

1) Find the coordinates of $M\left(-\frac{8\pi}{3}\right)$

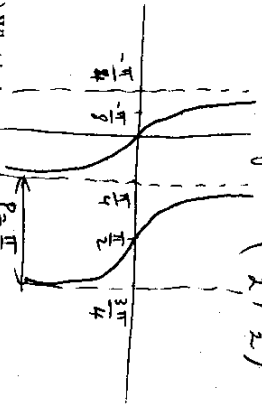
$$W\left(-\frac{8\pi}{3}\right) = \left(\cos\left(-\frac{8\pi}{3}\right), \sin\left(-\frac{8\pi}{3}\right)\right) = \left(\cos\frac{8\pi}{3}, -\sin\left(\frac{8\pi}{3}\right)\right)$$

$$\frac{8\pi}{3} = 2\pi + \frac{2\pi}{3} \rightarrow \text{reference angle} = \frac{2\pi}{3} \rightarrow \pi \leftarrow \cos \ominus \sin \oplus \Delta \theta < \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$= \left(-\cos\frac{\pi}{3}, -\left(\sin\frac{\pi}{3}\right)\right) = \boxed{\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$$

2) Sketch the graph of $f(x) = -2\tan(2x)$ over two periods

$P = \frac{\pi}{2}$, $a = -2$ decreasing
No PS, Cycle: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right) = \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$



3) What is the equation of the graph

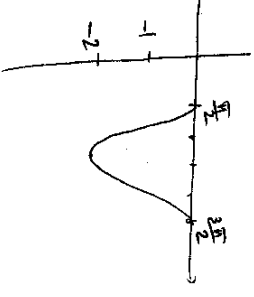
$$|a| = \text{amp} = \frac{M-m}{2} = \frac{0 - (-2)}{2} = 1 \Rightarrow \boxed{a=1}$$

Phase $|a+d| = M = 0$
 $\Rightarrow \boxed{d = 0 - |a| = -1}$

$\frac{2\pi}{b} = \text{Period} = \text{half pt} - 1^{st} \text{ pt} = \frac{3\pi}{2} - \frac{\pi}{2} = \pi$
 $\Rightarrow \frac{2\pi}{b} = \pi \Rightarrow \boxed{b=2}$

$-\frac{c}{b} = \text{PS} = 1^{st} \text{ pt of graph} = \frac{\pi}{2}$
 $\Rightarrow +c = -\frac{\pi}{2} \cdot b = -\frac{\pi}{2} \cdot 2 = \boxed{-\pi = c}$

$y = \cos(2x - \pi) + 1$



4) For the function $f(x) = \csc(3x + \frac{\pi}{2})$,
a) Give the general equation of the vertical asymptotes

$\csc = \frac{1}{\sin}$
 $3x + \frac{\pi}{2} = k\pi$
 $3x = k\pi - \frac{\pi}{2}$
 $x = k\frac{\pi}{3} - \frac{\pi}{6}$
 $x = \frac{(2k-1)\pi}{6}$

b) Find the number of V.A. in $\left[-\frac{\pi}{3}, \frac{5\pi}{6}\right]$

$$-\frac{\pi}{3} < (2k-1)\frac{\pi}{6} < \frac{5\pi}{6}$$

$$-\frac{2\pi}{6} < (2k-1)\frac{\pi}{6} < \frac{5\pi}{6}$$

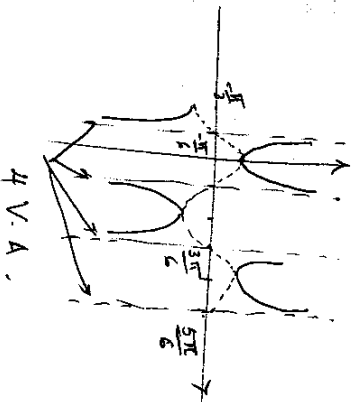
$$-2 < 2k-1 < 5$$

$$-1 < 2k < 6$$

$$-\frac{1}{2} < k < 3$$

$k = 0, 1, 2, 3 \rightarrow 4 \text{ V.A.}$

$P = \frac{2\pi}{3} = \frac{4\pi}{6}$ PS = $-\frac{\pi}{3} = -\frac{2\pi}{6} \Rightarrow \text{Cycle: } \left[-\frac{\pi}{6}, \frac{3\pi}{6}\right]$



1) Find the coordinates of $W\left(-\frac{17\pi}{6}\right)$

$$W\left(-\frac{17\pi}{6}\right) = \left(\cos\left(-\frac{17\pi}{6}\right), \sin\left(-\frac{17\pi}{6}\right)\right)$$

$$= \left(\cos\left(\frac{17\pi}{6}\right), -\sin\left(\frac{17\pi}{6}\right)\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$\frac{17\pi}{6} = 2\pi + \frac{5\pi}{6} \rightarrow \text{reference} = \frac{5\pi}{6} \rightarrow \text{II}$$

$$\cos \ominus, \sin \oplus \quad \theta \leq \frac{\pi}{6}$$

$$= \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

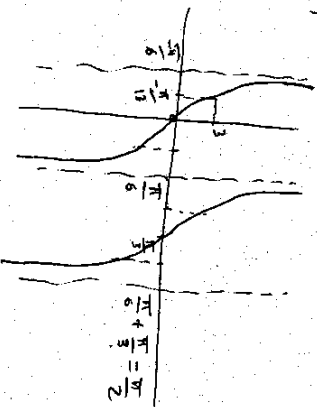
2) Sketch the graph of $f(x) = -3 \tan(3x)$ over two periods

$$a = -3 \rightarrow \text{slope of } -\text{tans}$$

$$-\frac{\pi}{6} \leq 3x \leq \frac{\pi}{6}$$

$$-\frac{\pi}{18} < x < \frac{\pi}{18}$$

$$P_2 = \frac{\pi}{2}$$



3) What is the equation of the graph

$$\text{Amplitude} = \frac{M-m}{2} = \frac{4-0}{2} = 2 = |a|$$

Slope of $\cos \Rightarrow a = 2$

$P =$ Left point - First pt

$$= 8\pi - \pi = \pi = \frac{2\pi}{b} \Rightarrow b = 2$$

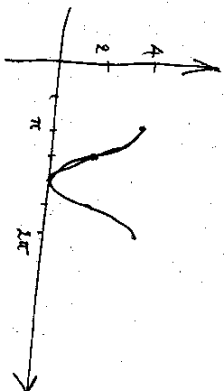
$$P_2 = 1^{st} \text{ pt of cycle} = \pi = -\frac{c}{b} = -\frac{c}{2}$$

$$\Rightarrow c = -2\pi$$

$$\text{Max} = |a| + d = 4 \Rightarrow d = 4 - |a|$$

$$= 4 - 2 = 2$$

$$y = 2 \cos(2x - 2\pi) + 2$$



4) For the function $f(x) = \csc(3x - \frac{\pi}{2})$,
a) Give the general equation of the vertical asymptotes

$$\csc = \frac{1}{\sin}$$

$$3x - \frac{\pi}{2} = k\pi$$

$$3x = k\pi + \frac{\pi}{2}$$

$$x = \frac{k\pi}{3} + \frac{\pi}{6} \quad \text{or} \quad x = (2k+1)\frac{\pi}{6}$$

b) Find the number of V.A. in $\left[-\frac{\pi}{6}, \frac{7\pi}{6}\right]$

$$-\frac{\pi}{6} \leq (2k+1)\frac{\pi}{6} \leq \frac{7\pi}{6}$$

$$-1 \leq 2k+1 \leq 7$$

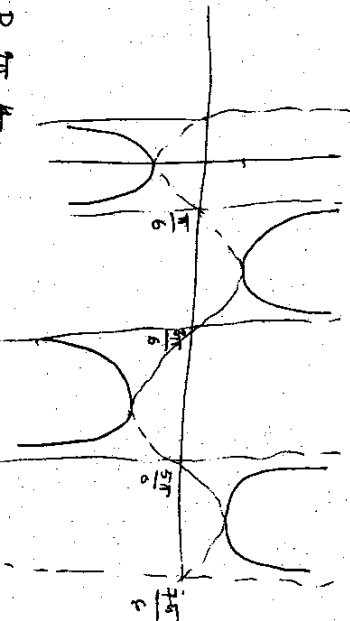
$$-2 \leq 2k \leq 6$$

$$-\frac{2}{2} \leq k \leq \frac{6}{2}$$

$$-1 \leq k \leq 3$$

$$-1, 0, 1, 2, 3$$

5 V.A.



$$P = \frac{7\pi}{6} - \frac{4\pi}{6}$$

$$P_2 = \frac{\pi}{6}$$