

King Fahd University of Petroleum and Minerals  
Mathematical Sciences Department

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- Important Instructions:**
- (1) Show All Necessary Steps
  - (2) Grading will be Based on Every Step
  - (3) Unverified Final Answers will not be Graded

(1) The radius  $R$  and the center  $C$  of the circle  $2x^2 + 2y^2 + 12x + 16y - 22 = 0$  are:

a)  $R = 6$  and  $C = (-3, -4)$

✓ b)  $R = 36$  and  $C = (3, 4)$

c)  $R = \sqrt{6}$  and  $C = (-3, -4)$

d)  $R = 36$  and  $C = (-3, -4)$

e)  $R = 6$  and  $C = (-4, -3)$

$$x^2 + y^2 + 6x + 8y = 11$$

$$(x^2 + 6x + 9) + (y^2 + 8y + 16) = 11 + 9 + 16$$

$$(x - (-3))^2 + (y - (-4))^2 = 36$$

(2) An equation of a circle that has a diameter with endpoints  $(2, 3)$  and  $(-4, 11)$  is:

a)  $(x+2)^2 + (y-3)^2 = 4$

b)  $(x-1)^2 + (y+7)^2 = 25$

c)  $(x+3)^2 + (y-3)^2 = 4$

✓ d)  $(x+1)^2 + (y-7)^2 = 25$

e)  $(x+7)^2 + (y-3)^2 = 4$

The diameter =  $\sqrt{(11-3)^2 + (-4-2)^2} = 10$

The center = Midpoint =  $\left(\frac{-4+2}{2}, \frac{11+3}{2}\right) = \boxed{(-1, 7)}$

The radius =  $\frac{1}{2} \cdot 10 = \boxed{5}$

(3) If  $(a, b)$  is the vertex and  $Max$  is the maximum value of the function  $y = -2x^2 + 4x - 4$ , then:

a)  $a+b=0$ ,  $Max = -1$

✓ b)  $a+b=-1$ ,  $Max = -2$

c)  $a+b=-3$ ,  $Max = -1$

d)  $a+b=-1$ ,  $Max = -3$

e)  $a+b=-1$ ,  $Max$  does not exist.

Rewriting the equation in standard form,

$$h = \frac{-4}{2(-2)} = 1$$

$$k = f(1) = -2 + 4 - 4 = -2 \quad \left. \vphantom{h} \right\} \text{Vertex} = \boxed{(1, -2)}$$

the eq. represent a downward open parabola.

$$\Rightarrow Max. = k = \boxed{-2}$$

4) The graph of the equation  $3x^2 = |2x - 5y|$  is symmetric with respect to:

a) the y-axis and origin

b) the x-axis only

c) the y-axis only

d) the origin only

e) the x-axis, the y-axis, and the origin

$$3(-x)^2 = |2(-x) - 5(-y)|$$

$$3x^2 = |-2x + 5y|$$

$$\Rightarrow 3x^2 = |-(2x - 5y)| = |2x - 5y|$$

$$\Rightarrow 3x^2 = |2x - 5y|$$

5) The x-intercept of the line that is passing through the point  $(2, -1)$  and parallel to the line  $3x + 4y = 5$  is:

a)  $\left(\frac{-1}{3}, 0\right)$

b)  $\left(\frac{2}{3}, 0\right)$

c)  $(4, 0)$

d)  $(-3, 0)$

e)  $\left(0, \frac{2}{3}\right)$

the slope  $m_2$  of  $3x + 4y = 5$  is  $-\frac{3}{4}$   
 $\Rightarrow$  the slope  $m_1$  of the line through  $(2, -1)$  is  $-\frac{3}{4}$  also

$\Rightarrow$  the eq. is  $y + 1 = -\frac{3}{4}(x - 2)$

$\Rightarrow$  the x-intercept is:  $0 + 1 = -\frac{3}{4}(x - 2)$

$$\Rightarrow x - 2 = -\frac{4}{3}$$

$$\Rightarrow x = \frac{2}{3}$$

6) Which one of the following relations defines  $y$  as a function of  $x$ ?

a)  $y^2 + 3x = 3$

b)  $\{(2, 3), (3, 5), (4, 6), (5, 3), (4, 10)\}$

c)  $x^2 + y^2 = 9$

d)  $|y| = |x| + 5$  if  $y < 0$  since  $y = -(5 - |x|)$  only.

e)  $y = 4 \pm \sqrt{x}$

(7) If  $f(x) = \begin{cases} \lfloor 1-2x \rfloor & \text{if } 0 \leq x \leq 5 \\ 5 & \text{if } 5 < x \leq 8 \\ \sqrt{x-7} & \text{if } 8 < x \leq 11 \end{cases}$ , then for  $1 < n \leq 2$ ,  $f(1.2) + f(8) + f(n^2 + 7) =$

a) 0

b) 3

c)  $-n + 3$

d)  $n + 3$

e)  $n + 4$

$f(1.2) = \lfloor 1 - 2(1.2) \rfloor = \lfloor -1.2 \rfloor = -2.$

$f(8) = 5.$

$f(n^2 + 7) = \sqrt{n^2 + 7 - 7} = |n| = n$  as  $1 < n \leq 2$

$\Rightarrow$  the sum is  $-2 + 5 + n = n + 3$

(8) The graph of the function  $f(x) = \begin{cases} 2 & \text{if } x < -2 \\ |x| + 1 & \text{if } -2 \leq x < 3 \\ 3 - x & \text{if } x \geq 3 \end{cases}$

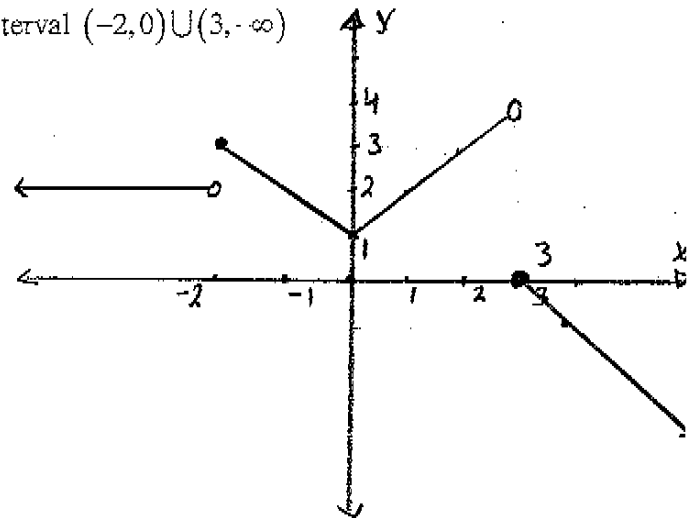
a) is decreasing over the interval  $(-\infty, 3)$

b) has no x-intercept and has the y-intercept  $y = 1$

c) has range  $(-\infty, 0] \cup [1, 4)$  and is decreasing over the interval  $(-2, 0) \cup (3, -\infty)$

d) is increasing over  $(0, \infty)$

See the below graph



(9) If the slope of the line passing through a point P on the x-axis and the point  $(-1, 3)$  is  $\frac{3}{2}$ , then

a) P is  $(3, 0)$

b) P is  $(-3, 0)$

c) P is  $(0, -3)$

d) P is  $(0, 0)$

Let the point be  $(x, 0)$

$\Rightarrow \frac{3 - 0}{-1 - x} = \frac{3}{2}$

$\Rightarrow -3x + 3 = -6$

$\Rightarrow x = -3$

