King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Code: Master

Prep-Year Math II
FINAL EXAM
Semester I, 2003-04
Saturday, January 10, 2004
Net Time Allowed: 130 minutes

Code: Master

Student's Name: ....Key..............Dr....Al.A.l.M./.H.../...

ID #: .... **Section #:** ....

Important Instructions:

1. All types of CALCULATORS, PAGERS, and OR MOBILES ARE NOT ALLOWED to be with you during the examination.

2. Use an HB $2\frac{1}{2}$ pencil.

3. Use a good eraser. Do not use the eraser attached to the pencil.

4. Write your name, ID number and Mathematics Section on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Math Section number, be sure that bubbles match with the number that you write.

6. The test Code Number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.

9. Check that the exam paper has 25 questions.

1. The coordinates of the focus of the parabola $8x - y^2 + 4y - 12 = 0$ are

(a) (3, 2)
(b) (−3, 2)
(c) (3, −2)
(d) (3, 4)
(e) (2, 4)

$8x - y^2 + 4y - 12 = 0$

$y^2 - 4y = 8x - 12$

$y^2 - 4y + 4 = 8x - 12 + 4 = 8x - 8$

$(y-2)^2 = 8(x-1)$

$(h,k) = (1,2)$ $4p = 8 \Rightarrow p = 2$

parabola opening to the right

$\Rightarrow F = (h+p, k) = (3,2)$

2. If $y = -3\sin x + 3\sqrt{3}\cos x$ is rewritten as $y = k\sin(x+\alpha)$, $0 \le \alpha < 360°$, then the values of k and α are given by

(a) $k = 6$, $\alpha = 120°$
(b) $k = 18$, $\alpha = 210°$
(c) $k = 6$, $\alpha = 240°$
(d) $k = 9$, $\alpha = 300°$
(e) $k = 6$, $\alpha = 330°$

$a = -3$, $b = 3\sqrt{3}$

$k = \sqrt{a^2+b^2} = \sqrt{9 + 9\cdot(3)} = 3\sqrt{4} = \boxed{6}$

$\cos\alpha = \frac{-3}{6} = -\frac{1}{2}$, $\sin\alpha = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha' = 60°$ & $\alpha \in Q\,II$

$\Rightarrow \alpha = 180 - \alpha' = 180 - 60 = \boxed{120°}$

3. If (a, b) is the solution of the system

$$\begin{cases} 6x + 7y = -4 \\ 2x + 5y = 4 \end{cases}$$

then $a + b$ is equal to

(a) -1

(b) 3

(c) -2

(d) $\frac{3}{2}$

(e) $-\frac{1}{2}$

4. If $3^{2+x} = 6^x$, then $x =$

(a) $\log_2 9$

(b) $\log_3 2$

(c) $\log_2 3$

(d) $\log_9 6$

(e) $\log_3 \frac{3}{2}$

$3^{2+x} = 6^x = (2 \cdot 3)^x = 2^x \cdot 3^x$

$\frac{3^{2+x}}{3^x} = 2^x$

$3^2 = 2^x \implies x = \log_2 3^2 = \boxed{\log_2 9}$

5. Given the vectors $\mathbf{v} = \langle 6, 7\rangle$ and $\mathbf{w} = \langle 3, 4\rangle$, the value of $\text{Proj}_{\mathbf{w}}\mathbf{v}$ is equal to

(a) -2

$Proj_w v =$

(b) $\frac{13}{5}$

(c) $-\frac{21}{5}$

(d) $\frac{41}{5}$

(e) -6

6. Let A, B, and C be any $n \times n$ matrices and let O be the $n \times n$ zero matrix. Which one of the following statements is **FALSE**?

(a) $AB - BA = O$

(b) $A(BC) = (AB)C$ True

(c) $OA = AO = O$ True

(d) $A(B + C) = AB + AC$ True

(e) $A^2 + B^2$ is an $n \times n$ matrix True

False because $AB \neq BA$ in general.

7. If $A = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 2 \\ m & 1 & -4 \\ \frac{1}{2} & 0 & k-\frac{1}{2} \end{bmatrix}$ are inverses of each other, then

(a) $k = 0$ and $m = 2$

(b) $k = -1$ and $m = 0$

(c) $k = 1$ and $m = 2$

(d) $k = 2$ and $m = 0$

(e) $k = 3$ and $m = -1$

$$A \cdot B = I_3 = \begin{pmatrix} 1 & 0 & 0 \\ \boxed{0} & 1 & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix}$$

elt in 2nd row, 1st col of AB is

$2\cdot(-1) + 1\,m + 0\cdot\frac{1}{2} = -2 + m = 0$

$\Rightarrow \boxed{m = 2}$

elt in 3rd row & 3rd col of AB is

$2\cdot 1 + 0(-4) + 2\left(k - \frac{1}{2}\right) = 1$

$2 + 2k - 1 = 1 \Rightarrow 2k = 2 - 2 = 0$

$\boxed{k = 0}$

8. The equation of the **directrix** of the parabola that has vertex $(-4, 1)$, has its axis of symmetry parallel to the y axis, and passes through the point $(-2, 2)$ is given by

(a) $y = 0$

(b) $y = -1$

(c) $x = -2$

(d) $y = -2$

(e) $x = 2$

axis of sym // to y-axis $\Rightarrow$ vertical.

$\Rightarrow (x-h)^2 = 4p(y-k)$

$(x+4)^2 = 4p(y-1)$

passes through $(-2, 2)$

$(-2+4)^2 = 4p(+2-1) = p \Rightarrow p = \frac{4}{4} = 1$

Directrix is hori $\Rightarrow y = k - p$

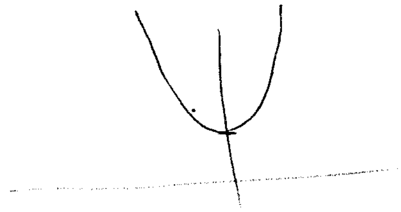
$\boxed{y = 1 - 1 = 0}$

9. Which one of the following statements is **FALSE** about the graph of $f(x) = \frac{e^{x} + e^{-x}}{2}$?

$f(x)$ is even

$f(0) = 1$

$x \to \infty \quad f(x) \longrightarrow \infty$



(a) the graph is increasing on $(-\infty, \infty)$

(b) $(0, 1)$ is the lowest point on the graph

(c) the graph is symmetric about the y-axis

(d) the graph has only one y-intercept

(e) the graph has no x-intercept

All true except a/

10. If A and B are 2×2 matrices with $|A| = 5$, $|B| = 2$, then $|3AB| + 2|B^{-1}|$ is equal to

(a) 91

(b) 63

(c) 84

(d) 103

(e) 37

$$|3AB| + 2|B^{-1}|$$

$$= 3^2 |A \cdot B| + 2 \frac{1}{|B|} = 9 |A| \cdot |B| + 2 \frac{1}{2}$$

$$= 9 \cdot 5 \cdot 2 + 1$$

$$= \boxed{91}$$

11. The expression

$$\log\left(y^{3/2} z^2\right) - 3\log(x\sqrt{y}) + 2\log\frac{x}{z}$$

simplifies to

(a) $-\log x$

(b) $\log\frac{y^2}{x}$

(c) $\frac{1}{2}\log(xy)$

(d) $-\log\left(\frac{y^3}{xz}\right)$

(e) $\log\left(\frac{y}{xz^3}\right)$

12. The number of solutions in the solution set of the equation

$$2\sin^2 x - \sin 2x = 0, \quad \text{where } 0 \le x < \frac{5\pi}{2},$$

is equal to

$2\sin^2 x - 2\sin x\cos x = 0$

(a) 6

$2\sin x(\sin x - \cos x) = 0$

(b) 5

$\sin x = 0 \rightarrow x = n\pi : 0, \pi, 2\pi \in \left[0, \frac{5\pi}{2}\right)$

(c) 4

(d) 3

$\sin x - \cos x = 0$

(e) 2

$\sin x = \cos x$

$\sin^2 x = \cos^2 x = 1 - \sin^2 x$

$2\sin^2 x = 1$

$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$

Check

$\frac{3\pi}{4}, \frac{7\pi}{4}$ rejected

$SS = \left\{0, \pi, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}\right\}$

6 sol's

13. The exact value of $\cos 157.5^\circ$ is equal to

(a) $-\dfrac{\sqrt{2+\sqrt{2}}}{2}$

(b) $-\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(c) $\dfrac{\sqrt{2+\sqrt{2}}}{2}$

(d) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(e) $2+\sqrt{2}$

$$\cos(157.5^\circ) = \cos\left(\frac{315^\circ}{2}\right)$$

Q II → cos ⊖ Q IV → cos ⊕

$$= -\sqrt{\frac{1+\cos(315^\circ)}{2}} = -\sqrt{\frac{1+\cos 45^\circ}{2}} = -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{-\frac{\sqrt{2+\sqrt{2}}}{2}}$$

14. One of the foci of the ellipse $\dfrac{(x-3)^2}{16}+\dfrac{(y+2)^2}{25}=1$ is at

(a) $(3,-5)$

(b) $(1,-2)$

(c) $(3,-9)$

(d) $(7,-2)$

(e) $(3,-1)$

$a^2 = 25$ $b^2 = 16$

$a = 5$ $b = 4$

$(h,k) = (3,-2)$

ellipse vertical.

$F = (h, k \pm c)$

$c = \sqrt{a^2-b^2} = \sqrt{9} = 3$

$F = (3, -2-3) = (3,-5)$ ✓

or $F(3, -2+3) = (3,1)$

15. Given the vectors $\mathbf{u} = 10\mathbf{i} + 8\mathbf{j}$ and $\mathbf{v} = 12\mathbf{i} - 6\mathbf{j}$. The direction angle of the vector $\frac{1}{2}\mathbf{u} - \frac{1}{6}\mathbf{v}$, in radians, is equal to

(a) $\frac{7\pi}{4}$

(b) $\frac{5\pi}{4}$

(c) $\frac{3\pi}{4}$

(d) $\frac{\pi}{4}$

(e) $\frac{5\pi}{3}$

$\frac{1}{2}u - \frac{1}{6}v = 5i + 4j - (2i - j)$

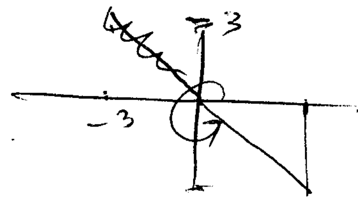
$= 3i - 3j = \langle 3, -3 \rangle$

$\cos\theta = \frac{a}{\| \|} = \frac{3}{\sqrt{18}} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{b}{\| \|} = -\frac{1}{\sqrt{2}}$

θ in Q IV, $\theta' = \frac{\pi}{4}$

$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$



16. The solution set of the equation

$$\log(-6 - 7x) = \log(3 + 2x) + \log(4 + 3x)$$

contains

(a) one negative integer only

(b) two negative integers

(c) one positive integer only

(d) one negative and one positive integers

(e) no real numbers

17. The equation of the asymptote with **positive slope** of the hyperbola $4x^2 - 9y^2 - 16x + 54y - 29 = 0$ is

(a) $2x - 3y + 5 = 0$

(b) $4x - 9y + 19 = 0$

(c) $2x - 6y + 10 = 0$

(d) $4x - 3y + 1 = 0$

(e) $3x - 4y + 6 = 0$

$4(x^2 - 4x) - 9(y^2 - 6y) = 29$

$4(x^2 - 4x + 4) - 9(y^2 - 6y + 9) = 29 + 16 - 81$

$4(x-2)^2 - 9(y-3)^2 = -36$

$\frac{(y-3)^2}{4} - \frac{(x-2)^2}{9} = 1$

$(h,k) = (2,3)$, $a^2 = 4$ $b^2 = 9$, $c^2 = 5$

Asymp. $(y-3) = \pm\frac{a}{b}(x-2) = \pm\frac{2}{3}(x-2)$

$3(y-3) = 2(x-2) \iff 3y - 9 = 2x - 4$

$\boxed{2x - 3y + 5 = 0}$

18. If $0 < x < \pi$, then $\csc x - \cot x$ simplifies to

(a) $\tan\frac{x}{2}$

(b) $\csc\frac{x}{2}$

(c) $\cos\frac{x}{2}$

(d) $\sin\frac{x}{2}$

(e) $\sec\frac{x}{2}$

$\csc x - \cot x$

$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \frac{1 - \cos x}{\sin x} = \tan\frac{x}{2}$

19. Let x be a nonzero real number and $A = \begin{bmatrix} x & x^4 & x \\ 0 & x^3 & 1 \\ x^5 & 2x^8 & -3x^5 \end{bmatrix}$. Then the determinant of A is equal to

Following the first row

(a) $-5x^9$

(b) $-3x^5$

(c) $-3x^9$

(d) $6(x^5 - x^3)$

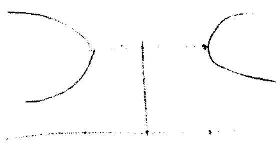
(e) $4x^5$

$$|A| = x \begin{vmatrix} x^3 & 1 \\ 2x^8 & -3x^5 \end{vmatrix} + x^5 \begin{vmatrix} x^4 & x \\ x^3 & 1 \end{vmatrix}$$

$$= x\left(-3x^8 - 2x^8\right) + x^5\underbrace{(x^4 - x^4)}_{0}$$

$$= \boxed{-5x^9}$$

20. The equation in standard form of the hyperbola with vertices $(2,3)$ and $(-2,3)$, and eccentricity $\frac{5}{2}$ is given by



(a) $\frac{x^2}{4} - \frac{(y-3)^2}{21} = 1$

(b) $\frac{(x-2)^2}{4} - \frac{(y-6)}{21} = 1$

(c) $\frac{x^2}{16} - \frac{(y-3)^2}{25} = 1$

(d) $\frac{(x-4)^2}{4} - \frac{(y-3)^2}{21} = 1$

(e) $\frac{x^2}{4} - \frac{(y-3)^2}{36} = 1$

$C = (h,k) = (0,3)$

form : $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$2a = |2-(-2)| = 4 \Rightarrow \boxed{a = 2}$

$e = \frac{c}{a} = \frac{5}{2} \Rightarrow \frac{c}{2} = 5/2$

$\Rightarrow c = 5 \Rightarrow b = \sqrt{c^2 - a^2}$
$= \sqrt{25-4} = \sqrt{21}$

$$\Rightarrow \boxed{\frac{(x-0)^2}{4} - \frac{(y-3)^2}{21} = 1}$$

21. If a wheel with a **diameter** of 16 centimeters is rotating at 10 radians per minute, then the linear speed of a point on the edge of the wheel is equal to

(a) $\frac{4}{3}$ centimeters/second

(b) $\frac{5}{3}$ centimeters/second

(c) $\frac{3}{2}$ centimeters/second

(d) $\frac{2}{3}$ centimeters/second

(e) $\frac{3}{10}$ centimeters/second

$v = w \cdot r$
$= 10 \cdot 8$ cm/min
$= \frac{80}{60}$ cm/sec
$= \frac{4}{3}$ cm/sec

22. $\sin 675° + \cos(-405°) + \tan\frac{8\pi}{3} =$ $\sin(-45°) + \cos(405) + \tan\left(2\pi + \frac{2\pi}{3}\right)$

(a) $-\sqrt{3}$

(b) $\sqrt{2} + \sqrt{3}$

(c) $3\sqrt{2} - \sqrt{3}$

(d) $-\sqrt{2} + 2\sqrt{3}$

(e) $2\sqrt{3}$

$= -\sin 45° + \cos 45° + \tan\frac{2\pi}{3}$
$= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \left(-\tan\frac{\pi}{3}\right)$
$= \boxed{-\sqrt{3}}$

$720 - 675 = 45$

$405 - 360 = 45$

π → $\tan \ominus$

23. A linear system written in a matrix form

$$\begin{bmatrix} -2 & 6 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

is

$$\left[\begin{array}{cc|c} -2 & 6 & 8 \\ -1 & 3 & 4 \end{array}\right]$$

$$\xrightarrow{\frac{R_1}{-2}} \left[\begin{array}{cc|c} 1 & -3 & -4 \\ -1 & 3 & 4 \end{array}\right]$$

$$\xrightarrow{R_1+R_2} \left[\begin{array}{cc|c} 1 & -3 & -4 \\ 0 & 0 & 0 \end{array}\right]$$

The system is dependent

(a) dependent
(b) inconsistent
(c) independent
(d) consistent with only two solutions
(e) consistent with only three solutions

$2a$?

24. The length of the major axis of the ellipse with center $(-4, 1)$, minor axis parallel to the y-axis and of length 8, and passing through $(0, 3)$ is equal to

$2b = 8 \Rightarrow b = 4$

minor axis vertical $\Rightarrow$ major axis is hor

$$\Rightarrow \frac{(x+4)^2}{a^2} + \frac{(y-1)^2}{16} = 1$$

$(0,3)$ pt of ellipse

$$\frac{4^2}{a^2} + \frac{(3-1)^2}{16} = 1$$

$$\frac{16}{a^2} + \frac{4}{16} = 1$$

$$\frac{16}{a^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow a^2 = \frac{4 \cdot 16}{3}$$

(a) $\frac{16\sqrt{3}}{3}$
(b) $32\sqrt{3}$
(c) $\frac{9}{2}\sqrt{3}$
(d) $\frac{3\sqrt{3}}{2}$
(e) $\frac{25\sqrt{3}}{3}$

25. Let $A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and I be the 3×3 identity matrix. Then the element in the second row and third column of the matrix $3A^2 - 2I$ is

$3A^2 - 2I = 3(b_{ij}) - 2(c_{ij})$

(a) 6

$b_{23} = (0 \ -1 \ 1) \cdot \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = -1 + 3 = 2$

(b) 4

(c) 7

$c_{23} = 0$

(d) 5

The element is then

(e) 10

$3(2) - 2(0) = 6$

26. The graph of the function

$$y = 3\cos 3(x - \pi), \quad \pi \le x \le \frac{5\pi}{3},$$

lies above the x-axis on the interval

$\hookrightarrow$ this function is $y = 3\cos 3x$ shifted by π to the right

(a) $\left[\pi, \frac{7\pi}{6}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right]$

(b) $\left(\frac{7\pi}{6}, \frac{5\pi}{3}\right)$

(c) $\left(\frac{7\pi}{6}, \frac{4\pi}{3}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$

(d) $\left[\pi, \frac{3\pi}{2}\right)$

(e) $\left[\pi, \frac{7\pi}{6}\right) \cup \left[\frac{4\pi}{3}, \frac{3\pi}{2}\right)$

27. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 3 \\ 1 & 5 & 1 \end{bmatrix}$, then the sum of the cofactors of the elements of the **third row** of the matrix A is

(a) -4

(b) 11

(c) 6

(d) -7

(e) 8

$M_{31} = \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} = 3 - 8 = -5 \Rightarrow C_{31} = -5$

$M_{32} = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 6 - 4 = 2 \Rightarrow C_{32} = -2$

$M_{33} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 \Rightarrow C_{33} = 3$

Sum $= -5 - 2 + 3 = -7 + 3 = -4$

28. The number of real solutions of the nonlinear system

$$\begin{cases} x^2 + y^2 = 1 \\ \dfrac{(x+1)^2}{4} - y^2 = 1 \end{cases}$$

is equal to

(a) 1

(b) 2

(c) 3

(d) 4

(e) 0

The sum of the eq^{ns} is

$x^2 + \dfrac{(x+1)^2}{4} = 2$

$4x^2 + x^2 + 2x + 1 = 8$

$5x^2 + 2x - 7 = 0$

$(5x + 7)(x - 1) = 0$

$x = -\frac{7}{5}$ or $x = 1$

$\underline{x = 1}$

$y^2 = 1 - x^2 = 0$

$y = 0$

$\boxed{(1, 0)} \longrightarrow 1$ sol^n

$x = -7/5$

$y^2 = 1 - \left(\frac{7}{5}\right)^2 < 0$

No sol^n.

29. The exact value of $\tan\left[\underbrace{\sin^{-1}\left(\frac{3}{5}\right)}_{\alpha} + \underbrace{\cos^{-1}\left(-\frac{5}{13}\right)}_{\beta}\right]$ is equal to

(a) $-\frac{33}{56}$

(b) $-\frac{17}{65}$

(c) $\frac{56}{33}$

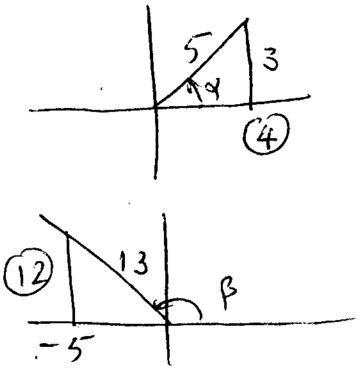
(d) $-\frac{15}{56}$

(e) $\frac{41}{65}$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$= \frac{\frac{3}{4} + \left(-\frac{12}{5}\right)}{1 - \frac{3}{4}\left(-\frac{12}{5}\right)}$$

$$= \frac{\frac{15-48}{20}}{\frac{20+36}{20}} = -\frac{33}{56}$$



30. The graph of $y = 6\sec\left(\pi x + \frac{\pi}{4}\right)$, $\frac{1}{4} < x < \frac{9}{4}$, is increasing on the interval

(a) $\left(\frac{1}{4}, \frac{3}{4}\right) \cup \left(\frac{7}{4}, \frac{9}{4}\right)$

(b) $\left(\frac{3}{4}, \frac{5}{4}\right) \cup \left(\frac{5}{4}, \frac{7}{4}\right)$

(c) $\left(\frac{1}{4}, \frac{5}{4}\right)$

(d) $\left(\frac{5}{4}, \frac{9}{4}\right)$

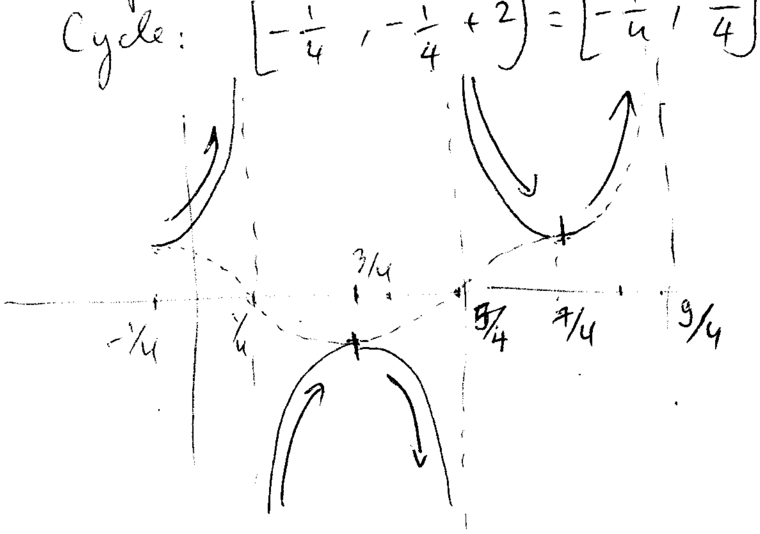
(e) $\left(\frac{1}{4}, \frac{5}{4}\right) \cup \left(\frac{5}{4}, \frac{9}{4}\right)$

$P = \frac{2\pi}{\pi} = 2$

$PS = -\frac{c}{b} = -\frac{\frac{\pi}{4}}{\pi} = \boxed{-\frac{1}{4}}$

Amp $= 6$

Cycle: $\left[-\frac{1}{4}, -\frac{1}{4} + 2\right] = \left[-\frac{1}{4}, \frac{7}{4}\right]$



31. The sum of all solutions of the equation

$$2\sin x\cos x - \sqrt{3}\sin x \quad 2\sqrt{2}\cos x + \sqrt{6} = 0, \quad \text{where } 0 \le x < 2\pi$$

is equal to

$\sin x(2\cos x - \sqrt{3}) \quad - \sqrt{2}(2\cos x - \sqrt{3}) = 0$

$(\sin x - \sqrt{2})(2\cos x - \sqrt{3}) = 0$

$\sin x = \sqrt{2}$ No solⁿ.

$\cos x = \frac{\sqrt{3}}{2}$ < QI, QIV

$x' = \frac{\pi}{6}$

$x = \pi/6$ or $\frac{11\pi}{6}$.

$\Rightarrow \boxed{\frac{\pi}{6} + \frac{11\pi}{6} = 2\pi}$

(a) 2π

(b) $\frac{9\pi}{4}$

(c) $\frac{5\pi}{6}$

(d) $\frac{11\pi}{4}$

(e) $\frac{17\pi}{6}$

32. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Then the **sum** of the elements of the second row of A^{-1} is equal to

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3/-1]{R_2/2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$

$\xrightarrow{-R_1+R_2} \left[\begin{array}{cccccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$

$\xrightarrow[R_3+R_2]{-R_3+R_1} \left[\begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & \boxed{-1 \quad \frac{1}{2} \quad -1} \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$

Sum is $\boxed{-3/2}$

(a) $-\frac{3}{2}$

(b) -4

(c) $\frac{5}{2}$

(d) $\frac{7}{2}$

(e) $-\frac{1}{2}$

33. Which one of the following statements is **TRUE**?

(a) $\tan\left(\tan^{-1} 100\right) = 100$ True

(b) $\sin\left(\sin^{-1}\left(\frac{3}{2}\right)\right) = \frac{3}{2}$ F $3/2 \notin [-1,1]$

(c) $\cos\left(\cos^{-1}\left(-\frac{3}{2}\right)\right) = \frac{3}{2}$ F $-3/2 \notin [-1,1]$

(d) $\sin^{-1}\left(\sin\left(-\frac{3\pi}{4}\right)\right) = \frac{\pi}{4}$ F $-\pi/4$

(e) $\tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right) = \frac{3\pi}{4}$ F $-\pi/4$

$\in \left(-\pi/2, \pi/2\right)$

34. If the **echelon** form of the augmented matrix for the linear system

$$\begin{cases} x - 3y + z = 8 \\ 2x - 5y - 3z = 6 \\ x - 6y + 7z = -7 \end{cases}$$

is $\left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 0 & 1 & m & n \\ 0 & 0 & 1 & p \end{array}\right]$, then

(a) $m = -5,\ n = -10,$ and $p = 5$

(b) $m = 3,\ n = -6,$ and $p = -3$

(c) $m = -5,\ n = 10,$ and $p = -3$

(d) $m = -2,\ n = 7,$ and $p = -1$

(e) $m = -3,\ n = 6,$ and $p = -2$

35) If $\tan\frac{\theta}{2} = -\frac{4}{3}$, $\pi < \theta < \frac{3\pi}{2}$, then $\csc\theta =$

(a) $-\frac{25}{24}$

(b) $\frac{5}{3}$

(c) $-\frac{25}{7}$

(d) $\frac{10}{3}$

(e) $-\frac{25}{12}$

$$\tan\theta = \tan 2\left(\frac{\theta}{2}\right) = \frac{2\tan\theta/2}{1-\tan^2\frac{\theta}{2}} = \frac{2\left(-\frac{4}{3}\right)}{1-\frac{16}{9}}$$

$$= \frac{-\frac{8}{3}}{\frac{9-16}{9}} = \frac{-8/3}{-7/9} = \frac{8}{\cancel{3}}\cdot\frac{\cancel{9}^{3}}{7} = \frac{24}{7}$$

$\theta \in Q\text{III}$

$\Rightarrow y = -24$, $x = -7$

$r^2 = (24)^2 + 7^2 = 576 + 49 = 625$

$\rightarrow r = 25$

$$\csc\theta = \frac{r}{y} = \boxed{-\frac{25}{24}}$$