

1) Factor $3x^2 + xy - 2y^2 - x - y$

$$\text{Sol:} = (3x^2 + xy - 2y^2) - (x + y)$$

$$= (x + y)(3x - 2y) - (x + y) = (x + y)(3x - 2y - 1)$$

2) Simplify

$$\frac{x^{-1} + y^{-1}}{x + y} - \frac{x^{-1} - y^{-1}}{x - y}$$

$$\text{Sol:} = \frac{\frac{1}{x} + \frac{1}{y}}{x + y} - \frac{\frac{1}{x} - \frac{1}{y}}{x - y} = \frac{\frac{y + x}{xy}}{x + y} - \frac{\frac{y - x}{xy}}{x - y}$$

$$= \frac{\cancel{y+x}}{xy} \frac{1}{\cancel{x+y}} - \frac{y-x}{xy} \frac{1}{x-y}$$

$$= \frac{1}{xy} - \frac{\cancel{y-x}}{xy} \frac{1}{-(\cancel{y-x})}$$

$$= \frac{1}{xy} + \frac{1}{xy} = \frac{2}{xy}$$

3) Find the conjugate of

$$z = \frac{\sqrt{-3}\sqrt{-27}i^{63}}{\sqrt[3]{-27} - \sqrt{-16}}$$

$$\text{Sol:} = \frac{(\sqrt{3}i)(\sqrt{27}i)(i^3)}{-3 - \sqrt{16}i} = \frac{\sqrt{3} \cdot 3\sqrt{3}i^5}{-3 - 4i}$$

$$= \frac{9i}{-3 - 4i} \frac{-3 + 4i}{-3 + 4i} = \frac{-27i + 36i^2}{(-3)^2 + (-4)^2} = \frac{-36 - 27i}{25} = -\frac{36}{25} - \frac{27}{25}i$$

$$\Rightarrow \text{The conjugate is } \bar{z} = -\frac{36}{25} + \frac{27}{25}i$$

4) Simplify

$$\frac{2}{x^2 - 1} - \frac{1}{x^2 - x} + \frac{x-1}{x^2 + x}$$

$$\text{Sol:} = \frac{2}{(x-1)(x+1)} - \frac{1}{x(x-1)} + \frac{x-1}{x(x+1)}$$

$$= \frac{2x - (x+1) + (x-1)(x-1)}{x(x-1)(x+1)} = \frac{2x - x - 1 + x^2 - 2x + 1}{x(x-1)(x+1)}$$

$$= \frac{x^2 - x}{x(x-1)(x+1)} = \frac{\cancel{x}(x-1)}{\cancel{x}(x-1)(x+1)} = \frac{1}{x+1}$$