Binomials, or Perfect Cubes (مكعب حدين)

$$(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$$

$$(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$$

Ex: Use the special product formulas to perform the indicated operation.

Sol.

a) $(3x + 5)(3x - 5)$ $= (3x)^2 - (5)^2$ $= 9x^2 - 25$

b) $(3x^2 - y)^2$ $= (3x^2)^2 - 2(3x^2)y + y^2$ $= 9x^4 - 6x^2y + y^2$

c) $[(x+5) + y][(x+5) - y]$ $= (x+5)^2 - y^2$ $= x^2 + 10x + 25 - y^2$

Ex: Find the coefficient of the term x^2y in the product $(2x - 3y)^3$

Sol.

$= (2x)^3 + 3(2x)^2(-3y) + 3(2x)(-3y)^2 + (-3y)^3$

$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$

The coefficient of the term x^2y *is -36*

Ex: Find the coefficient of the term $r^{1/2}$ without finding the entire product $r^{1/2}\left(r^{1/2} - r^{-1/2}\right)^2$

Sol.

$$= r^{1/2}\left(\left(r^{1/2}\right)^2 - 2r^{1/2}r^{-1/2} + \left(r^{-1/2}\right)^2\right) = r^{1/2}\left(r - 2 + r^{-1}\right)$$

The coefficient of $r^{1/2}$ is -2

to **evaluate a polynomial (حساب قيمة),** substitute the given value(s) for the variable(s) and then perform the indicated operations using the Order of Operation Agreement.

Ex: Evaluate the polynomial $2x^3-x^2+7$ for $x = 2$.

Sol

$$2(2)^3-(2)^2+7=16-4+7=19$$

Ex: Suppose that P and Q are both polynomials of degrees n and m, respectively, discuss the degrees of $P+Q$, PQ, *and* P-P

Polynomial	The degree $n = m$	The degree $n > m$	The degree $n < m$
$P+Q$	$\leq n\,(or\ m)$	$=n$	$=m$
PQ	$=2n(or\ 2m)$	$=n+m$	$=n+m$
P-P ***No degree ?***			