King Fahd University of Petroleum and Minerals
Prep-Year Math Program

CODE 002 | **Prep-Year Math II** | **CODE 002**

Prep-Year Math II
MIDTERM EXAM
Semester I, Term 061
Tuesday, November 14, 2006
Net Time Allowed: 120 minutes

Name: ______________________________

ID: ____________________ Sec: ____________________.

Check that this exam has 25 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The number of **intersection** points of the graphs of $y = \sin 2\pi x$ and $y = \cos 2\pi x$, over the interval $0 \le x \le \frac{3}{4}$, is

$P = \frac{2\pi}{2\pi} = 1$ Cycle : $[0, P] = [0, 1]$

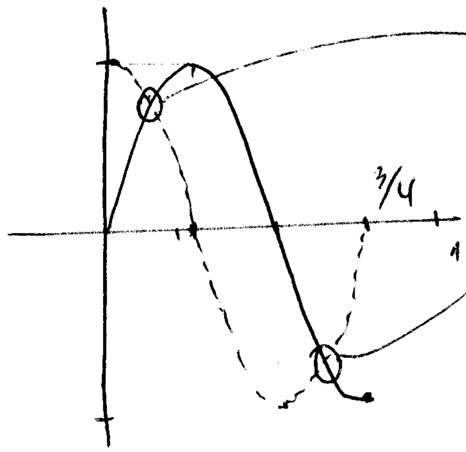
(a) 2

(b) 1

(c) 3

(d) 5

(e) 4



2 points of intersection.

2. If $\dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} = \dfrac{1}{2}$, then $x =$

(a) $\log \sqrt[3]{2}$

(b) $\dfrac{3}{\log 2}$

(c) $\log 3$

(d) $\log \sqrt{3}$

(e) $\log 2$

$2(10^x - 10^{-x}) = 10^x + 10^{-x}$

$2(10^x) - 10^x - 2 \cdot 10^{-x} - 10^{-x} = 0$

$10^x - 3 \cdot 10^{-x} = 0$

$(10^x)^2 - 3 = 0 \qquad \times 10^x$

$(10^{2x}) = 3$

$2x = \log 3$

$x = \frac{1}{2} \log 3 = \boxed{\log \sqrt{3}}$

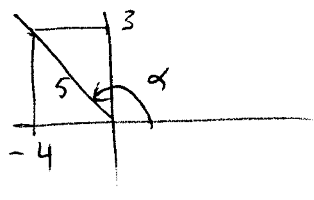
5. If $\sin\alpha = \frac{3}{5}$, α lies in second quadrant, and $\cos\beta = -\frac{5}{13}$, β lies in third quadrant, then $\sin\left(\frac{\pi}{2} - \alpha + \beta\right) =$

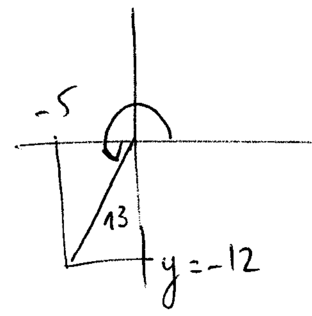
$= \sin\left(\frac{\pi}{2} - (\alpha - \beta)\right) = \cos(\alpha - \beta)$

$= \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$= \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right)$

$= \frac{20 - 36}{65} = \boxed{\frac{-16}{65}}$





(a) $\frac{48}{65}$

(b) $-\frac{36}{65}$

(c) $\frac{12}{65}$

(d) $-\frac{16}{65}$

(e) $-\frac{48}{65}$

6. If for the function $f(x) = -3\sin(\pi x - 2) + 5$, A is its amplitude, P is its period, M is its maximum value and m is its minimum vlaue, then $\frac{A+P}{M+m} =$

$A = \text{Amp} = |-3| = 3$

$P = \frac{2\pi}{\pi} = 2$

$M = |a| + d = 3 + 5 = 8$

$m = -|a| + d = -3 + 5 = 2$

$\frac{A+P}{M+m} = \frac{3+2}{8+2} = 1/2$

(a) 1

(b) 2

(c) 4

(d) $\frac{1}{2}$

(e) $\frac{1}{4}$

9. If f^{-1} is the inverse of the function $f(x) = 2^{-x+1} - 3$, then $f^{-1}(x) =$

(a) $-1 + \log_2(3 - x)$

(b) $3 + \log_2(x - 1)$

(c) $3 + \log_2(x + 3)$

(d) $1 - \log_2(x + 3)$

(e) $-1 + \log_2(x + 3)$

$$y = 2^{-x+1} - 3$$
$$x = 2^{-y+1} - 3$$
$$x + 3 = 2^{-(y-1)}$$
$$-(y-1) = \log_2(x+3)$$
$$1 - y = \log_2(x+3)$$
$$-y = -1 + \log_2(x+3)$$
$$\boxed{y = 1 - \log_2(x+3)}$$
$$\| \\ f^{-1}(x)$$

10. The line $y = 3$ intersects the graph of $y = -2\csc\dfrac{x}{3}$ over the interval $\left(-\dfrac{3\pi}{2}, 6\pi\right)$ at

(a) two points

(b) three points

(c) four points

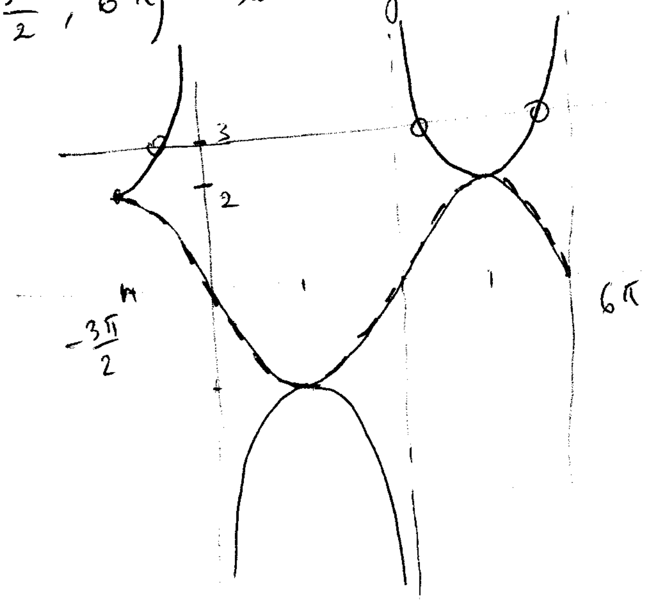
(d) no point

(e) one point

$$P = \frac{2\pi}{b} = \frac{2\pi}{(1/3)} = 6\pi$$

Cycle $= [0, 6\pi]$

$\left[-\frac{3\pi}{2}, 6\pi\right)$ is 1 cycle & quarter.



3 points

13. If $x = \frac{3}{2}\sin\theta$, where $0 < \theta < \frac{\pi}{2}$, then the expression $\frac{12x^2}{(9-4x^2)^{3/2}}$ simplifies to

$\left(\frac{3}{2}\sin\theta\right)^2 = \frac{9}{4}\sin^2\theta$

$\frac{12x^2}{(9-4x^2)^{3/2}} = \frac{3 \cdot 9\sin^2\theta}{(9-9\sin^2\theta)^{3/2}} = \frac{3 \cdot 9\sin^2\theta}{9^{3/2}(1-\sin^2\theta)^{3/2}}$

$= \frac{3\sin^2\theta}{3(\cos^2\theta)^{3/2}} = \frac{3}{3}\frac{\sin^2\theta}{\cos^2\theta\cos\theta} = \frac{3\tan^2\theta \cdot \sec\theta}{3}$

$= \boxed{\tan^2\theta \cdot \sec\theta}$

(a) $\cot^2\theta\sin\theta$

(b) $\tan^2\theta\sec\theta$

(c) $\cot^2\theta\sec\theta$

(d) $\tan^2\theta\cos\theta$

(e) $\tan^2\theta\sin\theta$

14. The **domain** of $f(x) = \log_{x-1} x$ is

$b = x - 1 > 0 \Rightarrow x > 1$

& $b \neq 1$ $\quad x \neq 2$

& $x > 0$

So The domain is $\boxed{(1,2)\cup(2,\infty)}$

(a) $(0,1)\cup(1,\infty)$

(b) $(1,2)\cup(2,\infty)$

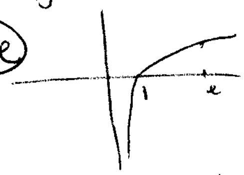
(c) $(1,\infty)$

(d) $(1,2)$

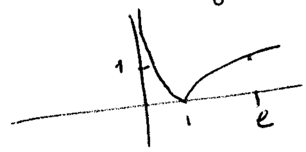
(e) $(2,\infty)$

17. Which one of the following statements is **TRUE** about the function $f(x) = -2|\ln x|$ and its graph?

$y = \ln(x)$



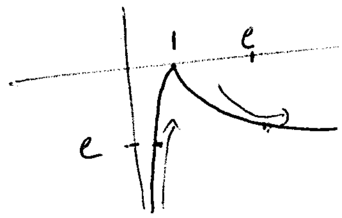
$y = |\ln x|$



(a) increases on $(0, 1)$ True

(b) increases on $(1, \infty)$ F

$y = -2|\ln x|$



(c) decreases on $\left(\frac{1}{2}, 2\right)$ F

(d) f has no maximum value F max = 0

(e) $x = 2$ is a vertical asymptote F, $x = 0$ V.A

18. If L is the distance between the two points $P_1(\cos\theta, \sin\theta)$ and $P_2(\cos 2\theta, \sin 2\theta)$, then $L^2 =$

x_1 y_1 x_2 y_2

(a) $2 - 2\cos\theta$

(b) $3 - \cos 3\theta$

(c) $2 + 2\sin\theta$

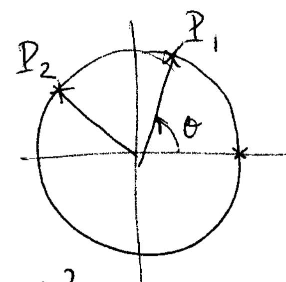
(d) $2 + 2\cos 3\theta$

(e) $3 - \cos\theta$

$\cancel{L^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\cancel{= (\cos 2\theta - \cos\theta)^2 + (\sin 2\theta - \sin\theta)^2}$

$\cancel{= \cos^2 2\theta + \cos^2\theta - 2\cos 2\theta \cos\theta + \sin^2\theta (2\cos\theta - 1)^2}$



$L^2 = (\text{dist}(P_2, P_1))^2$

$= (\text{dist}(P_1, (1,0)))^2$

$= (\cos\theta - 1)^2 + (\sin\theta - 0)^2$

$= \cos^2\theta - 2\cos\theta + 1 + \sin^2\theta$

$= \boxed{2 - 2\cos\theta}$

$= 2(1 - \cos\theta)$

21. If the terminal side of an angle θ lies on the line $3x + 4y = 0$, where $x > 0$, then the value of $\cot\theta + \cos\theta$ is

(a) $\frac{32}{15}$

(b) $-\frac{1}{5}$

(c) $-\frac{8}{15}$

(d) $-\frac{32}{15}$

(e) $\frac{1}{15}$

choose $x = 4 \Rightarrow 3(4) + 4y = 0$

$\Rightarrow 4y = -12 \Rightarrow y = -3$

$r = \sqrt{x^2 + y^2} = 5$

$\cot\theta = \frac{x}{y} = \frac{4}{-3} = -\frac{4}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5}$

$\cot\theta + \cos\theta = -\frac{4}{3} + \frac{4}{5} = \frac{-4\cdot 5 + 4\cdot 3}{15}$

$= \boxed{\frac{-8}{15}}$

22. If the points $(-1, 3)$ and $(2, 81)$ lie on the graph of the exponential function $f(x) = b^{x+c}$, then $b + c =$

(a) -1

$\boxed{\text{(b) } 5}$

(c) 3

(d) 4

(e) -2

$3 = b^{-1+c}$

$81 = b^{2+c}$

$3^4 = b^{2+c}$

$(b^{-1+c})^4 = b^{2+c}$

$b^{-4+4c} = b^{2+c}$

$\Rightarrow -4 + 4c = 2 + c$

$-6 = -3c$

$\boxed{c = 2}$

$\Rightarrow 3 = b^{-1+2} = b$

$\Rightarrow b = 3$

$b + c = 3 + 2 = \boxed{5}$

25. The range R and the period P of the function $y = -\left|3\sin\frac{x}{2}\right|$ are given by [Hint: Sketch]

(a) $R = [-3, 3]$, $P = 2\pi$

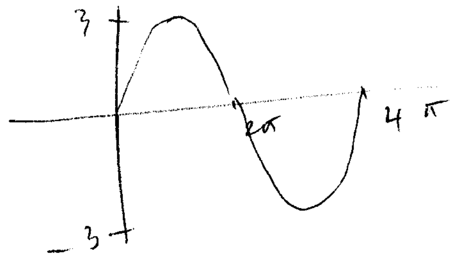
(b) $R = [-3, 0]$, $P = 4\pi$

(c) $R = [-3, 0]$, $P = 2\pi$

(d) $R = [-3, 0]$, $P = \frac{\pi}{2}$

(e) $R = [-3, 0]$, $P = \pi$

$y = 3\sin\frac{x}{2}$ $\quad P = \frac{2\pi}{\frac{1}{2}} = 4\pi$



$y = -3\sin\left(\frac{x}{2}\right)$

$P = 2\pi$

$R = [-3, 0]$ $\Leftarrow$