Work Sheet on the Material of Sections P1-1.5

1) Simplify each of the following expressions: (a) $\frac{x^3-y^3+x-y}{x-y}-\frac{x^4-1}{x^2-1}$ (b) $x-\frac{2}{1-\frac{1}{1-\frac{1}{x}}}$

(c) $\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}$ (d) $\frac{x-6}{x^2-4}+\frac{1+x}{2-x}$ (e) $\frac{1-(1-x)^{-1}}{x^{-1}+(1-x)^{-1}}$

2) Perform the following operations: (a) $\left(2\sqrt[3]{x}-\sqrt[3]{2}\right)\left(4\sqrt[3]{x^2}+2\sqrt[3]{2x}+\sqrt[3]{4}\right)$ (b) $12\sqrt[3]{4}-\frac{20}{\sqrt[3]{16}}$ (c) $\left|-2^{-3^2}\right|$, (d) $\frac{6\cdot 8\div 4-2}{5\cdot 4-9\cdot 2}$

3) Find the sum of all the coefficients in the product $(x-2)^2-(2x+1)^3$

4) Factor each of the following expressions completely over the set of integers:

(a) x^2-y^2+6y-9 (b) $3a^4b-6a^2b-24b$ (c) x^6-16x^3+64 (d) $6x^2-2y^2+xy-6x+4y$

5) Find the solution set of each of the following eqautions: (a) $\frac{3}{p+4}+\frac{4}{p+3}=\frac{4}{p^2+7p+12}$ (b) $\frac{3r+5}{r}=4+\frac{5}{r}$

(c) $\left(x^{-1}+5\right)^2-2x^{-1}-10=8$ (d) $\frac{2x-2}{3}-\frac{5-x}{x+3}=\frac{2x-2}{x+3}$

6) Find the sum of all solutions of the eqaution $\frac{9}{(r-2)^2}=\frac{11}{r-2}-2$

7) If the equation $\frac{3x}{x+2}=5+\frac{k}{x+2}$ has no solution, find the value of k.

8) Solve each of the following equations: (a) $-2|1-2x|+1=5$ (b) $|x|+3x-9=0$ (c) $|2x+1|=|-2x|$

9) Solve the following equations for the indicated varaibles: (a) $r^2m^2-2m+\frac{1}{r}=0$ for m (b) $s=\frac{n}{2}\left[2a+(n-1)d\right]$ for a

(d) $Z=\frac{A-XL}{1-X}$ for x (e) $m(x-n)=nx+k$ for x

10) Write each of the following expression without the abolute value symbols:

(a) $\frac{|\pi-3|+|\pi-4|+\left|\frac{|x|}{x}\right|}{|4-x|}$ where $x>4$ (b) $|2m+11|+|2m-22|$, where $-5<x<10$ (c) $|-3x|+|x-2|$, where $x<0$

11) If the equation $-3x^2+2x+5=0$ is transformed to the form $(x+a)^2=b$, find values of a and b.

12) (a) If $x+iy=2+\frac{i^{-43}}{2}$, find x and y. (b) Write the complex number $\frac{\sqrt[3]{125}-i^{11}-\sqrt{-4}\sqrt{-1}}{i-6}$ in standard form.

13) If 1 is a solution for the equation $(2x-1)(3x+2)=-5k$, find: (a) the value of k (b) the other solution

(14) If the discriminant of the equation $x^2+4kx-5=x$ is 29, find the values of k.

15) Find all real values of k such that the equation $4x^2-4(k+4)x+k=0$ has two equal solutions(one doubled solution).

16) Find all values of k such that the equation $x^2-x+\frac{1}{k+1}=0$ has two complex nonreal solutions.

17) Solve each of the following inequalities: (a) $(x+4)(x+2)<3$ (b) $x^2 \geq 4$ (c) $x^2-1<4$ (d) $x \leq \frac{1}{x}$

(e) $\frac{(x+1)^{10}(2-x)^{19}}{(x-7)^6} \geq 0$ (f) $4x^3+7x^2 \geq 2x$ (g) $\frac{1}{x^2+4x-5} \leq \frac{3}{x+5}$ (h) $1<|x+1|<4$ (i) $\frac{2}{x+1} > x-1$

18) Find all solutions for each the following equations: (a) $x+\sqrt[4]{3x^2+4}=0$ (b) $\sqrt{x+7}-\sqrt{2x}=1$

(c) $(x+2)^{\frac{2}{5}}-2(32x+64)^{\frac{1}{5}}+4=0$ (d) $|1-6x|^2-|24x-6|=45$ (e) $\frac{x}{4}-\frac{x}{2-x}=\frac{2}{x-2}$

19) If the sum of the roots of the quadratic equation $x^2+bx+c=0$ is -2 and the product of the roots is -8, find $b+c$.

20) Simplify each of the following expressions: (a) $\frac{\sqrt{(x-2)^2+2(x-2)+1}}{x-1}$, if $-2<x<-1$ (b) $(m^4)^{\frac{1}{4}}+(m^5)^{\frac{1}{5}}$, if $m<0$

21) Simplify each of the following expressionsand rationalize the denominator:

(a) $3\sqrt[3]{2}-5\sqrt[3]{16}+2\sqrt[3]{125}$ (b) $\sqrt[3]{\frac{16m^8n^9}{k^2p}}$ (c) $\frac{9^{\frac{1}{2}}-\left(3^{-\frac{4}{3}}\right)(3)}{(27)^{\frac{2}{9}}}$

(d) $\left[\frac{(xy)^{-1}z^5}{(-xz^{-2})^{-1}y^2}\right]^{\frac{-1}{3}}$ (e) $\frac{2(2x^n)^2(-x)^{2n+2}}{8x^{4n}x^2}$ (f) $\sqrt{a^4+a^4b^2}$ (g) $\frac{1}{(2+\sqrt{10})(1-\sqrt{5})}$ (h) $\sqrt{(4-\sqrt{17})^2}$

22) If $x+2yi=\frac{6+2i}{5-i}\cdot\frac{1-3i}{2-6i}$, then find the valiues of x and y.

Answers:

(1) (a)$y(x+1)$, (b)$3x-2$, (c) 1, (d)1, (e) $-x^2$ (2) (a)$2(x-4)$, (b) $7\sqrt[3]{4}$, (c) $\frac{1}{2^9}$, (d) 5, (e) $\frac{5-x}{3(x-2)}$ (3) -27

(4) (a) $(x-y+3)(x+y-3)$, (b) $3b(a-2)(a+2)(a^2 2)$, (c) $(x-2)^2(x^2+2x+4)^2$, (d)$(3x-2y)(x+y-2)$

(5) (a) no solution, (b) no solution, (c) $x=1$, $x=\frac{1}{7}$, (d) solution set contains $\frac{5}{2}$ only, (6) $r=\frac{19}{2}$, (7) $k=-6$

(8) (a) no solution, (b) $x=\frac{9}{4}$ only, (c) $x=-\frac{1}{4}$, (9) (a) $\frac{2s-n(n-1)d}{n}$, (b) $x=\frac{A-Z}{L-Z}$, (c) $x=\frac{k+mn}{m-n}$,

(10) (a) $\frac{2}{x-4}$, (b) 33, (c) $2-4x$, (11) $a=\frac{-1}{3}$, $b=\frac{16}{9}$, (12) (a) $x=2, y\ \frac{-1}{2}$,

(b) $\frac{-19}{37}+\frac{3}{37}i$ (13) (a) $k=-1$, (b) $\frac{-7}{6}$, (14) $k=\frac{-1}{2}$,1 (15) no real values, (16) $(-1,3)$,

(17) (a) $(-5,-1)$, (b) $(-\infty,-2]\cup[2,\infty)$, (c) $(-\sqrt{5},\sqrt{5})$, (d) $(-\infty,-1]\cup[0,1)$, (e) $(-\infty,1)\cup(1,2)$

(*f*) $\left[-\frac{1}{4},0\right]\cup[2,\infty)$ (g) $(-5,1)\cup\left[\frac{4}{3},\infty\right)$, (h) $(-5,-2)\cup(0,3)$, (i) $(-\infty,-\sqrt{3})\cup(-1,\sqrt{3})$

(18) (a) $x=\pm 2, x=\pm i$, (b) ,(c) 30 (d) $x=\frac{4}{3},\frac{5}{3}$, (e) $x=4$, (19) $b=2, c=-8$

(20)(*a*) -1, (b) 0, (21) (a) $10-7\sqrt[3]{2}$, (b) $\frac{2m^2n^3\sqrt[3]{2kp^2m^2}}{kp}$, (c) $-2\sqrt[3]{3}$

(*d*) $\frac{y}{z}$, (e) 1, (f) $a^2\sqrt{1+b^2}$ (g) $\frac{(2-\sqrt{10})(1+\sqrt{5})}{24}$, (h) $\sqrt{17}-4$, (22) $x=1$, $y=-5$