1.1 LINEAR AND ABSOLUTE VALUE EQUATIONS

Objectives:

In this section we will learn about:

- Definition of Linear Equation
- Contradiction, Conditional Equations and Identities
- Absolute Value Equations

Def. of An Equation:

An ***equation*** is a statement about the ***equality*** of two expressions

Definition of a Linear Equation المعادلات الخطية

A ***linear equation in the single variable x*** is an equation that can be written in the form

$$\boldsymbol{ax + b = 0}$$

where a and b are real numbers and $a \neq 0$.

Ex1. The following are not linear equations in one variable:

$2x + 3y = 11$ — Two variables

$(x - 1)^2 = 8$ — x is squared.

$\frac{2}{3x} + 5 = x - 7$ — Variable in the denominator

Equivalent Equations المعادلات المتكافئة

are equations that have **exactly** the **same** solution(s).

Procedures that Produce Equivalent Equations:

An equation can be transformed into an ***equivalent equation*** by one or more of the following operations:

- Simplify an expression by removing grouping symbols and combining like terms.
- Add (or subtract) the same real number or variable expression on both sides of the equation
- Multiply (or divide) on both sides of the equation by the same nonzero quantity.
- Interchange the two sides of the equation.

Ex2. The following equations are **equivalent** equations:

$$2x+3+5x=-11 \quad \textbf{and} \quad 7x+3=-11$$

$$3x-7=2 \quad \textbf{and} \quad 3x=9$$

$$\frac{5}{6}x=10 \quad \textbf{and} \quad x=12$$

A solution of a linear equation in one variable is a real number which, when substituted for the variable in the equation, makes the equation true. (***i.e.*** it ***satisfies*** the equation)

Ex3.: Is 4 a solution of $2x + 3 = 11$?

$$2x + 3 = 11$$ Original equation

$$2(4) + 3 = 11$$ Substitute 4 for x.

$$8 + 3 = 11$$ True equation, that is x=4 satisfies the equation

4 is **a solution** of $2x + 3 = 11$.

Ex4.: Is 3 a solution of $2x + 3 = 11$?

$$2x + 3 = 11$$ Original equation

$$2(3) + 3 = 11$$ Substitute 3 for x.

$$6 + 3 \neq 11$$ False equation, that is x=3 ***does not satisfy*** the equation

3 is not a solution of $2x + 3 = 11$.

To Solve a Linear Equation in One Variable:

1. Simplify both sides of the equation(open parenthesis, brackets, ….).
2. Use the addition and subtraction properties to get all ***variable*** terms on the ***left-hand side*** and all ***constant*** terms on the ***right-hand side***.
3. Simplify both sides of the equation.
4. ***Divide both sides*** of the equation by the ***coefficient*** of the variable.

Types of Linear Equations:

- **Conditional Equations** المعادلات المشروطة

Equations that are true for **some values** of the variable but **not true** for **other** values of the variable.

- **Contradiction Equations** المعادلات المتناقضة

Equations that has **no** solution.

- **Identity Equations** المعادلات المتطابقة

Equations that are true for **all values** of the variable for which all terms of the equation are defined.

Ex5. Solve the equation: $2(x-3)-17=13-3(x+2)$

$$2(x-3)-17=13-3(x+2)$$

$$2x-6-17=13-3x-6$$

$$2x-23=-3x+7$$

$$2x-23+3x=-3x+7+3x$$

$$5x-23+23=7+23$$

$$5x=30$$

$$x=6$$

The equation is conditional and has solution set { 6 }

To Solve a linear Equation in One Variable:

1. Simplify both sides of the equation(**open parenthesis, brackets..)**
2. 2. Use the addition and subtraction properties to get all **variable** terms on the **left-hand side** and all **constant** terms on the **right-hand side**.
3. Simplify both sides of the equation(**like terms**)
4. **Divide** both sides of the equation by the **coefficient** of the variable.
5. **Check the proposed** solution

Ex6. Solve the equation: $4x + 1 = 3x + 4 + x$

$$4x - 4x = 4 - 1$$ Simplify

$$0 = 3$$ **(what ?)**

Contradiction , the equation has **no solution**

Ex7. Solve the equation: $4[3(x-5)+7] = 12x - 32$

$$3(x-5)+7 = 3x-8$$ Divide by 4 and apply distribution property

$$3x-15+7 = 3x-8$$ Simplify

$$\underbrace{3x-8}\;=\;\underbrace{3x-8}$$

Identity , the equation has solution set **(−∞, ∞)**

Equations with ***fractions*** can be simplified by ***multiplying*** both sides by the LCD

Ex8. Solve the equation: $\frac{3x}{2} = \frac{x}{5} - \frac{39}{5}$

$$\frac{3x}{2} = \frac{x}{5} - \frac{39}{5}$$ Original equation

$$10 \cdot \frac{3x}{2} = 10\left(\frac{x}{5} - \frac{39}{5}\right)$$ Multiply both sides by LCD = 10

$$\overbrace{\cancel{10}}^{5} \cdot \frac{3x}{\underbrace{\cancel{2}}_{1}} = \overbrace{\cancel{10}}^{2} \cdot \frac{x}{\underbrace{\cancel{5}}_{1}} - \overbrace{\cancel{10}}^{2} \cdot \frac{39}{\underbrace{\cancel{5}}_{1}}$$ Simplify

$$15x = 2x - 78$$ Simplify

$$x = -6$$ Solution

Absolute Value Equations

If c is a any nonnegative real number and X represents any algebraic expression, then

$$|X| = c \qquad \text{if and only if} \qquad X = c \text{ or } X = -c.$$

Ex11. Solve the equation: $|2x - 3| = 11$

Use the absolute value definition to remove the absolute value bars

If $2x - 3 > 0$, then $2x - 3 = 11$

$x = 7$ (check in the original eq.)

If $2x - 3 < 0$, then $-(2x - 3) = 11$

$x = -4$ (check in the original eq.)

12 and − 18 satisfy the original eq., it is a solution.

The solutions set is $\{-4, 7\}$.

Ex12. Solve the following absolute value equations:

$$a)\quad |4x-2| = 0$$

$$|4x-2| = 0 \text{ if } 4x-2=0$$

which implies that $x = \frac{1}{2}$

$$\text{check: } \left|4\left(\frac{1}{2}\right)-2\right| = 0$$

The solutions set is $\left\{\frac{1}{2}\right\}$.

b) $|2-3x| = -17$

$$\underbrace{|2-3x|}_{\text{nonnegative}} = \underbrace{-17}_{\textit{negative}}$$

Which is **a contradiction**

Thus, the solution set is Ø, the empty set

c) $2|x+3|+4=34$

lets rewrite the equation as $|x+3|=15$

add -4 to both sides
divide both sides by 2

if $x+3>0$, then $x+3=15$

$\Rightarrow \quad x=12$ (check in the original eq.)

if $x+3<0$, then $-(x+3)=15$

$\Rightarrow \quad x=-18$

(check in the original eq.)

12 and -18 satisfy the original eq., they are both solutions.

The solution set is $\{ 12, -18 \}$

d) $|2-3x| = -2x$

$$\text{if } 2-3x > 0, \text{ then } 2-3x = -2x$$

$$\Rightarrow x = 2$$ (check in the original eq.)

2 does not satisfy the original eq., it is not solution.

discard x = 2

$$\text{if } 2-3x < 0, \text{ then } -(2-3x) = -2x$$

$$\Rightarrow x = \frac{2}{5}$$ (check in the original eq.)

2/5 does not satisfy the original eq., it is not a solution.

discard x = 2/5

The solution set is the empty set

e) $|x+2| = |x-1|$

$x + 2 = x - 1$

$\Rightarrow \; 2 = -1$

$or \;\; x + 2 = -(x-1)$

$\Rightarrow \; 2x = -1$

$\Rightarrow \; x = \dfrac{-1}{2}$ (check in the original eq.)

− 1 /5 satisfies the original eq., it is a solution.

The solution set is {−1/2 }