KFUPM
Math002
18 April 07

NAME:
ID:
Serial: Sec:

QUIZ # 3 (A)

1) Evaluate $\dfrac{\tan 84^\circ - \tan 54^\circ}{1+\cot 6^\circ \tan 54^\circ}$

$$\frac{\tan 84^\circ - \tan 54^\circ}{1 + \cot 6^\circ \tan 54^\circ} = \frac{\tan 84^\circ - \tan 54^\circ}{1 + \tan(90^\circ - 6^\circ)\tan(54^\circ)}$$

$$= \frac{\tan 84^\circ - \tan 54^\circ}{1 + \tan 84^\circ \tan 54^\circ} = \tan(84^\circ - 54) = \tan 30^\circ = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

2) Verify that $(\tan\theta - \sec\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

$$\frac{1-\sin\theta}{1+\sin\theta} = \frac{(1-\sin\theta)}{(1+\sin\theta)} \cdot \frac{(1-\sin\theta)}{(1-\sin\theta)} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = (\sec\theta - \tan\theta)^2$$

$$= (\tan\theta - \sec\theta)^2$$

3) Find the range of $y = 5\sin x + 12\cos x$ (you don't need to find α).

$$a = 5, \quad b = 12 \implies k = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169}$$
$$= 13$$

Hence $y = 5\sin x + 12\cos x$
$= 13\sin(x+\alpha)$

Hence the Range is $[-13, 13]$

4) Find the period of the $f(x) = 4(\cos^2 2x - \sin^2 2x)$

$= 4\left(\cos 2(2x)\right) = 4\cos(4x)$

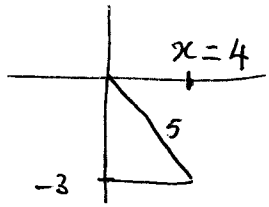
$P = \frac{2\pi}{b} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

5) Evaluate $\sin\left(2\sin^{-1}\left(-\frac{3}{5}\right)\right)$

$\theta = \sin^{-1}\left(-\frac{3}{5}\right) \in I \text{ or } IV$

$\Rightarrow \sin\theta = -\frac{3}{5} < 0 \Rightarrow III \text{ or } IV$

$\Rightarrow \theta \in IV$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{3}{5}\right)\cdot\left(\frac{4}{5}\right)$

$= \boxed{-\frac{24}{25}}$

6) Find the value of $\cos\frac{5\pi}{8}$

$\cos\frac{5\pi}{8} = \cos\left(\frac{\frac{5\pi}{4}}{2}\right) = -\sqrt{\frac{1+\cos\frac{5\pi}{4}}{2}}$

$II \rightarrow \cos\theta$

$III \rightarrow \cos -$

$\theta' = \frac{\pi}{4}$

$= -\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\left(\frac{2-\sqrt{2}}{2}\right)}{2}}$

$= -\sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}}$