King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Prep-Year Math II
MIDTERM EXAM
Semester II, Term 052
Tuesday March 28, 2006
Net Time Allowed: 120 minutes

Sources of Problems

MASTER VERSION

1. Which one of the following statements is TRUE for all positive real numbers $x \neq 1$ and $y \neq 1$?

(a) $\log_y \dfrac{1}{x} \cdot \log_x y = -1$

(b) $\log(x^3 + y^2) = 3\ln x + 2\ln y$

(c) $\log(xy) = \log x \cdot \log y$

(d) $(\log x)^n = n \log x$

(e) $\sqrt{\log \dfrac{x}{y}} = \dfrac{\sqrt{\log x}}{\sqrt{\log y}}$

Properties of Logarithms and change of base p. 395 and p. 397

See also problems #31 to 40 p.404

2. If P is the period and S is the phase shift of the function $y = 3\csc\left(\dfrac{\pi}{2} - \dfrac{x}{4}\right)$, then $P + S$ is equal to

(a) 10π

(b) 3π

(c) $\dfrac{\pi}{8}$

(d) $\dfrac{17\pi}{2}$

(e) -6π

See example 6 Page 533 and problems 11 to 14 p. 535

3. $\dfrac{\dfrac{1}{\sin x} + \csc x}{\dfrac{1}{\sin x} - \sin x} =$

It is problem #53 p. 560
and also similar to #25, 44
p. 560

(a) $2\sec^2 x$ ~~(a)~~

(b) $2\csc^2 x$

(c) $2\cos^2 x$

(d) $2\sin^2 x$

(e) $2\tan^2 x$

4. The number of vertical asymptotes of the graph of
$y = 2\cot 2x\,, \quad -\pi < x < \pi$ is

See examples 2 to 5 of section 5.6
and similar to problems #29, 30, 31, 32, 39, 40
p. 526-527

(a) 3 ~~(a)~~

(b) 5

(c) 4

(d) 1

(e) 2

5. If $\sec x - \tan x = A$ where $A \neq 0$, then $\sec x + \tan x =$

~~(a)~~ $\frac{1}{A}$

(b) $-A$

(c) A^2

(d) 0

(e) $-\frac{1}{A}$

An application of the identity
$\sec^2 x - \tan^2 x = 1$

6. Which one of the following is a factor of $\csc^2 x + \cot x - 31$?

~~(a)~~ $\cot x + 6$

(b) $\csc x - 5$

(c) $\cot x + 2$

(d) $\csc x - \cot x$

(e) $\cot x - 15$

The use of the identity
$\csc^2 x = 1 + \cot^2 x$
and factoring of a trinomial.

7. The length of an arc that subtends a central angle of 135° in a circle of radius 40 ft is

~~(a)~~ 30π feet

See example 5 p. 469

(b) 20π feet

(c) 25π feet

(d) 15π feet

(e) 35π feet

8. $\ln(x\sqrt{z}) - \ln(x\sqrt{y}) + \frac{1}{2}\ln\sqrt[3]{\frac{y}{z}} =$

~~(a)~~ $\frac{1}{3}\ln\left(\frac{z}{y}\right)$

See problems # 9 to 14 p. 403-404 and example 2 p. 396

(b) $\frac{2}{3}\ln\left(\frac{y}{z}\right)$

(c) $\frac{3}{2}\ln\left(\frac{z}{y}\right)$

(d) $\frac{1}{6}\ln(yz)$

(e) $\frac{4}{3}\ln\left(\frac{z}{y}\right)$

9. If the hypotenuse of a $30°$, $60°$, and $90°$ triangle is 10 cm, then the perimeter of the triangle is equal to

~~(a)~~ $(15+5\sqrt{3})$ cm

(b) $(15+5\sqrt{2})$ cm

(c) $(2+2\sqrt{10})$ cm

(d) $(10+10\sqrt{2})$ cm

(e) $(10+5\sqrt{2}+5\sqrt{3})$ cm

Properties of a $30°, 60°$, and $90°$ triangle See p. 479

10. If $\cos\alpha = \frac{15}{17}$, α in Quadrant IV, and $\sin\beta = -\frac{3}{5}$, β in Quadrant III, then $\tan(\alpha-\beta) =$

~~(a)~~ $-\frac{77}{36}$

(b) $-\frac{13}{84}$

(c) $\frac{84}{77}$

(d) $-\frac{49}{36}$

(e) $\frac{77}{84}$

It is problem #43 p. 571
and similar to example 4 p. 567
and problems # 37 to 48 p. 571

11. The domain, in interval notation, of the function $f(x) = \log_2(6x^2 + x - 2)$ is

~~(a)~~ $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$

(b) $\left(-\frac{2}{3}, \frac{1}{2}\right)$

(c) $\left(-\frac{2}{3}, \infty\right)$

(d) $\left(-\infty, \frac{1}{2}\right)$

(e) $\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

See example 5 P.388
and problems # 39 & 48

12. Which one of the following statements is TRUE about the function $f(x) = -3\sec\frac{\pi x}{2}$ on the interval $[0, 2]$? [Hint: sketch]

~~(a)~~ f is decreasing on $[0, 1) \cup (1, 2]$

(b) the minimum value of f is -3

(c) the maximum value of f is 3

(d) there are two vertical asymptotes for the graph of f

(e) f is decreasing on $[0, 1)$ and increasing on $(1, 2]$

See example 4 P. 524
and example 5 p. 525
and problems # 36,
37, 38 p. 527

13. If a car with a wheel of radius 40 cm is moving with a speed of 120 kilometers per hour, then the **angular** speed of the wheel of the car in **radian per minute** is

(a) 5000

(b) 4000

(c) 500

(d) 3000

(e) 50000

See example 7 p.470

and problems # 67 to 72 p.473

14. If $0 \leq x \leq \dfrac{2\pi}{3}$, then the graph of $y = |-2\cos 3x|$ is increasing on the interval

(a) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right] \cup \left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$

(b) $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

(c) $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$

(d) $\left[0, \frac{\pi}{3}\right]$

(e) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

It is Problem # 49 p. 518

and similar to problems

47 to 54 p. 518

and example 6 p. 517

15. $\log_4 16 + \log_{\frac{3}{2}} \dfrac{8}{27} + \left(\dfrac{1}{2}\right)^{\log_2 3} =$

(a) $-\dfrac{2}{3}$

(b) $\dfrac{4}{3}$

(c) $\dfrac{2}{3}$

(d) $-\dfrac{4}{3}$

(e) $\dfrac{1}{3}$

See example 3 p. 385
problems 1 to 30 p.391

16. If $\sin 20^\circ = x$, then $\tan 160^\circ =$

(a) $-\dfrac{x}{\sqrt{1-x^2}}$

(b) $\dfrac{\sqrt{1-x^2}}{x}$

(c) $\dfrac{x}{\sqrt{1+x^2}}$

(d) $\dfrac{x}{\sqrt{x^2-1}}$

(e) $-\dfrac{\sqrt{1+x^2}}{x}$

To use the fact that $\tan(180^\circ - \alpha) = \tan\alpha$.

17. The exact value of $\cos\frac{7\pi}{4}\tan\frac{4\pi}{3}+3\sqrt{2}\cos\frac{7\pi}{6}$ is

(a) $-\sqrt{6}$

(b) $3\sqrt{6}$

(c) $-2\sqrt{6}$

(d) $\frac{\sqrt{3}-\sqrt{2}}{2}$

(e) $\frac{\sqrt{6}-\sqrt{3}}{2}$

It is similar to Problem #66 p.498
and similar to #61 to 66
p. 497–498

18. The solution of the equation $\frac{e^x+e^{-x}}{e^x-e^{-x}}=3$ is equal to

(a) $\ln\sqrt{2}$

(b) $\ln 2$

(c) $\ln 6$

(d) $\ln\sqrt{6}$

(e) $\ln\sqrt{3}$

It is problem #46 p. 415
and similar to 39 to 46 p. 415
and example 4 p. 410

19. Which one of the following statements is TRUE about the function $f(x) = 2^{-x} + 2^{x}$?

(a) the graph of f **goes up to its far left** and **up to its far right**

(b) f **increases on** $(-\infty, 0)$

(c) f **decreases on** $(0, \infty)$

(d) the graph of f is **asymptotic to** the x-axis

(e) **the maximum value** of f is 2

See problems # 35 to 38 p. 377

20. Two buildings are 240 meters apart. The angle of elevation from the top of the shorter building to the top of the taller building is 30°. If the shorter building is 8 meters high, then the taller building is

(a) $(8 + 80\sqrt{3})$ meters high

(b) $(8 + 8\sqrt{3})$ meters high

(c) $(8 + 80\sqrt{2})$ meters high

(d) 88 meters high

(e) $(8 + 8\sqrt{2})$ meters high

It is problem # 74 p. 487 (after choosing suitable numbers)
and similar to examples 4 and 6 p. 482-483
and problems # 76 p. 487

21. The graph of $f(x) = \left|\log_{\frac{1}{2}}(2x+5)\right|$ is decreasing on the interval

~~(a)~~ $\left(-\frac{5}{2}, -2\right)$

(b) $(-2, \infty)$

(c) $\left(-\frac{5}{2}, \infty\right)$

(d) $\left(-\infty, -\frac{5}{2}\right)$

(e) $\left(-\frac{5}{2}, 0\right) \cup (0, \infty)$

See problems #49 to 56 p.391
and #59 to 68 p.392

22. Let W be the wrapping function with $W(t) = \left(\frac{3}{5}, -\frac{4}{5}\right)$, then $\sin\left(\frac{3\pi}{2} - t\right) + \tan(7\pi + t) =$

~~(a)~~ $-\frac{29}{15}$

(b) $\frac{11}{15}$

(c) $-\frac{11}{15}$

(d) $\frac{29}{15}$

(e) $-\frac{8}{15}$

See example 1 p. 500
See problems # 1 to 12 p. 508

23. If f is an exponential function of the form $f(x) = a^x$, and $f(-2) = 9$, then $f(-1) =$

(a) 3

(b) -3

(c) 5

(d) -4

(e) -7

To find the base of an exponential function

24. The graph of the function $f(x) = -2\sin \pi x + 1$ on the interval $[0, 3]$ intersects the x-axis at

(a) four points

(b) three points

(c) two points

(d) one point

(e) five points

See example 4 p.531 and problems # 33 & 44 p.535

25. If x is the solution of the equation $\ln x = 4 - x^2$, then x is in the interval [Hint: Use the graphs of $f(x) = \ln x$ and $g(x) = 4 - x^2$]

(a) $[1, 2]$

(b) $[0, 1]$

(c) $\left[0, \frac{1}{\sqrt{2}}\right]$

(d) $\left[\frac{1}{\sqrt{2}}, \frac{1}{2}\right]$

(e) $[2, 4]$

It is problem #56 p. 415 and similar to problems # 51 to 56 p. 415