King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Prep-Year Math II
SECOND EXAM
Semester I, Term 061
Tuesday, December 05, 2006
Net Time Allowed: 75 minutes

Sources Of Problems

MASTER VERSION

1. **The coordinates of the focus** of the parabola that passes through the origin and the points $(-3, 12)$ and $(3, 12)$ are

(a) $\left(0, \frac{3}{16}\right)$

See example 2 p. 687

See problems 27 and 28 p. 692

(b) $\left(0, \frac{3}{4}\right)$

(c) $\left(0, \frac{5}{16}\right)$

(d) $\left(0, -\frac{3}{4}\right)$

(e) $\left(0, \frac{1}{16}\right)$

2. If $\tan\alpha = -\frac{4}{3}, \frac{3\pi}{2} < \alpha < 2\pi,$ then $\sec\frac{\alpha}{2} =$

(a) $-\frac{\sqrt{5}}{2}$

See Problems 37 to 48 p. 579

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{5}}$

(d) $-\frac{\sqrt{3}}{2}$

(e) $-\sqrt{5}$

3. The expression $\dfrac{\sin 2x - \sin x}{2\cos^2 x + \cos x - 1}$ simplifies to

(a) $\tan \dfrac{x}{2}$

See problems 55, 56, 58 and 89
p. 579–580

(b) $\cot \dfrac{x}{2}$

(c) $\cos \dfrac{x}{2}$

(d) $\sin \dfrac{x}{2}$

(e) $\sec \dfrac{x}{2}$

4. The equation of the parabola with focus at $(-8, 1)$ and directrix $x - 4 = 0$ is

See example 4 p. 689

See problems 29 to 32 p. 692

(a) $(y-1)^2 = -24(x+2)$

(b) $(y-1)^2 = -6(x+2)$

(c) $(x+2)^2 = -24(y-1)$

(d) $(x+2)^2 = 24(y-1)$

(e) $(y+1)^2 = -24(x-2)$

5. If the function $y = -3\sin 2x - 3\cos 2x$ is written in the form $y = k\sin(2x+\beta)$, $0 < \beta < 2\pi$, then the values of k and β are

(a) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{4}$

(b) $k = -6,\ \beta = \frac{5\pi}{4}$

(c) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{8}$

(d) $k = -6,\ \beta = \frac{3\pi}{4}$

(e) $k = 3\sqrt{2},\ \beta = \frac{7\pi}{4}$

See example 5 P. 585

See problems 49 to 66 P. 588

6. The exact value of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right) + \tan\left(\cos^{-1} -\frac{1}{2}\right)$ is

(a) $-\frac{\pi}{6} - \sqrt{3}$

(b) $\frac{7\pi}{6} + \frac{\sqrt{3}}{3}$

(c) $\frac{\pi}{6} + \sqrt{3}$

(d) $\frac{5\pi}{6} - 1$

(e) $-\frac{\pi}{6} + \sqrt{3}$

See examples 2 and 3 P. 596

See problems 21 to 52 P. 602

7. The eccentricity of the ellipse $9x^2+4y^2+36x-8y+4=0$ is equal to

See example 2 p. 699
and example 4 p. 702
See problems 49 and 50 p. 705

(a) $\frac{\sqrt{5}}{3}$

(b) $\frac{\sqrt{5}}{4}$

(c) $\frac{\sqrt{11}}{3}$

(d) $\frac{\sqrt{13}}{4}$

(e) $\frac{4}{5}$

8. The equation of the ellipse in the standard form with vertices $(-2,4)$ and $(-2,-2)$, and passing through $(0,1)$ is

See example 3 p. 701
See problems 35 to 44 p. 705

(a) $\frac{(x+2)^2}{4}+\frac{(y-1)^2}{9}=1$

(b) $\frac{(x+2)^2}{4}+\frac{(y-1)^2}{25}=1$

(c) $\frac{(x-2)^2}{4}+\frac{(y-1)^2}{25}=1$

(d) $\frac{(x+2)^2}{3}+\frac{(y-2)^2}{12}=1$

(e) $\frac{(x-2)^2}{4}+\frac{(y+1)^2}{9}=1$

9. If $0 \leq x < 2\pi$, then the sum of all solutions of the equation $2\cos^2 x + \cos x - 1 = 0$ is qual to

~~(a)~~ 3π

(b) π

(c) 2π

(d) $\frac{4\pi}{3}$

(e) $\frac{5\pi}{3}$

See example 1 p. 605
and example 3 p. 606
See problems 43 and 44 p. 614

10. If θ is the smallest positive angle between the vectors $u = 3\mathrm{i} - 4\mathrm{j}$ and $v = -2\mathrm{i} + \mathrm{j}$, then $\tan\theta$ is equal to

~~(a)~~ $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{3}$

(e) $-\frac{1}{5}$

See example 9 p. 658
See problems 53 to 60 p. 662

11. Given the vectors $\mathbf{u} = \langle -4, 10 \rangle$ and $\mathbf{v} = \langle -5, 1 \rangle$. If the vector $\mathbf{w} = \langle a, b \rangle$ is a **unit** vector **in the opposite direction** of $\frac{1}{2}\mathbf{u} - \mathbf{v}$, then $a + b$ is equal to

See example 3 p. 653
and example 4 p. 654
See problems 7 to 20 p. 661

(a) $-\frac{7}{5}$

(b) $-\frac{3}{5}$

(c) $-\frac{4}{5}$

(d) $-\frac{2}{5}$

(e) $-\frac{9}{5}$

12. Which one of the following statements is **FALSE** about the graph of $y = \tan^{-1}(x+1) - \frac{\pi}{2}$?

See example 7 p. 599 and
problem 81 p. 602

(a) the graph has only one x-intercept

(b) the graph has only one y-intercept

(c) the graph has two asymptotes $y = -\pi$ and $y = 0$

(d) the graph increases for all real numbers x

(e) the graph lies completely under the x-axis

13. The exact value of the expression

$$\sin\frac{13\pi}{12}\cos\frac{\pi}{12}$$

is equal to

An application of the Identity

$$\sin\theta = \frac{1}{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}.$$

~~(a)~~ $-\frac{1}{4}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

(d) $-\frac{1}{8}$

(e) $\frac{1}{2}$

14. If $\cos^{-1}\frac{x}{2}+\sin^{-1}\left(-\frac{3}{5}\right)=\frac{\pi}{3}$, then $x=$

See example 5 p. 598

See problems 57 & 66 P. 602

~~(a)~~ $\frac{4-3\sqrt{3}}{5}$

(b) $\frac{3-4\sqrt{3}}{5}$

(c) $\frac{8-6\sqrt{3}}{5}$

(d) $\frac{4+3\sqrt{3}}{10}$

(e) $\frac{4-3\sqrt{3}}{20}$

15. If $0 \leq x < 2\pi$, then the number of all solutions of the equation $2\sin\left(2x + \frac{\pi}{6}\right) - 1 = 0$ is

~~(a)~~ 4

See problems 61 to 70 p. 615

(b) 6

(c) 8

(d) 2

(e) 10