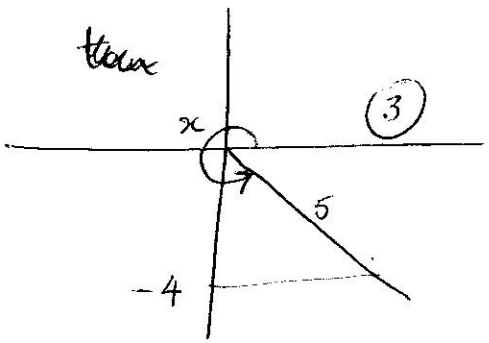
Solution of Recitation Hour Problems
6.3 & 6.4.

Question 1.

$\csc x = -\frac{5}{4}$, $\frac{3\pi}{2} < x < 2\pi$, $\cot 2x$? , $\sec\left(\frac{x}{2}\right)$?



$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2}$$

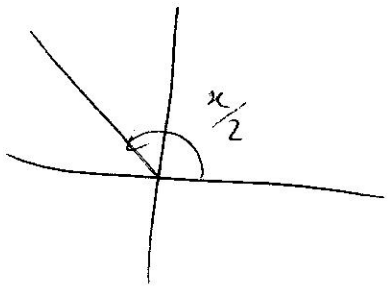
$$= \frac{-8/3}{1-\frac{16}{9}} = \frac{-8/3}{-7/9} = \frac{8}{3}\cdot\frac{3}{7} = \frac{24}{7}$$

$$\Rightarrow \boxed{\cot 2x = \frac{7}{24}}$$

$\frac{3\pi}{2} < x < 2\pi$

$\Downarrow$

$\frac{3\pi}{4} < \frac{x}{2} < \pi \rightarrow \frac{x}{2} \in$ QII



$$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}}$$

$\frac{x}{2} \in$ II

$$= -\sqrt{\frac{1+3/5}{2}}$$

$$= -\sqrt{\frac{8/5}{2}} = -\sqrt{\frac{4}{5}}$$

$$= -\frac{2}{\sqrt{5}} = \boxed{-\frac{2\sqrt{5}}{5}}$$

Question 2. a) $\tan(22.5°)$?

$$= \tan\frac{45°}{2} = \frac{\sin 45°}{1+\cos 45°} = \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$$

$$= \frac{\sqrt{2}}{2} \div \frac{2+\sqrt{2}}{2} = \frac{2}{2+\sqrt{2}} = \frac{2(2-\sqrt{2})}{4-2} = \boxed{2-\sqrt{2}}$$

Q2 b/ $\dfrac{1}{1-2\sin^2\left(\frac{7\pi}{12}\right)} = \dfrac{1}{\cos\left(2\left(\frac{7\pi}{12}\right)\right)} = \dfrac{1}{\cos\frac{7\pi}{6}} = \dfrac{1}{-\cos\frac{\pi}{6}}$

$\cos 2\theta = 1 - 2\sin^2\theta$

III

$= -\dfrac{1}{\frac{\sqrt{3}}{2}} = -\dfrac{2}{\sqrt{3}} = \boxed{-\dfrac{2\sqrt{3}}{3}}$

Q3 a/ Find k & t

$4\sin^2 8\theta - 4\cos^2 8\theta = k\cos t\theta$

$4\left(\sin^2(8\theta) - \cos^2(8\theta)\right) = -4\left(\cos^2(8\theta) - \sin^2(8\theta)\right)$

$= -4\left(\cos 2(8\theta)\right) = -4\cos 16\theta = k\cos t\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow \boxed{k = -4}$, $\boxed{t = 16}$

b/ $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = \dfrac{(\sin x + \cos x)(\sin^2 x - \sin x\cos x + \cos^2 x)}{(\sin x + \cos x)}$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= \underbrace{\sin^2 x - \sin x\cos x + \cos^2 x}_{1} = 1 - \sin x\cos x$

$= 1 - \frac{1}{2}\sin 2x$

$= 1 + k\sin tx$

$\Rightarrow \boxed{k = -1/2} \quad \boxed{t = 2}$

Question 4

$f(x) = 2\sin 2x - 2\sqrt{3}\cos 2x = k\sin(2x + \alpha)$

$a = 2, \quad b = -2\sqrt{3} \Rightarrow k = \sqrt{2^2 + (-2\sqrt{3})^2} = 2\sqrt{1+3} = 4.$

$\cos\alpha = \frac{2}{4} = \frac{1}{2}$ → I, IV

$\sin\alpha = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$ → III, IV

$\Rightarrow \alpha \in$ IV

$\sin\alpha' = \frac{\sqrt{3}}{2} \Rightarrow \alpha' = \frac{\pi}{3} \xrightarrow{\alpha \in \text{IV}} \alpha = -\frac{\pi}{3}$ or $\alpha = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\Rightarrow f(x) = 4\sin\left(3x - \frac{\pi}{3}\right)$

Amp $= |a| = 4$, $PS = -\left(\frac{-\frac{\pi}{3}}{2}\right) = \frac{\pi}{6}$, $P = \frac{2\pi}{2} = \pi$.

Range : $[-4, 4]$

Cycle : $[PS, PS + \pi] = \left[\frac{\pi}{6}, \frac{7\pi}{6}\right]$