

4.5 EXPONENTIAL AND LOGARITHMIC EQUATIONS

Equations with variables in the exponent are Exponential Equations

Example 1

Solve the exponential equations;

a) $9^{2x} = 3^{3x+1}$

b) $\frac{e^x + 1}{3e^{-x} - e^x} = 1$

c) $(\sqrt{2})^{12x-8} = 4\left(\frac{1}{2}\right)^{5(x-1)}$

Solution

a) $9^{2x} = 3^{3x+1} \Leftrightarrow 3^{4x} = 3^{3x+1}$
 $\Rightarrow 3x+1 = 4x$
 $\Rightarrow x = 1$

b) $\frac{e^x + 1}{3e^{-x} - e^x} = 1 \Leftrightarrow e^{2x} + e^x = 3 - e^{2x}$
 $\Rightarrow 2e^{2x} + e^x - 3 = 0$
 $\Rightarrow (2e^{2x} + 3)(e^x - 1) = 0$
 $\Rightarrow e^x = -\frac{3}{2} \quad \text{or} \quad e^x = 1$
Since $e^x > 0$, $e^x \neq -\frac{3}{2}$
 $\Rightarrow e^x = 1 \Rightarrow x = 0$

c) $(\sqrt{2})^{12x-8} = 4\left(\frac{1}{2}\right)^{5(x-1)} \Leftrightarrow 2^{6x-4} = 2^{7-5x}$
 $\Rightarrow 6x-4 = 7-5x$
 $\Rightarrow x = 1$

Equations involving logarithms are called logarithmic equations.

Example 2

Solve the logarithmic equations;

a) $10^{\log \log(7x-12)} = \log 2 + \log(3-x)$

b) $\log_3 \log_3(x^2 + 2) = e^{\ln 1}$

c) $\ln(x+3) + \log_{\frac{1}{e}}(x-1) = \ln 3 - \ln(x-2)$

$$d) 10^{\log|2x-1|} = \log_3 6^{3\log_6 3}$$

Solution

Note: *Solutions to logarithmic equations MUST be checked*

$$a) \log(7x-12) = \log 2 + \log(3-x) \Leftrightarrow \log(7x-12) = \log[2(3-x)]$$

$$\Rightarrow 7x-12 = 6-2x$$

$$\Rightarrow x = 2$$

Check: if $x = 2$ $LHS = \log 2 = RHS$

Solution set is $\{2\}$

$$b) \log_3 \log_3(x^2 + 2) = 1 \Leftrightarrow \log_3(x^2 + 2) = 3$$

$$\Rightarrow x^2 + 2 = 3^3$$

$$\Rightarrow x = \pm 5$$

Check: if $x = \pm 5$ $LHS = 1 = RHS$

Solution set is $\{-5, 5\}$

$$c) \ln(x+3) + \log_{\frac{1}{e}}(x-1) = \ln 3 - \ln(x-2) \Leftrightarrow \ln(x+3) - \ln(x-1) = \ln\left(\frac{3}{x-2}\right)$$

$$\Rightarrow \frac{x+3}{x-1} = \frac{3}{x-2}$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

Check: if $x = -1$ $LHS = \log(-1) = \text{undef}$

if $x = 3$ $LHS = \log 3 = RHS$

Solution set is $\{3\}$

$$d) 10^{\log|2x-1|} = \log_3 6^{3\log_6 3} \Leftrightarrow |2x-1| = 3$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

Check: if $x = -1$ $LHS = 3 = RHS$

if $x = 2$ $LHS = 3 = RHS$

Solution set is $\{-1, 2\}$

Example 3

Find the values of a and b if $5^{(2+x)} = 3^{(2x-1)}$ and $x = \log_b a$

Solution

$$5^{(2+x)} = 3^{(2x-1)} \Leftrightarrow (2+x)\ln 5 = (2x-1)\ln 3$$

$$\Rightarrow 2\ln 5 + x\ln 5 = 2x\ln 3 - \ln 3$$

$$\Rightarrow x\ln 5 - 2x\ln 3 = -\ln 3 - 2\ln 5$$

$$\Rightarrow x = \frac{-\ln 3 - 2\ln 5}{\ln 5 - 2\ln 3}$$

$$\Rightarrow x = \frac{-\ln 75}{\ln \frac{5}{9}} = \log_{\frac{5}{9}} \frac{1}{75} = \log_{\frac{9}{5}} 75$$

$$\Rightarrow b = \frac{5}{9} \text{ and } a = \frac{1}{75} \text{ OR } \Rightarrow b = \frac{9}{5} \text{ and } a = 75$$

Example 4

The n th number in sequence is given by the formula $f(n) = 2^{n-1} + 3$. If the n th term is 131 find the value of n .

Solution

$$131 = 2^{n-1} + 3$$

$$\Rightarrow 2^{n-1} = 128$$

$$\Rightarrow 2^{n-1} = 2^7$$

$$\Rightarrow n = 8$$