King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Code: 003

Prep-Year Math II
EXAM II
Term 063
Wednesday, August 08, 2007
Net Time Allowed: 110 minutes

Code: 003

Student's Name: ....Key Solution..............

ID #: .... **Section #:** ....

Important Instructions:

1. All types of CALCULATORS, PAGERS, OR MOBILES ARE NOT ALLOWED to be with you during the examination.

2. Use an HB $2\frac{1}{2}$ pencil.

3. Use a good eraser. Do not use the eraser attached to the pencil.

4. Write your name, ID number and Mathematics Section on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Math Section number, be sure that bubbles match with the number that you write.

6. The test Code Number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

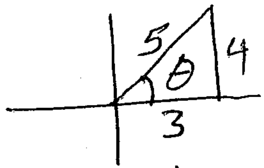
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

9. Check that the exam paper has 22 questions.

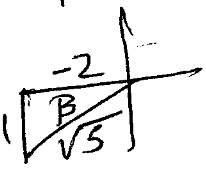
Q1.

If $\sin\theta = \frac{4}{5}$, θ in Quadrant I, and $\cos\beta = -\frac{2}{\sqrt{5}}$, β in Quadrant III, then $\cot(\theta+2\beta) =$

$$\cot(\theta+2\beta) = \frac{1}{\tan(\theta+2\beta)}$$



$$\tan(\theta+2\beta) = \frac{\tan\theta + \tan 2\beta}{1-\tan\theta\tan 2\beta}$$



$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2(\frac{1}{2})}{1-(\frac{1}{2})^2} = \frac{4}{3}$$

$$\therefore \tan(\theta+2\beta) = \frac{\frac{4}{3}+\frac{4}{3}}{1-\frac{4}{3}(\frac{4}{3})} = \frac{8/3}{1-\frac{16}{9}} = \frac{8/3}{-\frac{7}{9}}$$

$$= \frac{8}{3} \times \frac{9}{-7} = -\frac{24}{7}$$

$$\therefore \cot(\theta+2\beta) = \frac{1}{-\frac{24}{7}} = \boxed{\frac{-7}{24}}$$

See EX 3b, p. 576 and probs 37-48, p. 570-71

A) $-\frac{7}{24}$ ✓

B) $\frac{7}{12}$

C) $\frac{-1}{\sqrt{5}}$

D) $-\frac{14}{3}$

E) $\frac{5}{24}$

Q2.

The sum of the solutions of the equation $\cos 4\theta + 3 = 5\cos 2\theta$ for $0 \le \theta < \frac{3\pi}{2}$

$$\cos 4\theta = 2\cos^2 2\theta - 1$$

$$\therefore 2\cos^2 2\theta - 5\cos 2\theta + 2 = 0$$

$$(2\cos 2\theta - 1)(\cos 2\theta - 2) = 0$$

$\cos 2\theta = \frac{1}{2}$ or $\cos 2\theta = 2$ (rejected)

$2\theta = \frac{\pi}{3}$ or $2\theta = \frac{5\pi}{3}$, $2\theta = \frac{7\pi}{3}$

$\theta = \frac{\pi}{6}$, $\theta = \frac{5\pi}{6}$, $\theta = \frac{7\pi}{6}$

Sum $= \frac{\pi}{6} + \frac{5\pi}{6} + \frac{7\pi}{6} = \boxed{\frac{13\pi}{6}}$

See EX 5, P 608 and probs on p. 614-615

A) $\frac{13\pi}{6}$ ✓

B) $\frac{13\pi}{3}$

C) 2π

D) $\frac{11\pi}{6}$

E) π

Q3.

The expression $\sin^{-1}(\sin\frac{5\pi}{3})$ is:

$\frac{5\pi}{3} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\therefore \sin\frac{5\pi}{3} = \sin(-\frac{\pi}{3})$

$\therefore \sin^{-1}(\sin(-\frac{\pi}{3})) = -\frac{\pi}{3}$

See Ex 2, p. 596. and probs 21-48 (prob 32)

A) $-\frac{5\pi}{3}$

B) $\frac{5\pi}{3}$

C) $\frac{\pi}{3}$

✓ D) $-\frac{\pi}{3}$

E) undefined

Q4.

$\frac{\tan x - \cot x}{\tan x + \cot x} =$

$\frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{\sin^2 x - \cos^2 x}{\sin x \cos x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}}$

$\sin^2 x - \cos^2 x = -\cos 2x$

See Ex 5, p. 558 & probs of set 6.1

A) $2 \sec x$

B) $2 \csc x$

C) $\csc 2x$

✓ D) $-\cos 2x$

E) $-\sec 2x$

Q5.

If (h,k) is the center of the ellipse $25x^2+16y^2-150x+64y-111=0$ and e is its eccentricity, then $h+k+e$ is equal to:

$25(x^2-6x+9)+16(y^2+4y+4)=111+225+64$

$25(x-3)^2+16(y+2)^2=400$

$\frac{25(x-3)^2}{400}+\frac{16(y+2)^2}{400}=\frac{400}{400}$

$\frac{(x-3)^2}{16}+\frac{(y+2)^2}{25}=1$, $c=\sqrt{a^2-b^2}=\sqrt{25-16}=3$

$h=3,\ k=-2,\ e=\frac{c}{a}=\frac{3}{5}$

$h+k+e=3-2+\frac{3}{5}=1+\frac{3}{5}=\boxed{\frac{8}{5}}$

See EX 2, p. 699, & probs 17-32, p. 705

A) $\frac{8}{5}$ ✓

B) $-\frac{2}{5}$

C) $\frac{3}{5}$

D) $\frac{7}{5}$

E) $-\frac{1}{5}$

Q6.

Given the vectors $\mathbf{u}=\langle -2,2\rangle$, $\mathbf{v}=\langle 2,-4\rangle$. If the vector $\mathbf{w}=\langle a,b\rangle$ is a unit vector in the opposite direction of $\mathbf{u}-\frac{1}{2}\mathbf{v}$, then $a-b=$

$\vec{u}-\frac{1}{2}\vec{v}=\langle -2,2\rangle+\langle -1,2\rangle=\langle -3,4\rangle$

Unit vector opposite to $\vec{u}-\frac{1}{2}\vec{v}$ is

$w=\langle \frac{3}{5}, \frac{-4}{5}\rangle=\langle a,b\rangle$

$a=\frac{3}{5},\ b=-\frac{4}{5}$

$a-b=\frac{3}{5}-(-\frac{4}{5})=\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$

A) $\frac{3}{5}$

B) $\frac{7}{5}$ ✓

C) $-\frac{3}{5}$

D) $\frac{1}{5}$

E) $-\frac{7}{5}$

Q7.

If $V=(a,b)$ and $F=(c,d)$ are the vertex and focus of the parabola given by the equation $3x^2+6x-4y+15=0$, then $a+b+c+d=$

$3x^2+6x=4y+15$

$3(x^2+2x+1)=4y+15+3$

$3(x+1)^2=4(y-3)$

$(x+1)^2=\frac{4}{3}(y-3)$

$4P=\frac{4}{3}\Rightarrow P=\frac{4}{4\times3}=\frac{1}{3}>0\Rightarrow$ upward

Vertex at $(-1,3)=(a,b)$

focus $=F=(-1,3+\frac{1}{3})=(-1,\frac{10}{3})=(c,d)$

$a+b+c+d=-1+3\cancel{+}-1+\frac{10}{3}=1+\frac{10}{3}=\boxed{\frac{13}{3}}$

A) $\frac{11}{3}$

B) $\frac{23}{3}$

C) $\frac{15}{3}$

D) $\frac{13}{3}$

E) $\frac{14}{3}$

Q8.

The sum of the solutions of the equation $\sin(\frac{6x-\pi}{3})+\frac{\sqrt{3}}{2}=0$ in the interval $[0,2\pi)$ is:

$\sin\left(\frac{6x-\pi}{3}\right)=-\frac{\sqrt{3}}{2}$

$\frac{6x-\pi}{3}=\frac{4\pi}{3},\frac{5\pi}{3},\frac{10\pi}{3},\frac{11\pi}{3}$

$0\le x<2\pi\Rightarrow 0\le 2x<4\pi$

$\frac{6x-\pi}{3}=\frac{4\pi}{3}\Rightarrow x=\frac{5\pi}{6}$

$\frac{6x-\pi}{3}=\frac{5\pi}{3}\Rightarrow x=\pi$

$\frac{6x-\pi}{3}=\frac{10\pi}{3}\Rightarrow x=\frac{11\pi}{6}$

$\frac{6x-\pi}{3}=\frac{11\pi}{6}\Rightarrow x=2\pi\notin[0,2\pi)$ (not included)

$\therefore$ Sum of solutions $=\frac{5\pi}{6}+\pi+\frac{11\pi}{6}=\frac{(5+6+11)\pi}{6}=\frac{22\pi}{6}=\frac{11\pi}{3}$

A) $\frac{7\pi}{3}$

B) $-\frac{2\pi}{3}$

C) $\frac{11\pi}{3}$

D) $\frac{9\pi}{4}$

E) $-\frac{\pi}{4}$

Q9.

If k is an integer, then $\frac{\cos(\theta+(2k+1)\pi)+\sin(\theta+2k\pi)}{\sin\theta} =$ -1

$\cos(\theta+(2k+1)\pi) = \cos\theta\cos(2k+1)\pi - \sin\theta\cancel{\sin(2k+1)\pi}$ 0

$= -\cos\theta$

$\sin(\theta+2k\pi) = \sin\theta\cos 2k\pi + \cos\theta\cancel{\sin 2k\pi}$ 1 0

$= \sin\theta$

$\therefore \frac{-\cos\theta+\sin\theta}{\sin\theta} = -\cot\theta+1 = \boxed{1-\cot\theta}$

A) $1-\cot\theta$ ✓

B) $-\cos\theta$

C) $\tan\theta-\cot\theta$

D) $\sin\theta$

E) $\sin\theta+\cos\theta$

Q10.

If the function $f(x) = -\sin 2x + \sqrt{3}\cos 2x$ is written in the form $f(x) = k\sin(bx+\alpha)$ then the phase shift of $f(x)$ is:

$\sqrt{a^2+b^2} = \sqrt{(-1)^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}} = \frac{-1}{2}$ & $\sin\alpha = \frac{\sqrt{3}}{2}$

$\therefore \alpha = \frac{2\pi}{3}$

$f(x) = 2\sin(2x+\alpha) = 2\sin\left(2x+\frac{2\pi}{3}\right)$

Phase shift $2x+\frac{2\pi}{3} = 0 \Rightarrow 2x = -\frac{2\pi}{3}$

$x = \frac{-2\pi}{3\times 2} = \boxed{-\frac{\pi}{3}}$

See Ex 5, p. 585 and probs 49-56 p. 588

A) $-\frac{\pi}{6}$

B) $\frac{2\pi}{3}$

C) $\frac{\pi}{6}$

D) $-\frac{\pi}{3}$ ✓

E) $\frac{\pi}{3}$

Q11.
Which one of the following statements is **false** for nonzero vectors **u** and **v**: $\vec{u} \perp \vec{v}$

T A) If **u** and **v** are orthogonal, then $proj_u \mathbf{v} = 0$ $\quad proj_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\|u\|} = \frac{0}{\|u\|} = 0$ ok

T B) If $proj_u \mathbf{v} = 0$, then **u** and **v** are orthogonal $\quad \vec{u} \cdot \vec{v} = 0 \to \vec{u} \perp \vec{v}$ T

T C) If $proj_u \mathbf{v} = \|\mathbf{v}\|$, then **u** and **v** are parallel $\quad proj_{\vec{u}}^{\vec{v}} = \|\vec{v}\| = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|} \to \vec{u} \cdot \vec{v} = \|\vec{v}\| \, \|\vec{u}\|$

$\therefore \cos\theta = 0$

$\therefore \theta = 0$

$\therefore$ parallel

T D) $proj_v \mathbf{v} = \|\mathbf{v}\| = \frac{V \cdot V}{\|V\|} = \frac{\|V\|^2}{\|V\|} = \|\bar{V}\|$ T

F E) $proj_u \mathbf{v} - proj_v \mathbf{u} = 0$ $\quad , proj_u^V \neq proj_V^U$

$\therefore proj_u^V - proj_V^U \neq 0$ False

Q12.
If the lines whose equations are $2x + 3y = 1$, $x = 3y + 5$ and $kx + 3y = 3$ all intersect at the same point. Then the value of k is:

C A) 3

B) -2

C) 4

D) 0

E) -1

Find the point of intersection

$$\begin{aligned} 2x + 3y &= 1 \\ x - 3y &= 5 \\ \hline 3x = 6 &\Rightarrow x = 2 \end{aligned}$$

$y = \frac{x-5}{3} = \frac{2-5}{3} = -1$

The point of intersection is $(2,-1)$ which is also on the line $Kx + 3y = 3$

$\therefore K(2) + 3(-1) = 3$

$2K - 3 = 3$

$2K = 6$

$\boxed{K = 3}$

Q13.

If $\dfrac{\tan 4x}{1-\tan^2 4x} = k \tan bx$, then $2k + b =$

A) 10

B) 8

C) 4

D) 9

E) 5

$\frac{1}{2}\frac{(2\tan 4x)}{1-\tan^2 4x} = \frac{1}{2}\tan 8x$

$k = \frac{1}{2}, b = 8$

$2k + b = 2(\frac{1}{2}) + 8 = 1 + 8 = \boxed{9}$

Q14.

The asymptotes of the hyperbola $81x^2 + 162x - 4y^2 + 16y + 29 = 0$ are:

A) $3y - 2x - 12 = 0$ and $3y + 2x + 6 = 0$

B) $2y - 9x - 13 = 0$ and $2y + 9x + 5 = 0$

C) $9y - 2x - 8 = 0$ and $9y + 2x + 3 = 0$

D) $2y - 3x - 11 = 0$ and $2y + 3x + 9 = 0$

E) $9y - 2x - 10 = 0$ and $9y + 2x + 4 = 0$

$81(x^2+2x+1) - 4(y^2-4y+4) = -29+81-16$

$81(x+1)^2 - 4(y-2)^2 = 36$

$\frac{81(x+1)^2}{36} - \frac{4(y-2)^2}{36} = \frac{36}{36}$

$\frac{9(x+1)^2}{4} - \frac{(y-2)^2}{9} = 1$

equns of asympt, $y-k = \pm\frac{b}{a}(x-h)$, $a = \frac{2}{3}$, $b = 3$

$y-2 = \frac{3}{2/3}(x+1) \Rightarrow 2y-4 = 9x+9 \rightarrow \boxed{2y-9x-13=0}$

$y-2 = -\frac{9}{2}(x+1) \rightarrow 2y-4 = -9x-9 \rightarrow \boxed{2y+9x+5=0}$

Q15.
The equation of a parabola with vertex $(4,2)$ which passes through the point $(5,-3)$ is:

The equn $(x-4)^2 = 4P(y-2)$

A) $y=-\frac{1}{81}(x+4)^2+2$

$(5,-3) \Rightarrow (5-4)^2 = 4P(-3-2)$

$1 = -20P$

B) $y=\frac{1}{81}(x-4)^2-2$

$P = -\frac{1}{20}$

C) $y=-\frac{5}{81}(x+4)^2+2$

$\therefore$ the equn $(x-4)^2 = 4(-\frac{1}{20})(y-2)$

$(x-4)^2 = -\frac{1}{5}y+\frac{2}{5}$

D) $y=-5(x-4)^2+2$

multipy by $(-5) \Rightarrow -5(x-4)^2 = y-2$

E) $y=-(x-4)^2+2$

$\therefore \boxed{y = -5(x-4)^2+2}$

Q16.
The equation of the hyperbola with vertices $(-2,10)$ and $(-2,2)$ and eccentricity $e=\frac{5}{4}$ is:

Center $(-2,6)$, $a = \frac{10-2}{2} = 4$, V. axis

$e = \frac{c}{a} = \frac{c}{4} = \frac{5}{4} \rightarrow c = 5$

$c^2 = a^2+b^2 \Rightarrow b^2 = 25-16 = 9$

A) $\frac{(y-6)^2}{9}-\frac{(x+2)^2}{16}=1$

$\therefore$ The eequation is

B) $\frac{(y+2)^2}{16}-\frac{(x-2)^2}{9}=1$

$\frac{(y-6)^2}{16} - \frac{(x+2)^2}{9} = 1$

C) $\frac{(y-6)^2}{25}-\frac{(x+2)^2}{16}=1$

D) $\frac{(y-10)^2}{16}-\frac{(x+2)^2}{9}=1$

E) $\frac{(y-6)^2}{16}-\frac{(x+2)^2}{9}=1$

Q17.

If (A,B) is the solution of the system of equations:

$3y = 2x - 18$

$7x = -3 - 6y$

Then $A + B =$

A) 14

B) 0

C) $-\frac{3}{10}$

D) -13

E) -1

$2x - 3y = 18$ — (1)

$7x + 6y = -3$ — (2)

multiply equn (1) by 2

$4x - 6y = 36$

$7x + 6y = -3$

$11x = 33 \rightarrow \boxed{x = 3}$ -- (3)

$\therefore y = \frac{2x-18}{3} = \frac{2(3)-18}{3} = -\frac{12}{3} = -4$

$\therefore$ the solution is $(3, -4)$

$A = 3$ & $B = -4$

$A + B = 3 - 4 = -1$

Q18.

The value of the expression $\sin(\frac{3\pi}{2} + \beta)\cos(\pi - \beta) + \cos(\frac{\pi}{2} - \beta)\sin\beta$ is equal to:

A) 0

B) $\frac{\pi}{2}$

C) π

D) 1

E) $\frac{3\pi}{2}$

$\sin(\frac{3\pi}{2} + \beta) = \sin\frac{3\pi}{2}\cos\beta + \cancel{\cos\frac{3\pi}{2}}^{0}\sin\beta$

$= -\cos\beta$

$\cos(\pi - \beta) = \cos\pi\cos\beta - \cancel{\sin\pi}^{0}\sin\beta$

$= -\cos\beta$

$\cos(\frac{\pi}{2} - \beta) = \sin\beta$

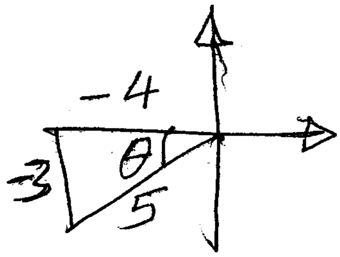
$\therefore$ the expression $(-\cos\beta)(-\cos\beta) + \sin^2\beta$

$= \cos^2\beta + \sin^2\beta = 1$

Q19.

If $\csc\theta = -\frac{5}{3}$, $\pi < \theta < \frac{3\pi}{2}$ then $\tan\frac{\theta}{2}$ is

$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{-3/5}{1-4/5} = \frac{-3/\cancel{5}}{\frac{5-4}{\cancel{5}}} = -3$



c A) -3

B) $\sqrt{3}$

C) 1

D) $-\sqrt{3}$

E) 3

Q20.

The expression $\sin(\tan^{-1}\frac{1}{2} - \cos^{-1}\frac{4}{5})$ is:

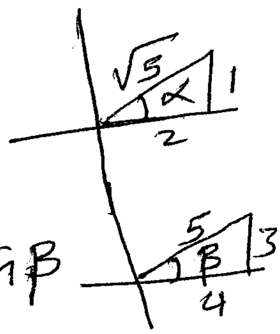
Let $\tan^{-1}\frac{1}{2} = \alpha \rightarrow \tan\alpha = \frac{1}{2}$

$\cos^{-1}\frac{4}{5} = \beta \rightarrow \cos\beta = \frac{4}{5}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{1}{\sqrt{5}}\cdot\frac{4}{5} - \frac{2}{\sqrt{5}}\cdot\frac{3}{5}$

$= \frac{4}{5\sqrt{5}} - \frac{6}{5\sqrt{5}} = -\frac{2}{5\sqrt{5}} = -\frac{2\sqrt{5}}{25}$



A) $\frac{4\sqrt{5}}{25}$

c B) $-\frac{2\sqrt{5}}{25}$

C) $-\frac{4\sqrt{5}}{5}$

D) $\frac{2\sqrt{5}}{25}$

E) $\frac{1}{2}$

Q21.

The equation of an ellipse in the standard form with foci $(-2,3)$ and $(2,3)$ and major axis of length 8 is equal to:

$a = \frac{8}{2} = 4$, center $(0,3)$

$c = 2 \to b^2 = a^2 - c^2 = 16 - 4 = 12$

The equation is $\frac{x^2}{16} + \frac{(y-3)^2}{12} = 1$

A) $\frac{x^2}{4} + \frac{(y-3)^2}{16} = 1$

B) $\frac{x^2}{16} + \frac{(y-3)^2}{12} = 1$ ✓

C) $\frac{(x-3)^2}{16} + \frac{y^2}{9} = 1$

D) $\frac{x^2}{12} + \frac{(y-3)^2}{16} = 1$

E) $\frac{x^2}{16} + \frac{(y-3)^2}{9} = 1$

Q22.

The **cosine** of the smallest positive angle between the vectors $\mathbf{u} = \langle -1, 1 \rangle$ and $\mathbf{v} = \langle 1, 7 \rangle$ is equal to:

$$\cos\alpha = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{\langle -1,1 \rangle \cdot \langle 1,7 \rangle}{\sqrt{1+1}\sqrt{1+49}}$$

$$= \frac{-1+7}{\sqrt{2}\sqrt{50}} = \frac{6}{\sqrt{100}} = \frac{6}{10} = \frac{3}{5}$$

A) $\frac{7}{10}$

B) $-\frac{1}{5}$

C) $\frac{4}{5}$

D) $\frac{6}{5\sqrt{2}}$

E) $\frac{3}{5}$ ✓