3. The exact value of $\cos\frac{7\pi}{4}\tan\frac{4\pi}{3} + 3\sqrt{2}\cos\frac{7\pi}{6}$ is

IV

cos $\oplus$

$\theta' = \pi/4$

III

tan $\ominus$

$\theta' = \pi/3$

III

cos $-$

$\theta' = \pi/6$

(a) $3\sqrt{6}$

(b) $-2\sqrt{6}$

(c) $-\sqrt{6}$

(d) $\frac{\sqrt{6}-\sqrt{3}}{2}$

(e) $\frac{\sqrt{3}-\sqrt{2}}{2}$

$$\left(\cos\frac{\pi}{4}\right)\left(-\tan\frac{\pi}{3}\right) + 3\sqrt{2}\cdot\left(-\cos\frac{\pi}{6}\right)$$

$$-\frac{\sqrt{2}}{2}\left(\sqrt{3}\right) - 3\sqrt{2}\cdot\frac{\sqrt{3}}{2}$$

$$\frac{-\sqrt{6} - 3\sqrt{6}}{2} = \frac{-4\sqrt{6}}{2} = \boxed{-2\sqrt{6}}$$

4. If a car with a wheel of radius 40 cm is moving with a speed of 120 kilometers per hour, then the **angular** speed of the wheel of the car in **radian per minute** is

(a) 4000

(b) 3000

(c) 5000

(d) 500

(e) 50000

$v = 120\ km/h$

$= 120\ 10^3\ m/h$

$= \cancel{120}\cdot 10^3\ \frac{m}{\cancel{60}\,min}$

$= 2\cdot 10^3\ m/min$

$r = 40\ cm$

$= 0.4\ m$

$v = \underbrace{\omega}\cdot r$ angular speed in radian/time

$\omega = \frac{v}{r}$

$$= \frac{2\cdot 10^3}{0.4} = \frac{2\cdot 10^3}{4\cdot 10^{-1}} = \frac{2\cdot 10^4}{4}$$

$$= \frac{20\,000}{4} = 5000\ rad/min$$

7. Which one of the following statements is TRUE for all positive real numbers $x \neq 1$ and $y \neq 1$?

(a) $\log(x^3 + y^2) = 3\ln x + 2\ln y$ False $\log(u+v) \neq \log u + \log v$

True   (b) $\log_y \frac{1}{x} \cdot \log_x y = -1$ $\log_y\left(\frac{1}{x}\right) \cdot \frac{\log_y y}{\log_y x} = -\cancel{\log_y x} \cdot \frac{1}{\cancel{\log_y x}} = \boxed{-1}$

(c) $\sqrt{\log \frac{x}{y}} = \frac{\sqrt{\log x}}{\sqrt{\log y}}$ No $\log \frac{x}{y} \neq \frac{\log x}{\log y}$

(d) $\log(xy) = \log x \cdot \log y$ No $\log(xy) = \log x + \log y$

(e) $(\log x)^n = n \log x$ No , $\log(x^n) = n \log x$.

8. The domain, in interval notation, of the function $f(x) = \log_2(6x^2 + x - 2)$ is

(a) $\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

(b) $\left(-\infty, \frac{1}{2}\right)$

(c) $\left(-\frac{2}{3}, \infty\right)$

(d) $\left(-\frac{2}{3}, \frac{1}{2}\right)$

(e) $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$

$6x^2 + x - 2 > 0$

$(3x + 2)(2x - 1) > 0$

x		$-2/3$		$1/2$	
$6x^2+x-2$	$+$		$-$		$+$

Domain $= \left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$

11. The solution of the equation $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$ is equal to

(a) $\ln 6$

(b) $\ln 2$

(c) $\ln \sqrt{6}$

(d) $\ln \sqrt{3}$

(e) $\ln \sqrt{2}$

$e^x + e^{-x} = 3e^x - 3e^{-x}$

$e^x + 4e^{-x} - 3e^x = 0$

$-2e^x + 4e^{-x} = 0$

$e^x - 2e^{-x} = 0$ $\quad \div -2$

$\times\, e^x$

$(e^x)^2 - 2 = 0$

$(e^x)^2 = 2$

$e^x = \pm\sqrt{2}$ $\Rightarrow \boxed{x = \ln\sqrt{2}}$

rejected

12. $\ln(x\sqrt{z}) - \ln(x\sqrt{y}) + \dfrac{1}{2}\ln\sqrt[3]{\dfrac{y}{z}} =$

(a) $\dfrac{1}{6}\ln(yz)$

(b) $\dfrac{3}{2}\ln\left(\dfrac{z}{y}\right)$

(c) $\dfrac{4}{3}\ln\left(\dfrac{z}{y}\right)$

(d) $\dfrac{1}{3}\ln\left(\dfrac{z}{y}\right)$

(e) $\dfrac{2}{3}\ln\left(\dfrac{y}{z}\right)$

$= \ln \dfrac{\cancel{x}\sqrt{z}\cdot\left(\sqrt[3]{\frac{y}{z}}\right)^{1/2}}{\cancel{x}\sqrt{y}} =$

$= \ln \dfrac{\sqrt{z}}{\sqrt{y}} \cdot \left(\dfrac{y}{z}\right)^{1/6}$

$= \ln \left(\dfrac{z}{y}\right)^{1/2}\left(\dfrac{y}{z}\right)^{1/6} = \ln\left(\dfrac{z}{y}\right)^{1/2}\left(\dfrac{z}{y}\right)^{-1/6}$

$= \ln\left(\dfrac{z}{y}\right)^{1/2 - 1/6} = \ln\left(\dfrac{z}{y}\right)^{\frac{3}{6}-\frac{1}{6}}$

$= \ln\left(\dfrac{z}{y}\right)^{\frac{2}{6}} = \ln\left(\dfrac{z}{y}\right)^{1/3} = \boxed{\dfrac{1}{3}\ln\left(\dfrac{z}{y}\right)}$

15. The number of vertical asymptotes of the graph of

$y = 2\cot 2x\,, \quad -\pi < x < \pi$ is

(a) 3

(b) 4

(c) 5

(d) 2

(e) 1

$2x = n\pi$

$x = n\frac{\pi}{2}$

$-\pi < n\frac{\pi}{2} < \pi$

$-1 < \frac{n}{2} < 1$

$-2 < n < 2$

$n = -1, 0, 1$

3 V.A.

16. If the hypotenuse of a $30°$, $60°$, and $90°$ triangle is 10 cm, then the perimeter of the triangle is equal to

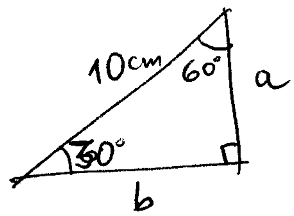
(a) $(15 + 5\sqrt{2})$ cm

(b) $(10 + 5\sqrt{2} + 5\sqrt{3})$ cm

(c) $(15 + 5\sqrt{3})$ cm

(d) $(10 + 10\sqrt{2})$ cm

(e) $(2 + 2\sqrt{10})$ cm



$\sin 30° = \frac{a}{10} \Rightarrow a = 10 \sin 30 = 10 \cdot \frac{1}{2} = 5$ cm

$\cos 30° = \frac{b}{10} \Rightarrow b = 10 \cos 30° = 10\,\frac{\sqrt{3}}{2} = 5\sqrt{3}$

Perimeter $= 10 + a + b$

$= 10 + 5 + 5\sqrt{3} = \boxed{15 + 5\sqrt{3}}$

19. The graph of the function $f(x) = -2\sin \pi x + 1$ on the interval $[0, 3]$ intersects the x-axis at

$P = \frac{2\pi}{\pi} = 2 \Rightarrow [0,3]$ one & half period

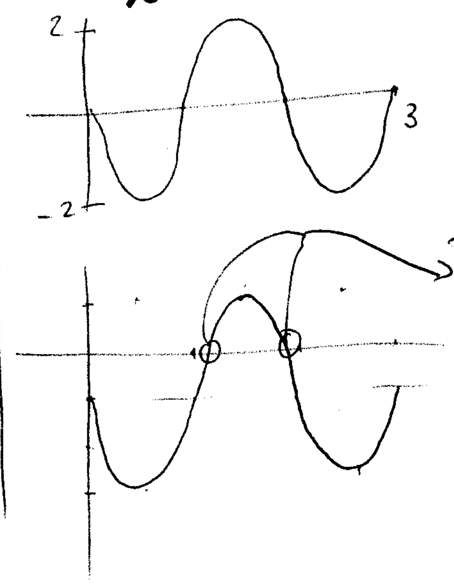
(a) five points

(b) four points

(c) one point

(d) three points

(e) $\boxed{\text{two points}}$



$\boxed{2 \; x\text{-intersepts}}$

20. Which one of the following statements is TRUE about the function $f(x) = 2^{-x} + 2^{x}$?

means the x-axis is an asymptote for f.

(a) the graph of f is asymptotic to the x-axis No

(b) f decreases on $(0, \infty)$ No

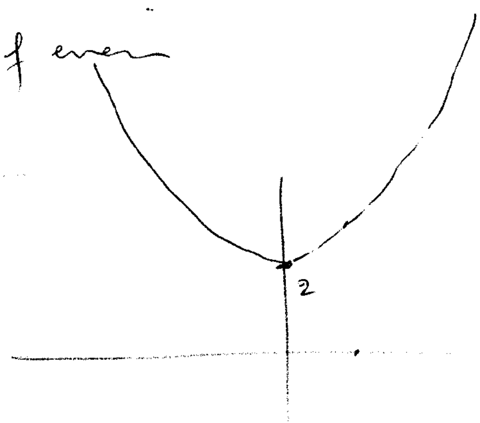
(c) f increases on $(-\infty, 0)$ No

(d) the maximum value of f is 2 No the min = 2

(e) the graph of f goes up to its far left and up to its far right $\boxed{\text{Yes}}$

$f(x) = 2^{-x} + 2^{x} = f(-x) \rightarrow f$ even

x	0	1	2
	2	$2+\frac{1}{2}$	$4+\frac{1}{4}$



23. If f is an exponential function of the form $f(x) = a^x$, and $f(-2) = 9$, then $f(-1) =$

(a) -3

(b) 5

(c) 3

(d) -4

(e) -7

$a^{-2} = 9$

$(a^{-1})^2 = 3^2$

$\frac{1}{a} = 3$

$a = \frac{1}{3}$

$f(-1) = \left(\frac{1}{3}\right)^{-1} = \boxed{3}$

24. The graph of $f(x) = \left|\log_{\frac{1}{2}}(2x+5)\right|$ is decreasing on the interval

(a) $\left(-\frac{5}{2}, -2\right)$

(b) $\left(-\infty, -\frac{5}{2}\right)$

(c) $\left(-\frac{5}{2}, \infty\right)$

(d) $(-2, \infty)$

(e) $\left(-\frac{5}{2}, 0\right) \cup (0, \infty)$

$y = 2x + 5$ is increasing

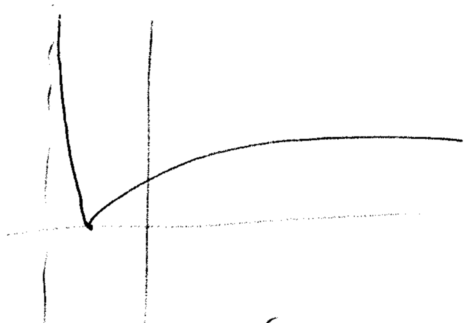
$\log_{1/2} x$ is decreasing

$\Rightarrow \log_{1/2}(2x+5)$ is decreasing.

$2x + 5 > 0 \Rightarrow x > -5/2$

$2x + 5 = 1 \Rightarrow 2x = -4$

$x = -2$



f is decreasing on $(-5/2, -2)$