

QUIZ # 4

- 1) a) Show that the equation $|x-1| + (y-3)^3 = 1$ defines a function.

$$(y-3)^3 = 1 - |x-1|$$

$$y-3 = \sqrt[3]{1 - |x-1|} \Rightarrow y = 3 + \sqrt[3]{1 - |x-1|}$$

1 solution y only \Rightarrow function

- 2) Let $f(x) = \lfloor 2x - 1 \rfloor + 2$,

a) Evaluate $f(-3.7) = \lfloor 2(-3.7) - 1 \rfloor + 2 = \lfloor -7.4 - 1 \rfloor + 2 = \lfloor -8.4 \rfloor + 2 = -9 + 2 = -7$

- b) Find the set of x such that $f(x) = 0$

$$\begin{array}{l} \lfloor 2x - 1 \rfloor + 2 = 0 \\ \lfloor 2x - 1 \rfloor = -2 \\ -2 \leq 2x - 1 < -2 + 1 \end{array} \quad \left| \quad \begin{array}{l} -2 \leq 2x - 1 < -1 \\ -1 \leq 2x < 0 \\ \boxed{-\frac{1}{2} \leq x < 0} \end{array} \right.$$

- 3) Find the domain of $f(x) = \frac{3}{\sqrt{x^2-1}}$

$$x^2 - 1 > 0$$

	-1	1	
$x+1$	-	0	+
$x-1$	-	0	+
x^2-1	+	0	+

Domain = $(-\infty, -1) \cup (1, \infty)$

- 4) A linear function has y -intercept at $(0, -2)$ and its x -intercept at $(-4, 0)$. Find the equation of the function. (you can use the graph)

y -int is $(0, -2)$

Slope = $m = \frac{0 - (-2)}{-4 - 0} = \frac{2}{-4} = -\frac{1}{2}$

$\Rightarrow \boxed{y = -\frac{1}{2}x - 2}$

5) If 5 is the maximum of $f(x) = -x^2 + 2mx + 1$. Find the values of m .

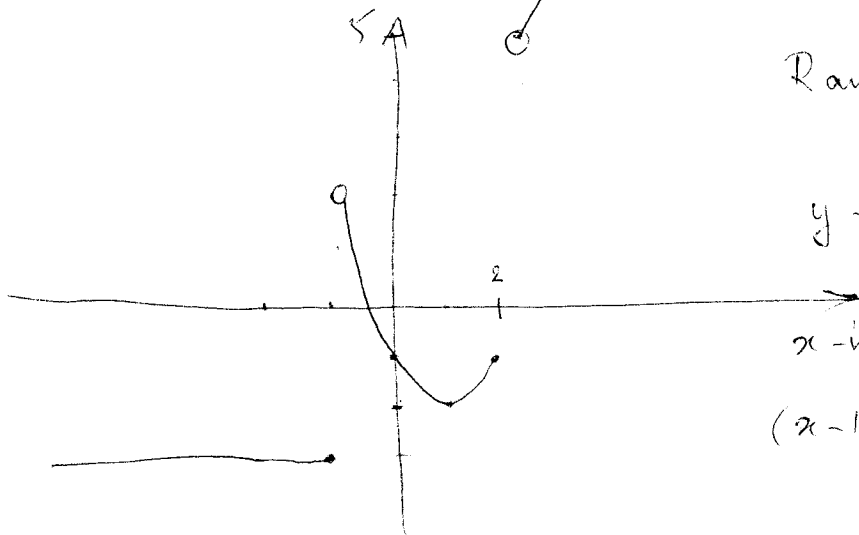
$$\begin{aligned} y &= -x^2 + 2mx + 1 \\ &= -(x^2 - 2mx) + 1 \\ &= -(x^2 - 2mx + m^2) + 1 + m^2 \\ &= -(x - m)^2 + 1 + m^2 \end{aligned}$$

$$k = 1 + m^2 = 5 \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$$

6) a) Draw the graph of $f(x) = \begin{cases} -3 & \text{if } x \leq -1 \\ (x-1)^2 - 2 & \text{if } -1 < x \leq 2 \\ 3x - 1 & \text{if } 2 < x \end{cases}$

b) Find its range, x-intercepts and y-intercept

	-2	-1	1	2	3
-3	-3	-3			
$(x-1)^2 - 2$		2	-2	-1	
$3x - 1$				5	8



$$\text{Range} = \{-3\} \cup [-1, 2) \cup (5, \infty)$$

$$\begin{aligned} \text{y-int } f(0) &= (0-1)^2 - 2 = -1 \\ &(0, -1) \end{aligned}$$

$$\begin{aligned} (x-1)^2 - 2 &= 0 \Rightarrow (x-1)^2 = 2 \\ x-1 &= \pm\sqrt{2} \\ x &= 1 \pm \sqrt{2} \end{aligned}$$

$$x = 1 - \sqrt{2} \in (-1, 2]$$

$$\boxed{(1 - \sqrt{2}, 0)}$$

QUIZ # 4 c

1) a) Determine if the equation $|y-1| + (x-3)^3 = 1$ defines a function or not.

$$|y-1| = 1 - (x-3)^3$$

$$y-1 = \pm (1 - (x-3)^3)$$

$$y = 1 \pm (1 - (x-3)^3) \rightarrow 2 \text{ sol}^n \Rightarrow \text{not function}$$

2) Let $f(x) = \lfloor 3x-1 \rfloor + 2$,

a) Evaluate $f(-3.4) = \lfloor 3(-3.4) - 1 \rfloor + 2 = \lfloor -10.2 - 1 \rfloor + 2$

$$= \lfloor -11.2 \rfloor + 2 = -12 + 2 = \boxed{-10}$$

b) Find the set of x such that $f(x) = 1$

$$\lfloor 3x-1 \rfloor + 2 = 1$$

$$\lfloor 3x-1 \rfloor = -1$$

$$-1 \leq 3x-1 < 0$$

$$0 \leq 3x < 1$$

$$\boxed{0 \leq x < \frac{1}{3}}$$

3) Find the domain of $f(x) = \sqrt{\frac{x+3}{x-1}}$

$$\frac{x+3}{x-1} \geq 0 \quad \& \quad x-1 \neq 0$$

$$D = (-\infty, -3] \cup (1, \infty)$$

x		-3		1	
$x+3$	-	0	+		+
$x-1$	-		-	0	+
		+	0	-	+

4) A line L passes through $(0, -2)$ and its perpendicular to line L_2 with equation $2x - 4y - 1 = 0$. Find the equation of the function. (you can use the graph)

$$m_2 = ? \quad 4y = 2x - 1 \Rightarrow y = \frac{2}{4}x - \frac{1}{4} = \frac{x}{2} - \frac{1}{4}$$

$$\Rightarrow m_2 = \frac{1}{2}$$

$$\Rightarrow m = -\frac{1}{m_2} = -2$$

$$y\text{-int } (0, -2)$$

$$\Rightarrow \boxed{y = -2x - 2}$$

- 5) If the minimum of $f(x) = -x^2 + 2mx + 1$ is reached at $x = 2$. What is the minimum value of the function

The minimum is reached at $x = h = 2$

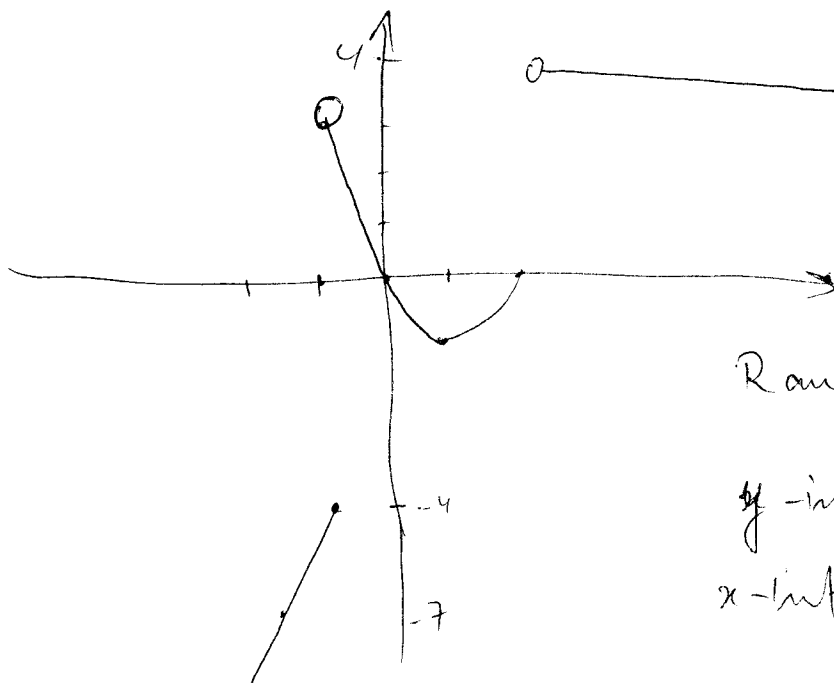
$$h = -\frac{b}{2a} = -\frac{2m}{-2} = m = 2$$

minimum is $k = f(h) = -(2)^2 + 2(2)(2) + 1 = -4 + 8 + 1 = 5$

- 6) a) Draw the graph of $f(x) = \begin{cases} 3x-1 & \text{if } x \leq -1 \\ (x-1)^2 - 1 & \text{if } -1 < x \leq 2 \\ 4 & \text{if } 2 < x \end{cases}$

- b) Find its range, x-intercepts and y-intercept

	-2	-1	1	2	3
$3x-1$	-7	-4			
$(x-1)^2 - 1$		3	-1	0	
4				4	4



Range: $(-\infty, -4] \cup [-1, 3) \cup \{4\}$

y-int $(0, 0)$

x-int $(0, 0), (2, 0)$