(1) **(11 points)** Find the value of each of the following limits:

3 pts (a) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x \to 3} (x+3) = 6.$

2 pts (b) $\lim_{x \to \infty} \frac{x^{100} - \frac{1}{x^2}}{e - x^{98}} = \lim_{x \to \infty} \frac{x^{100}}{-x^{98}} = -\lim_{x \to \infty} x^2 = -\infty$ (D.N.E)

2 pts (c) $\lim_{x \to -\infty} \frac{\pi x^2 - x^5}{23x - 3x^4} = \lim_{x \to -\infty} \frac{-x^5}{-3x^4} = \lim_{x \to -\infty} \frac{x}{3} = -\infty$, (D.N.E)

4 pts (d) $\lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x - 1} \cdot \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2}$

$= \lim_{x \to 1} \frac{(x-1)}{(x-1)\sqrt{x+3} + 2} = \frac{1}{4}$

(2) **(4 points)** Is the function $f(x) = \begin{cases} x + 4 & \text{if } x > -2 \\ 3x + 6 & \text{if } x \le -2 \end{cases}$ continuous everywhere? Explain your answer.

No it is not. $f(x)$ is not continous at $x = -2$. 1 pt

Proof: $f(-2) = 3(-2) + 6 = 0$

$$\left.\begin{array}{l} \lim_{x \to -2^+} f(x) = -2 + 4 = 2 \\ \lim_{x \to -2^-} f(x) = 3(-2) + 6 = 0 \end{array}\right\} \Rightarrow \lim_{x \to -2} f(x) \text{ D.N.E.}$$ 3 pts

(3) **(4 points)** Given the function $f(x) = 2x^2 - 3$, use the **derivative definition** to find $f'(x)$.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 - 3 - 2x^2 + 3}{h}$

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-3} \cancel{-2x^2} \cancel{+3}}{h} = \lim_{h \to 0} (4x + 2h)$

3 pts $= 4x.$ 1 pt

(4) **(3 points)** Given that $y = \dfrac{10x}{1+0.1x}$, if x were to increase without bound, what value would y approach?

Discuss $\lim\limits_{x \to \infty} y = \lim\limits_{x \to \infty} \dfrac{10x}{1+0.1x} = \dfrac{10}{0.1} = 100.$ 1 pt

2 pts

(5) **(5 points)** Given that $f(x) = \dfrac{2x+3}{x+4}$, then find the percentage rate of change of f when $x = 1$.

2.5 pts $\dfrac{dy}{dx} = \dfrac{2x+8-2x-3}{(x+4)^2} = \dfrac{5}{(x+4)^2}$

when $x=1$, the % rate of change of f is

2.5 pts $\left.\dfrac{\frac{df}{dx}}{f}\right|_{x=1} \times 100 = \dfrac{0.2}{1} \times 100 = 20\%$

(6) **(4 points)** If the total cost function is $c = \dfrac{(q^5+q)\sqrt{3q+1}}{q^4+1}$, then find the marginal cost when $q = 1$.

2 pts $C = \dfrac{q(q^4+1)\sqrt{3q+1}}{q^4+1} = q\sqrt{3q+1}$

2 pts $\Rightarrow \dfrac{dC}{dq} = \sqrt{3q+1} + \dfrac{3q}{2\sqrt{3q+1}} = \dfrac{9q+2}{2\sqrt{3q+1}} = \dfrac{11}{4}$ when $q=1$

(7) **(5 points)** If $p = -0.5q + 450$, then find the approximate revenue received from selling one additional unit of some product.

2.5 pts $r = p \cdot q = -0.5q^2 + 450q$

2.5 pts $\frac{dr}{dq} = -q + 450$

(8) **(5 points)** Given that $C = 7 + 0.6I - 0.25\sqrt{I}$ is a consumption function. Find the marginal propensity to save when $I = 16$.

2.5 pts $\frac{dC}{dI} = 0.6 - \frac{0.25}{2\sqrt{I}} = 0.6 - \frac{0.25}{2\sqrt{16}} = \frac{6}{10} - \frac{25}{800} = \frac{455}{800} \simeq 0.5688$

2.5 pts $\Rightarrow \frac{dS}{dI} = 1 - \frac{455}{800} = \frac{345}{800} = 0.43125$

i.e if a current income of \$16 billion increases by \$1 billion, the nation consumes $\simeq$ 56.7% & saves $\simeq$ 43.13% of that increase.

(9) **(4 points)** If $z = 2y^2 - 4y + 5$, $y = 6x - 5$, and $x = 2t$ find $\frac{dz}{dt}$ when $t = 1$.

1 pt each

$\frac{dz}{dt} = \frac{dz}{dy} \cdot \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= (4y-4)(6)(2)$

$= (28-4) \cdot 12$

$= 288$

$t = 1 \Rightarrow x = 2$

$\Rightarrow y = 7$