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King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Prep-Year Math II
MIDTERM EXAM
Semester II, Term 062
Saturday, April 21, 2007
Net Time Allowed: 120 minutes

Sources of Problems

MASTER VERSION

1. Let $f(x) = a\sin bx$, where $b > 0$. If the period of f is 12 and $f(3) = 4$, then $f(25) =$

~~(a)~~ 2

(b) 6

(c) 4

(d) 0

(e) 8

See problems 1 to 16 p. 518

$P = 12 = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{12} = \boxed{\frac{\pi}{6}}$

$f(3) = a\sin\left(\frac{\pi}{6}\cdot 3\right) = 4$

$= a\sin\frac{\pi}{2} = 4$

$\boxed{a = 4}$

$f(25) = 4\sin\left(\frac{\pi}{6}\cdot 25\right) = 4\sin\left(4\pi + \frac{\pi}{6}\right)$

$4\sin\left(\frac{\pi}{6}\right) = 2\ \frac{1}{2} = \boxed{2}$

2. The adjacent figure represents a part of the graph of

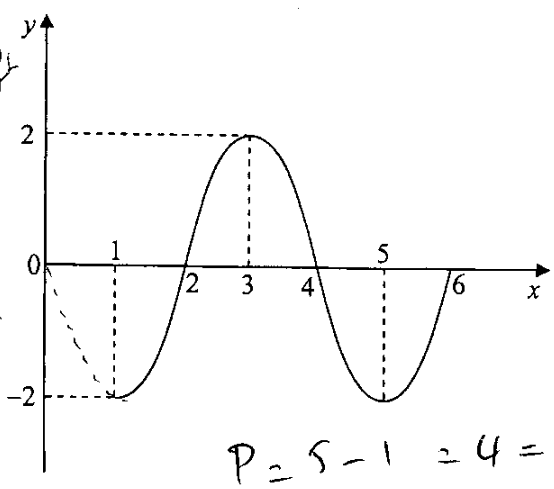
~~(a)~~ $y = -2\sin\frac{\pi}{2}x$

(b) $y = -2\cos\frac{\pi}{2}x - 2$ → V. shift No

(c) $y = 2\sin\frac{\pi}{2}x$

(d) $y = 2\cos\frac{\pi}{2}x + 2$ → Vert shift No

(e) $y = 2\sin\left(\frac{\pi}{2}x - 1\right)$



$P = 5 - 1 = 4 = \frac{2\pi}{b}$

$\Rightarrow b = \boxed{\frac{\pi}{2}}$

PS = 1 → $a = -2$, cos

Extend to 0

$c = 0$, $a = -2$

So $\boxed{y = -2\sin\frac{\pi}{2}x}$

See example 2 p. 514

and problem 30 p. 518

and example 7 p. 531

and problems 33 to 44 p. 535

3. $\log_4(\log_2(\log_{\sqrt{3}} 9)) =$

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) 2

(d) 1

(e) 0

See problems 21 to 30 P. 391

$\text{Log}_4\left(\log_2\left(\log_{\sqrt{3}}(\sqrt{3})^4\right)\right)$

$\log_2(4)$

$\log_4(2) = \log_4 4^{\frac{1}{2}} = \boxed{\frac{1}{2}}$

4. The following figure represents the graph of

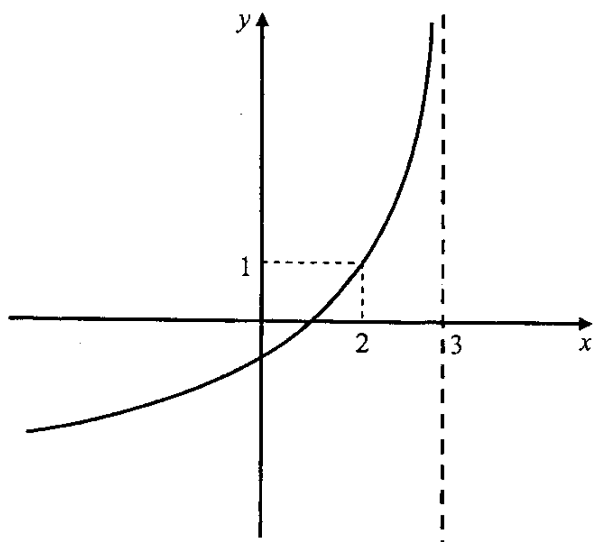
(a) $f(x) = 1 - \ln(-x+3)$

(b) $f(x) = 1 + \ln(-x+3)$

(c) $f(x) = 1 - \ln|x-3|$

(d) $f(x) = |-1 + \ln(-x+3)|$

(e) $f(x) = -1 - \ln(-x+3)$



See example 6 P. 388

See problems 57, 58 P. 391–392

$D = (-\infty, 3) \Rightarrow$ (c) rejected

Graph up & down $\Rightarrow$ (d) eliminated

x-int (a) $\ln(-x+3) = 1 \Rightarrow -x+3 = e \Rightarrow x = 3 - e \in [0,1]$

(b) $\ln(-x+3) = -1 \quad -x+3 = 1/e \Rightarrow x = 3 - 1/e > 2$ ✗

(e) $\ln(-x+3) = -1 \quad x = 3 - 1/e > 2$ ✗

So (a) $f(x) = 1 - \ln(-x+3)$

5. Which one of the following statements is **FALSE**?

$x = (\sqrt{2})^y$

~~(a)~~ $\log_{\sqrt{2}} x = y$ if and only if $y = (\sqrt{2})^x$

See examples 1, 2 p.384 and problems 1 to 20 p.391

(b) $5^{3+x} = y$ if and only if $\log_5 y = x + 3$

(c) $\log_3 \frac{1}{81} = x$ if $3^x = 3^{-4}$

(d) $e^{x+2} = y - 3$ if and only if $\ln(y-3) = x + 2$ $\rightarrow (x+2) = \ln(y-3)$

(e) $2^{\frac{1}{2}\log_2 x} = y$ if and only if $y = \sqrt{x}$ $\rightarrow 2^{\log_2 x^{1/2}} = x^{1/2} = \sqrt{x} \Rightarrow T$

6. A car has wheels that are 3.6 feet in diameter. If the wheels, rolling without slipping, turn through 72° degrees, then the distance moved by the car is equal to

~~(a)~~ $\frac{18\pi}{25}$ feet

(b) $\frac{21\pi}{25}$ feet

(c) $\frac{13\pi}{25}$ feet

(d) $\frac{19\pi}{25}$ feet

(e) $\frac{17\pi}{25}$ feet

See discussion opposite to Figure 5.28 p.471

$d = 3.6$ ft $\quad r = 1.8$ ft

$s = \theta \cdot r = \underbrace{\left(72 \times \frac{\pi}{180}\right)}_{\text{in rad}} \times 1.8 = \frac{72\pi}{100}$

$\boxed{s = \frac{18\pi}{25} \text{ ft}}$

7. The expression $\dfrac{1-\sin\theta}{1+\sin\theta}$ is identical to

(a) $(\sec\theta - \tan\theta)^2$

(b) $(\sec\theta + 1)^2$

(c) $(1+\tan\theta)^2$

(d) $(1-\sec\theta)^2$

(e) $(\csc\theta + 1)^2$

See problem 65 p. 561

$$\frac{(1-\sin\theta)}{(1+\sin\theta)}\cdot\frac{(1-\sin\theta)}{(1-\sin\theta)} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\sec\theta - \tan\theta\right)^2$$

8. If $\sec\theta = -\dfrac{13}{5}$, where $\dfrac{5\pi}{2} < \theta < 3\pi$, then the exact value of $\sin\left(\dfrac{\theta}{2}\right)$ is

(a) $\dfrac{-3\sqrt{13}}{13}$

(b) $\dfrac{-2\sqrt{13}}{13}$

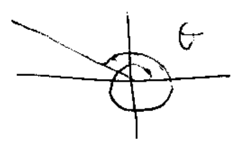
(c) $\dfrac{6}{13}$

(d) $\dfrac{3\sqrt{13}}{13}$

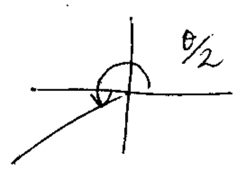
(e) $\dfrac{-1}{13}$

See problems 37 to 48 p. 379

$$\frac{5\pi}{2} < \theta < 3\pi$$



$$\frac{5\pi}{4} < \frac{\theta}{2} < \frac{3\pi}{2}$$



$$\sec\theta = -\frac{13}{5} \Rightarrow \cos\theta = -\frac{5}{13}$$

$$\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-\cos\theta}{2}} = -\sqrt{\frac{1-\left(-\frac{5}{13}\right)}{2}}$$

III

$$= -\sqrt{\frac{13+5}{2(13)}} = -\sqrt{\frac{\cancel{18}\,9}{\cancel{2}(13)}} = -\frac{3}{\sqrt{13}} = \boxed{\frac{-3\sqrt{13}}{13}}$$

9. Given that $\sin\alpha = \frac{4}{5}$, α in Quadrant I, and $\tan\beta = \frac{5}{12}$, β in Quadrant III, then $\sin\left(\frac{\pi}{2} + \alpha - \beta\right)$ is equal to

~~(a)~~ $-\frac{56}{65}$

(b) $-\frac{16}{65}$

(c) $-\frac{4}{13}$

(d) $\frac{56}{65}$

(e) $\frac{36}{65}$

See example 4 P. 567
and problems 37 to 48 P. 570–571

$\sin\left(\frac{\pi}{2} - (\beta - \alpha)\right) = \cos(\beta - \alpha)$

$= \cos\beta\cos\alpha + \sin\beta\sin\alpha$

$= \ldots$

10. If we use a trigonometric identity of the difference of two angles, then the exact value of $\sin 165°$ is equal to

~~(a)~~ $\frac{1}{4}(\sqrt{6} - \sqrt{2})$

(b) $\frac{1}{4}(\sqrt{6} + \sqrt{2})$

(c) $\frac{1}{4}(\sqrt{3} - 1)$

(d) $-\frac{1}{4}(\sqrt{6} + \sqrt{2})$

(e) $-\frac{1}{4}(\sqrt{3} + 1)$

See example 1 P. 564
and problems 1 to 6 P. 570

$\sin 165° = \sin 15° = \sin(45° - 30°)$

Q II → $\sin\theta$
$\theta' = 15°$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4} = \boxed{\frac{1}{4}(\sqrt{6} - \sqrt{2})}$

11. Which one of the following statements is **NOT** an identity?

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See example 1 Page 556 and problems 1 to 10 P. 559

(a) $\sqrt{1+\tan^2 x} = \sec x$

$\sqrt{1+\tan^2} = |\sec x|$

(b) $\dfrac{1-\cos x}{\sin x} = \dfrac{\sin x}{1+\cos x}$

(c) $(\sin x + \cos x)^2 = 2\sin x \cos x + 1$

(d) $2\cos^2 x - 1 = \cos^2 x - \sin^2 x$

(e) $\sin\left(x+\dfrac{\pi}{3}\right) = \dfrac{1}{2}\sin x + \dfrac{\sqrt{3}}{2}\cos x$

I

12. Let θ be an acute angle of a right triangle for which $\sec\theta = \dfrac{3}{2}$, then the exact value of $\dfrac{\tan\theta - \sin\theta}{\csc\theta + \cot\theta}$ is

See example 2 P. 492 and problems 15 to 24 P. 497

(a) $\dfrac{1}{6}$

(b) 0

(c) $\dfrac{1}{30}$

(d) $\dfrac{\sqrt{5}}{6}$

(e) $\dfrac{5}{6}$

$\sec\theta = 3/2 = \dfrac{r}{x} \Rightarrow r = 3,\ x = 2,\ y = \sqrt{5}$

$\tan\theta = \dfrac{\sqrt{5}}{2}$, $\sin\theta = \dfrac{\sqrt{5}}{3}$, $\csc\theta = \dfrac{3}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$

$\cot\theta = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$.

$\dfrac{\tan\theta - \sin\theta}{\csc\theta + \cot\theta} = \dfrac{\frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{3}}{\frac{3\sqrt{5}}{5} + \frac{2\sqrt{5}}{5}} = \dfrac{\frac{3\sqrt{5}-2\sqrt{5}}{6}}{\sqrt{5}}$

$= \dfrac{\sqrt{5}}{6} \div \sqrt{5} = \boxed{\dfrac{1}{6}}$

13. The sum of the solutions of the equation

$$\log(2x-1) = \log(4x-5) + \log_{1/10}(x-1)$$

is equal to

See example 7 p. 412

and problems 26 to 30 p. 415

(a) $\frac{7}{2}$

(b) 3

(c) $\frac{1}{2}$

(d) 2

(e) $\frac{5}{2}$

$\log(2x-1) = \log(4x-5) + \frac{\log(x-1)}{\log 1/10}$

$\log(2x-1) = \log(4x-5) - \log(x-1)$

$\log(2x-1) + \log(x-1) = \log(4x-5)$

$(2x-1)(x-1) = 4x-5 \Rightarrow 2x^2-3x+1 = 4x-5$

$2x^2-7x+6=0$, $(2x-3)(x-2)=0$

$x = 3/2$, $x=2$ $\Rightarrow$ The sum $3/2 + 2 = 7/2$

14. If the terminal side of an angle θ lies on the line $3x+4y=0$, where $x>0$, then the value of $5\sin\theta + 10\cos\theta$ is equal to

See example 1 p.491

and problems 1 to 8 p. 497

(a) 5

(b) 9.5

(c) 11

(d) -4

(e) -3

$y = -\frac{3}{4}x \underset{\text{take } \boxed{x=4}}{=} -\frac{3}{4}(4) = \boxed{-3 = y}$

$r = 5$

$5\sin\theta + 10\cos\theta =$

$5\left(\frac{-3}{5}\right) + 10\left(\frac{4}{5}\right) = -3+8 = \boxed{5}$

15. If $\cos^{-1} x + 2\sin^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$, then $x =$

(a) $\frac{24}{25}$

(b) $\frac{18}{25}$

(c) $\frac{4}{5}$

(d) $-\frac{21}{25}$

(e) $\frac{6}{5}$

See example 5 p. 598
and problems 49 to 52 and
63 to 66 p. 602

16. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\sin\frac{5\pi}{7}\right) =$

(a) $-\frac{\pi}{21}$

(b) $\frac{7\pi}{6}$

(c) $\frac{2\pi}{21}$

(d) $-\frac{3\pi}{14}$

(e) $-\frac{5\pi}{21}$

See example 1 p. 593
See example 2(f) p. 596
and problems 1 to 46 p. 601

17. If T is the vertical translation, S is the phase shift and P is the period of the graph of $y = -\frac{3}{4}\left[\cot\left(\frac{x}{4} + 3\pi\right) + 12\pi\right]$, then $T + S + P$ is equal to

(a) -17π

(b) -12π

(c) 15π

(d) 18π

(e) -10π

See problems 15, 16 P. 535

$y = \underbrace{-\frac{3}{4}}_{a} \cot\left(\underbrace{\frac{x}{4}}_{b} + \underbrace{3\pi}_{c}\right) \underbrace{-9\pi}_{d}$

$T = -9\pi$, $S = -\frac{c}{b} = -\frac{3\pi}{\frac{1}{4}} = -12\pi$

$P = \frac{\pi}{\frac{1}{4}} = 4\pi$.

$T + S + P = -9\pi - 12\pi + 4\pi = \boxed{-17\pi}$

18. The number of the vertical asymptotes of the graph of $y = -3\cot\left(\frac{2x}{3}\right)$ on the interval $\left[-\frac{3\pi}{4}, \frac{15\pi}{4}\right]$ is

(a) 3

(b) 2

(c) 4

(d) 5

(e) 6

See example 3 P. 523

and problems 31, 32, 40, 48 and 49 P. 527

$b = \frac{2}{3}$ VA: $x = n\frac{\pi}{b} - \frac{c}{b}$

$= n\frac{\pi}{\frac{2}{3}} - 0 = \frac{3n\pi}{2}$

$-\frac{\cancel{3}\cancel{\pi}}{4} \leq \frac{\cancel{3}n\cancel{\pi}}{2} \leq \frac{\cancel{15}^{5}\cancel{\pi}}{4}$

$\times 2$ $\quad -\frac{1}{2} \leq n \leq \frac{5}{2} \Rightarrow n = 0, 1, 2$

3 V.A.

19. If $A = (\log_3 125) \cdot \left(\log_5 \sqrt[3]{3}\right)$ and $B = \left(\sqrt[4]{5}\right)^{-2\log_5 9}$, then $A - B =$

(a) $\frac{2}{3}$

(b) $-\frac{1}{3}$

(c) $-\frac{4}{5}$

(d) 0

(e) $\frac{1}{3}$

See examples 1 and 2 p. 384 and problems 41, 42 p. 404

$\log_3 5^3 \cdot \log_5 3^{1/3} = \cancel{3}(\log_3 5)\left(\frac{1}{\cancel{3}}\log_5 3\right)$

$= \frac{\log_5 5}{\log_5 3} \cdot \log_5 3 = 1$

$\left(\sqrt[4]{5}\right)^{-2\log_5 9} = 5^{\frac{1}{4}(-2)\log_5 9} = 5^{\log_5 9^{-1/2}} = \frac{1}{3}$

$A - B = 1 - \frac{1}{3} = \boxed{\frac{2}{3}}$

20. The graph of the function $f(x) = \left|2^{x-1} - 4\right|$ is increasing on the interval

(a) $(3, \infty)$

(b) $(-\infty, \infty)$

(c) $(-\infty, 3)$

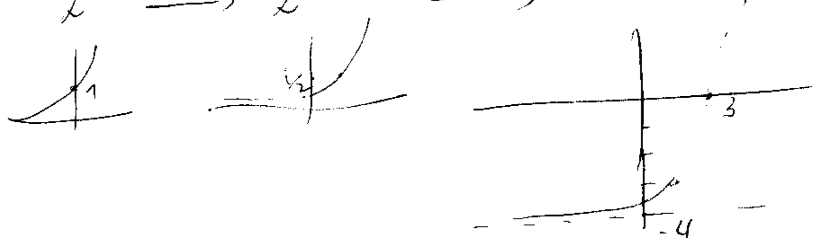
(d) $(0, \infty)$

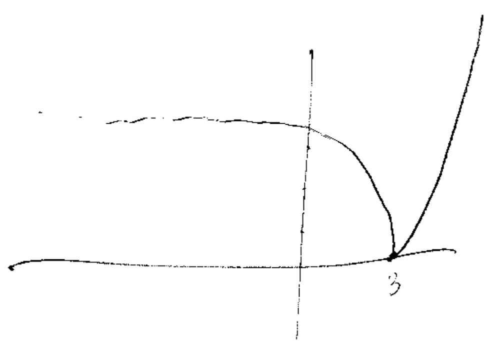
(e) $(-\infty, 0)$

See problem 15 p. 377

$f(x) = 2^{x-1}$

$2^x \longrightarrow 2^{(x-1)} \longrightarrow 2^{x-1} - 4$





$2^{x-1} = 4$

$x - 1 = 2$

$\boxed{x = 3}$

21. The expression $4\sin x\cos^3 x - 4\cos x\sin^3 x$ simplifies to

(a) $\sin 4x$

(b) $2\sin 4x$

(c) $2\cos 4x$

(d) $\cos 4x$

(e) $4\sin^2 4x$

$= 4\sin x\cos x\,(\cos^2 x - \sin^2 x)$

$= 2\,(\sin 2x)\,(\cos 2x)$

$= \sin 4x$.

#62 p. 579

22. The graph of the function $y = 3\sec\left(\pi x - \dfrac{\pi}{3}\right)$, **on the interval** $\left[-\dfrac{1}{6}, \dfrac{7}{3}\right]$, intersects the line $y = 5$ at [Hint: sketch]

See problems 25, 28 p. 535 and 63 p. 536

(a) three points

(b) four points

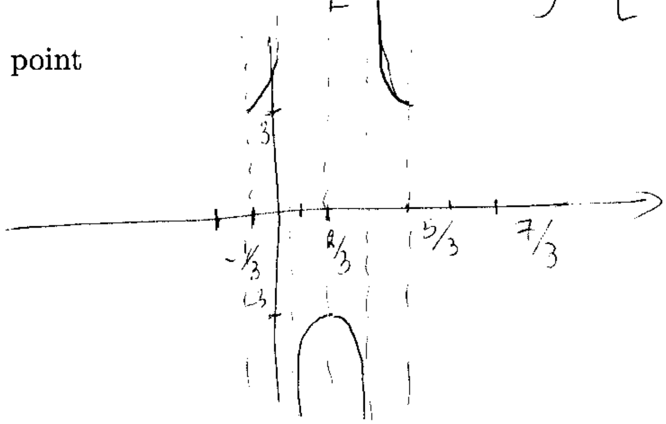
(c) two points

(d) one point

(e) no point

$P = \dfrac{2\pi}{\pi} = 2$ $\quad PS = \dfrac{-\pi/3}{\pi} = -1/3$

Cycle: $\left[-1/3,\ 2 - 1/3\right] = \left[-1/3,\ 5/3\right]$



23. The exact value of $2\sin^2 5^\circ + 2\sin^2 85^\circ + 5\sin 217^\circ + 5\cos 307^\circ$ is equal to

See example 2 p. 565
and problems 19 to 24 p. 570

(a) 2

(b) 1

(c) -3

(d) 7

(e) 0

$2\sin^2 5^\circ + 2\cos^2(90-85^\circ) - 5\sin(37^\circ) + 5\cos(53^\circ)$

$2\sin^2 5^\circ + 2\cos^2(5^\circ) - 5\sin 37^\circ + 5\sin(90^\circ - 53^\circ)$

$2\underbrace{(\sin^2 5^\circ + \cos^2 5^\circ)}_{1} - \cancel{5\sin 37^\circ} + \cancel{5\sin 37^\circ}$

$= \boxed{2}$

24. If from the top of a tower 200 feet high, the angles of depression of the top and bottom of a building opposite to the tower are observed to be 30° and 60°, respectively, then the height of the building is equal to

(a) $\frac{400}{3}$ feet

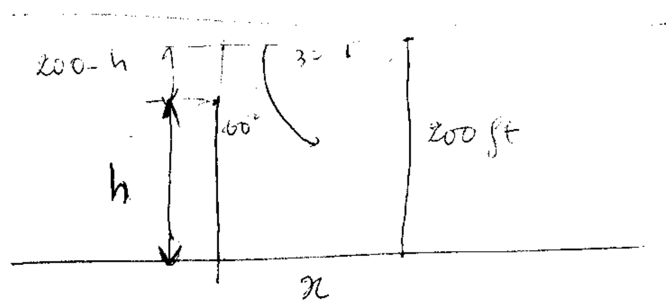
(b) $\frac{400\sqrt{3}}{3}$ feet

(c) $100\sqrt{3}$ feet

(d) $\frac{200\sqrt{3}}{3}$ feet

(e) $\frac{350}{3}$ feet

See examples 5, 6 p. 483
and problem 76 p. 487



$\tan 30^\circ = \frac{200-h}{x}$, $\tan 60^\circ = \frac{200}{x}$

$\sqrt{3} = \frac{200}{x}$

$x = \frac{200}{\sqrt{3}} = \boxed{\frac{200\sqrt{3}}{3}}$

$\frac{1}{\sqrt{3}} = \frac{(200-h)\,3}{200\sqrt{3}}$

$1 = 3 - \frac{3h}{200}$

$\Rightarrow \frac{3h}{200} = 2 \Rightarrow \boxed{h = \frac{400}{3}\text{ ft}}$

25. $\dfrac{\sin 5^\circ + \cos 5^\circ}{\sqrt{2}} =$

see example 4, 5 P. 585
and problems 59 to 66
P. 588 .

(a) $\sin 50^\circ$

(b) $\sin 40^\circ$

(c) $\sin 10^\circ$

(d) $\dfrac{\sqrt{2}}{2}\sin 10^\circ$

(e) $\dfrac{\sqrt{2}}{2}\sin 50^\circ$

$\sin 5^\circ + \cos 5^\circ$

$a = 1, \quad b = 1. \Rightarrow k = \sqrt{1+1} = \sqrt{2}$

$\cos\alpha = \frac{1}{\sqrt{2}} \qquad \sin\alpha = \frac{1}{\sqrt{2}}$

$\Rightarrow \alpha = \pi/4 = 45^\circ$

$\sin 5^\circ + \cos 5^\circ = \sqrt{2}\left(\sin(5^\circ + 45^\circ)\right)$

$= \sqrt{2}\,\sin(50^\circ)$

$\Rightarrow \dfrac{\sin 5^\circ + \cos 5^\circ}{\sqrt{2}} = \dfrac{\sqrt{2}\sin 50^\circ}{\sqrt{2}} = \boxed{\sin 50^\circ}$