20/ 21/ 22/ 25/

1. The period of the function

$$f(x) = -3\cos\frac{2x}{3}$$

is

$b = \frac{2}{3}$

(a) 2π $\quad P = \frac{2\pi}{2/3} = 3\pi$

(b) $\frac{3\pi}{4}$

✓(c) 3π

(d) 4π

(e) $\frac{4\pi}{3}$

2. If $\sec\theta = \frac{2\sqrt{3}}{3}$ and $\sin\theta = -\frac{1}{2}$, then $\cot\theta =$

(a) $\sqrt{3}$

(b) $-\frac{\sqrt{3}}{3}$

(c) $\frac{\sqrt{3}}{2}$

✓(d) $-\sqrt{3}$

(e) $\frac{3}{2}$

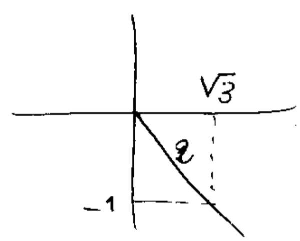
$\sec\theta = \frac{2\sqrt{3}}{3} \Rightarrow \cos\theta = \frac{3}{2\sqrt{3}} > 0 \Rightarrow \mathrm{I}, \mathrm{IV}$

$\sin\theta = -\frac{1}{2} < 0 \Rightarrow \mathrm{III}, \mathrm{IV}$ $\quad \Big\}\ \mathrm{IV}$

$\sin\theta = -\frac{1}{2} = \frac{-1}{2}$

we can take $\boxed{y = -1\ , r = 2}$

$\cancel{x} = \pm\sqrt{3} \underset{\mathrm{IV}}{=} \sqrt{3}$



$\cot\theta = \frac{x}{y} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

3. The reference angle θ' of $\theta = \underbrace{217°15'}_{\text{III}}$ is

(a) $152°45'$

(b) $46°25'$

(c) $36°45'$

✓(d) $37°15'$

(e) $27°15'$

$\theta' = 217°15° - 180° = \boxed{37°15'}$

4. When written as a single logarithm with coefficient 1, the expression

$$\log(x+2) - \log(2x-1) + \log_{\frac{1}{10}}\left(\frac{1}{x-2}\right)$$

becomes

(a) $\log\left(\dfrac{x^2-1}{2x-1}\right)$

(b) $\log\left(\dfrac{2x^2-1}{x-1}\right)$

(c) $\log\left(\dfrac{x+2}{(2x-1)(x-2)}\right)$

✓(d) $\log\left(\dfrac{x^2-4}{2x-1}\right)$

(e) $\log\left[(x^2-4)(2x-1)\right]$

$\log(x+2) - \log(2x-1) + \dfrac{\log\left(\frac{1}{x-2}\right)}{\log\frac{1}{10}}$ (-1)

$\log(x+2) - \log(2x-1) - \log\left(\frac{1}{x-2}\right)$

$\log(x+2) - \log(2x-1) + \log(x-2)$

$\log\dfrac{(x+2)(x-2)}{2x-1}$

$= \log\left(\dfrac{x^2-4}{2x-1}\right)$

5. The value of $3\tan\frac{\pi}{4} + \sec 60^\circ - \sin 30^\circ \cos\frac{\pi}{3}$ is equal to

(a) $\frac{19}{4}$

(b) $-\frac{3}{4}$

(c) $\frac{15}{2}$

(d) $-\frac{1}{2}$

(e) $\frac{3}{4}$

$3(1) + \frac{1}{\cos 60^\circ} - \frac{1}{2} \cdot \frac{1}{2}$

$3 + \frac{1}{\left(\frac{1}{2}\right)} - \frac{1}{4} = 3 + 2 - \frac{1}{4}$

$= \frac{19}{4}$

6. $3\log_3 36 - 6\log_3 2 =$

(a) 6

(b) 18

(c) 12

(d) 9

(e) 3

$= 3\log_3 3^2 \cdot 2^2 - 6\log_3 2$

$= 6\left(\log_3 3 + \log_3 2\right) - 6\log_3 2$

$= 6\log_3 3 + \cancel{6\log_3 2} - \cancel{6\log_3 2}$

$= \boxed{6}$

7. The period P and the phase shift S of the function

$$f(x) = -2\sec\left(\frac{\pi}{3} - \frac{x}{4}\right) + 5$$

are

Routine.

(a) $P = 2\pi$, $S = \frac{2\pi}{3}$

✓ (b) $P = 8\pi$, $S = \frac{4\pi}{3}$

(c) $P = 4\pi$, $S = \frac{\pi}{3}$

(d) $P = 8\pi$, $S = -\frac{4\pi}{3}$

(e) $P = -8\pi$, $S = \frac{4\pi}{3}$

8. $\frac{\sin x}{1+\cos x} + \cot x =$

$$\frac{\sin}{1+\cos x} \cdot \frac{(1-\cos x)}{(1-\cos x)} + \cot x.$$

$$\frac{\cancel{\sin x}(1-\cos x)}{\sin^{\cancel{2}} x} + \frac{\cos x}{\sin x}$$

$$\frac{1}{\sin x} - \cancel{\frac{\cos x}{\sin x}} + \cancel{\frac{\cos x}{\sin x}}$$

$$= \boxed{\csc x}$$

(a) $-\cos x$

(b) $\sin x$

✓ (c) $\csc x$

(d) $\csc x + \cot x$

(e) $\tan x$

9. If k is an **odd** integer and m is an **even** integer, then $\cos[\theta + 2k\pi] + \sin[\theta + (2m+1)\pi] =$

(a) $-\cos\theta - \sin\theta$

✓ (b) $\cos\theta - \sin\theta$

(c) $\cos\theta + \sin\theta$

(d) $-\cos\theta + \sin\theta$

(e) 0

$\cos\theta + \sin(\theta + \pi)$

$\boxed{\cos\theta - \sin\theta}$

10. The solution of the equation $\dfrac{4^x - 4^{-x}}{4^x + 4^{-x}} = \dfrac{1}{2}$ is

✓ (a) $\log_4 \sqrt{3}$

(b) $-1 + \log_4 3$

(c) $\sqrt[3]{4}$

(d) $-\ln 4$

(e) $\log_3 4$

$2.(4^x - 4^{-x}) = 4^x + 4^{-x}$

$2.4^x - 2.4^{-x} = 4^x + 4^{-x}$

$4^x - 3.4^{-x} = 0$ $\quad \times 4^x$

$(4^x)^2 - 3 = 0$

$4^{2x} = 3$

$2x = \log_4 3$

$x = \frac{1}{2}\log_4 3 = \log_4 3^{1/2} = \boxed{\log_4 \sqrt{3}}$

11. If a belt runs a drive wheel of $\overbrace{\text{radius 8}}^{r}$ centimeters at 15 revolutions per minute, then the linear speed of the belt in centimeter per second is v

angular speed

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{16}$

✓ (c) 4π

(d) 16π

(e) $\frac{15}{4}\pi$

$v = \omega \cdot r \quad = 15 \text{ rev/min} \cdot 8 \text{ cm}$

has to be in rad/t. $= 15 \cdot 2\pi \text{ rad/min} \cdot 8 \text{ cm}$

$= 240\pi \text{ cm/min}$

need conversion to cm/sec

$= 240\pi \dfrac{\text{cm}}{60\text{ sec}} = \boxed{4\pi \text{ cm/s}}$

12. If A is the solution of the equation

$$2\log\sqrt{4-x} + \log\left(\frac{1}{x+8}\right) = \log(2x+13),$$

then $2A+1 =$

(a) -15

(b) -19

(c) 10

(d) 5

✓ (e) -9

$\log(\sqrt{4-x})^2 + \log\frac{1}{x+8} = \log(2x+13)$

$\log\frac{4-x}{x+8} = \log(2x+13)$

$\frac{4-x}{x+8} = 2x + 13$

$4 - x = (2x+13)(x+8) = 2x^2 + 29x + 104$

$2x^2 + 30x + 100 = 0$

$(2x+10)(x+10) = 0 \quad \Rightarrow x = -5 \quad x = -10$

Check $x = -5$ ✓ $\quad x = -10$ ✗

$2A + 1 = -10 + 1 = -9$

13. Which one of the following is **TRUE**?

$\frac{\pi}{2} < 2 < 3 < \pi$

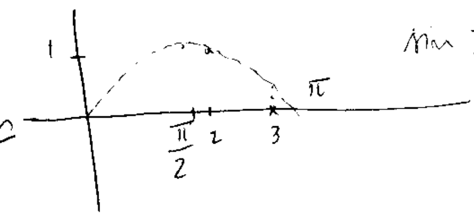
(a) $\sin 2 < \cos 3$ F

(b) $\sin 2 < \sin 3$ F graph

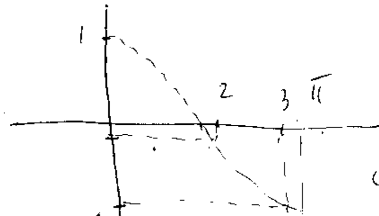
✓ (c) $\sin 2 > \sin 3$ T graph

(d) $\cos 3 > \cos 2$ F graph

(e) $\cos 2 > \sin 3$ F



sin > 0 in II



cos < 0 in II

14. A 22 foot ladder is resting against a vertical wall and makes an angle of 60° with ground. Find the exact height to which the ladder will reach the wall

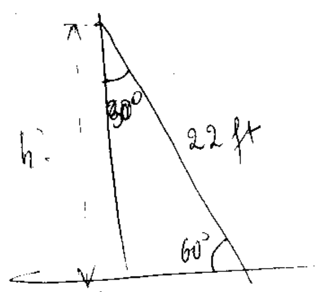
(a) $22\sqrt{3}$ feet

(b) 11 feet

(c) $11\sqrt{2}$ feet

(d) 22 feet

✓ (e) $11\sqrt{3}$ feet



$\sin 60 = \frac{h}{22}$

$h = 22 \sin 60°$

$= 22 \cdot \frac{\sqrt{3}}{2} = \boxed{11\sqrt{3} \text{ ft}}$

15. Given $\sin\alpha = \dfrac{\sqrt{2}}{2}$, α in Quadrant I and $\cos\beta = \dfrac{\sqrt{3}}{2}$, β in Quadrant IV, find $\cos(\alpha-\beta)$

Routine

(a) $\dfrac{\sqrt{3}-\sqrt{2}}{2}$

(b) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

✓ (c) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(d) $\sqrt{2}-\sqrt{6}$

(e) $\dfrac{\sqrt{3}+\sqrt{2}}{2}$

16. If $\overbrace{\dfrac{\pi}{2} < x < \pi}^{\to \text{II}}$, then $\tan x =$

II → cos −

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sin x}{\pm\sqrt{1-\sin^2 x}} = -\frac{\sin x}{\sqrt{1-\sin^2 x}}$$

(a) $\dfrac{-\sin x}{1-\sin x}$

✓ (b) $-\dfrac{\sin x}{\sqrt{1-\sin^2 x}}$

(c) $-\dfrac{\sin x}{\sqrt{1+\sin^2 x}}$

(d) $\dfrac{\sin x}{\sqrt{1-\sin^2 x}}$

(e) $\dfrac{\sin x}{1-\sin^2 x}$

all choices are in sin

⇒ we need to write tan x as sin x

17. The domain of the function $f(x) = \ln\left(\dfrac{x^3}{x-2}\right)$ is

The argument of log should be > 0

The sign of x^3 is the same as sign of x

$$\frac{x^3}{x-2} > 0$$

x		0		2	
x^3	$-$	0	$+$		$+$
$x-2$	$-$		$-$	0	$+$
$\left(\frac{x^3}{x-2}\right)$	$+$	0	$-$	U	$+$

$D = (-\infty, 0) \cup (2, \infty)$

✓ (a) $(-\infty, 0) \cup (2, \infty)$

(b) $(\infty, -2) \cup (2, \infty)$

(c) $(-\infty, 0) \cup (0, \infty)$

(d) $(-2, 0) \cup (0, 2)$

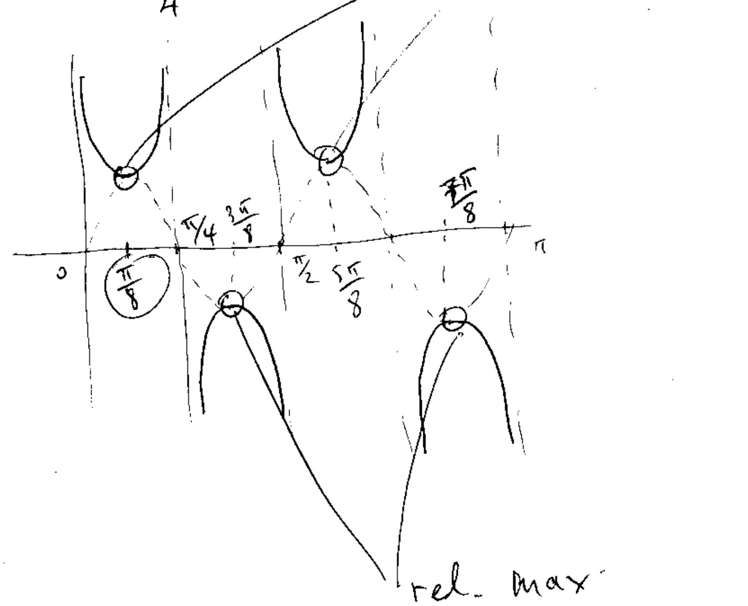
(e) $(-2, 2)$

18. The graph of $y = 2\csc 4x$ has **relative minimum** in the interval $[0, \pi]$ at

rel min

Sketch

$P = \frac{2\pi}{4} = \frac{\pi}{2}$



(a) $x = \frac{\pi}{2}$ and $x = \frac{\pi}{4}$

(b) $x = \frac{3\pi}{8}$ and $x = \frac{7\pi}{8}$

(c) $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$

✓ (d) $x = \frac{\pi}{8}$ and $x = \frac{5\pi}{8}$

(e) $x = 0$ and $x = \frac{\pi}{4}$

$\frac{\pi}{8}, \frac{5\pi}{8}$.

19. If α and β are, respectively, the complement and the supplement of the angle $\theta = 64°15'$, then $2\alpha + \beta$ is equal to

$\alpha = 90 - 64°15' =$

$2\alpha =$

✓ (a) $167°15'$

(b) $154°15'$

(c) $167°45'$

(d) $152°15'$

(e) $25°45'$

20. The range, in interval notation, of the function

$$f(x) = 5 - \left|2\cos\frac{\pi x}{2}\right|$$

is

draw $2\cos\frac{\pi x}{2}$ & reflect down what is above the x-axis

✓ (a) $[3, 5]$

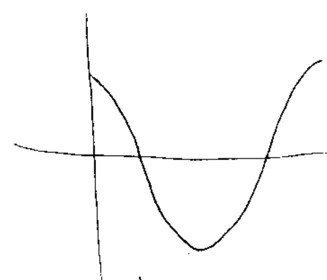
(b) $[-2, 5]$

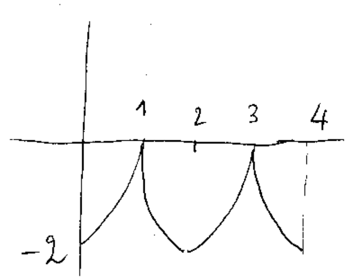
(c) $[1, 3]$

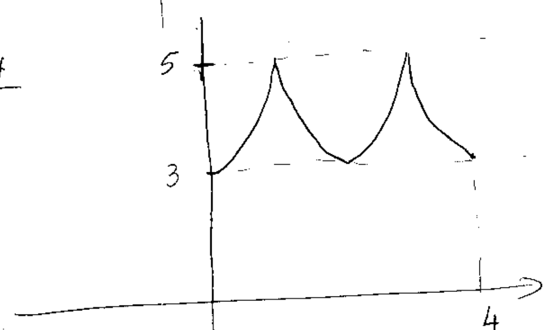
(d) $[2, 4]$

(e) $[2, 5]$

$P = \frac{2\pi}{\left(\frac{\pi}{2}\right)} = 4$







$R = [3, 5]$

OR

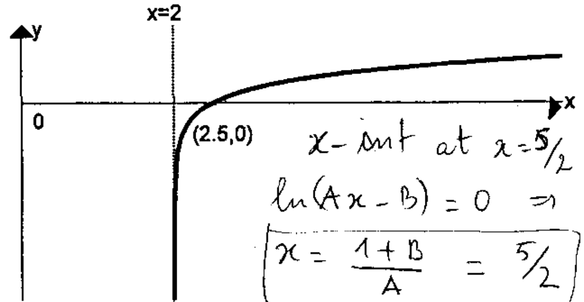
$-\left|2\cos\frac{\pi x}{2}\right| \to [-2, 0]$

$5 - \left|2\cos\frac{\pi x}{2}\right| \to [5-2, 5+0]$

$[3, 5]$

Simpler

21. The adjacent figure is the graph of $y = \ln(Ax - B)$, then the value of $5A - 2B$ is



(a) 6

(b) 5

(c) 4

✓ (d) 2

(e) 3

x- int at $x = 5/2$

$\ln(Ax - B) = 0 \Rightarrow Ax - B = 1$

$\boxed{x = \frac{1+B}{A} = 5/2}$

V.A at $x = 2$, $Ax - B = 0$

$\boxed{x = \frac{B}{A} = 2}$

$\Rightarrow \frac{1}{A} + \frac{B}{A} = 5/2$

$\Rightarrow \frac{1}{A} + 2 = 5/2 \Rightarrow \frac{1}{A} = \frac{1}{2} \Rightarrow \boxed{A = 2} \Rightarrow \boxed{B = 4}$

$5A - 2B$

$10 - 8 = 2$

22. If (x, y) are the coordinates of the <u>highest</u> point on the graph of $f(x) = -3\sin\left(\frac{3x}{2}\right) - 1$ in the interval $\left[0, \frac{4\pi}{3}\right]$, then $5x - \pi y =$

$P = \frac{2\pi}{\left(\frac{3}{2}\right)} = 2\pi \cdot \frac{2}{3} = \frac{4\pi}{3}$

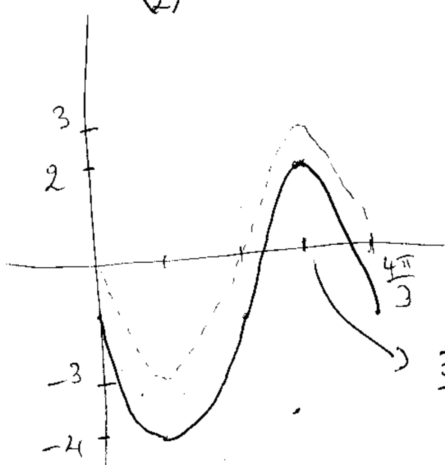
(a) 2π

(b) -2π

(c) $-\pi$

✓ (d) 3π

(e) π



$(x, y) = (\pi, 2)$

$5x - \pi y$

$5\pi - 2\pi = \boxed{3\pi}$

23. If $a = (\sqrt{2})^{\log_4 9}$ and $b = 2\sin(\ln e^{\pi/3})$, then $a - b =$

(a) $\sqrt{2}$

(b) 1

(c) $\sqrt{3}$

(d) 2

✓ (e) 0

$\sqrt{2}^{\log_4 9}$ try to write it as $2^{\log_2(\text{something})}$

$= 2^{\frac{1}{2}\log_4 9} = 2^{\frac{1}{2}\frac{\log_2 9}{\log_2 4}} = 2^{\frac{1}{4}\log_2 9}$ ($\log_2 4 = 2$)

$= 2^{\log_2 9^{1/4}} = 9^{1/4} = \boxed{\sqrt{3}}$,

$b = 2\sin\left(\pi/3\right) = 2\sin 60^\circ = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

$a - b = \sqrt{3} - \sqrt{3} = 0$

24. Which one of the following statements is **FALSE** about the graph of the function $f(x) = 2 - \left(\frac{1}{2}\right)^{x+3}$?

(a) The graph increases on the interval $(-\infty, \infty)$. T

(b) The graph has x-intercept at $(-4, 0)$. T

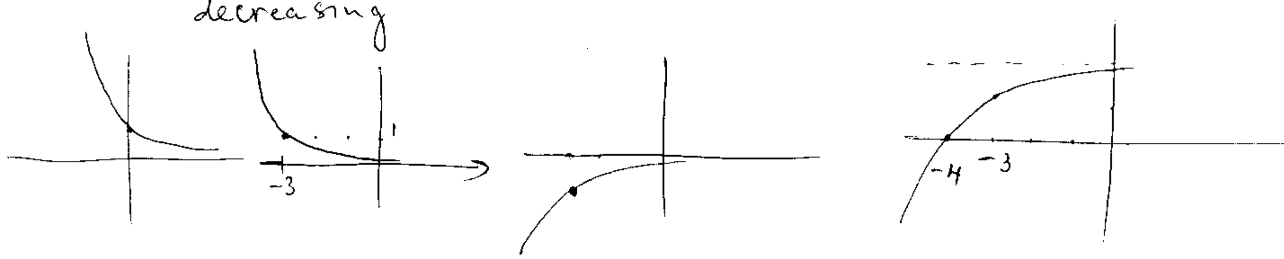
✓ (c) The graph decreases on the interval $(-\infty, \infty)$. F

(d) The graph is below the line $y = 2$. T

(e) The graph has $y = 2$ as a horizontal asymptote. T

$y = 2 - \left(\frac{1}{2}\right)^{x+3}$ because $\left(\frac{1}{2}\right)^{x+3}$ decre $\Rightarrow$ $-\left(\frac{1}{2}\right)^{x+3}$ increasi

decreasing



25. Which one of the following statements is **TRUE** about the graph of $y = 3\tan\left(2x + \frac{\pi}{2}\right)$, where $-\frac{5\pi}{4} \le x \le \frac{3\pi}{4}$? ✓

(a) The graph is decreasing on $\left(0, \frac{\pi}{2}\right)$. F

(b) The graph has three vertical asymptotes. F 4 V.A

(c) The graph has one y-intercept. F No y-int

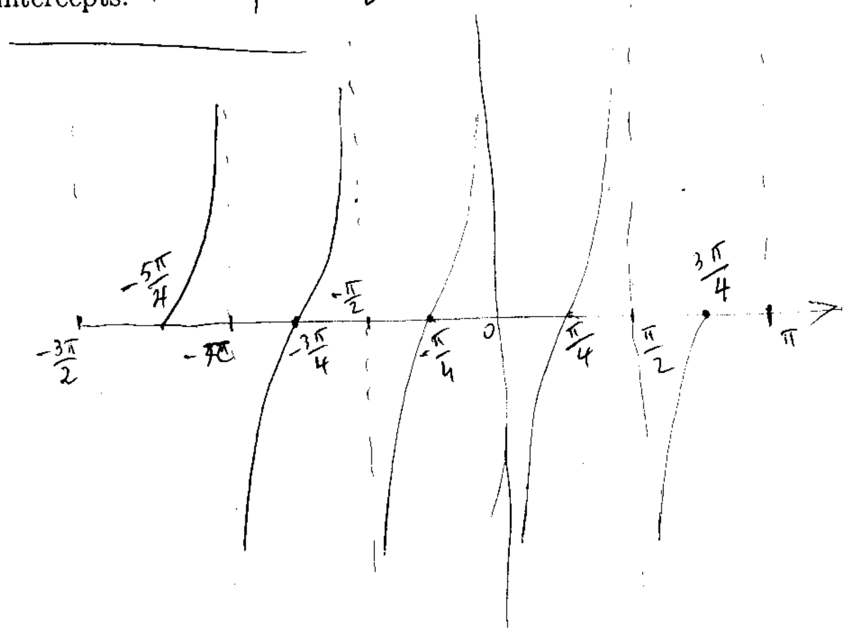
(d) The graph has five vertical asymptotes. F 4 VA

✓ (e) The graph has 5 x-intercepts. T ✓

$-\frac{\pi}{2} < 2x + \frac{\pi}{2} < \frac{\pi}{2}$

$-\frac{\pi}{2} - \frac{\pi}{2} < 2x < 0$

$-\frac{\pi}{2} < x < 0$



4 V A