1. Let $f(x) = -1 + 5^{-x+1}$.

   (a) (1 point) Find the x-intercept of f.

$x\text{-int} \Rightarrow y = 0 \Rightarrow -1 + 5^{-x+1} = 0 \Rightarrow 5^{-x+1} = 1$ $\frac{1}{2}$

$\Rightarrow -x + 1 = 0$

$\Rightarrow \boxed{x = 1}$ $\frac{1}{2}$

   (b) (1 point) Find the y-intercept of f. x-int is $(1,0)$

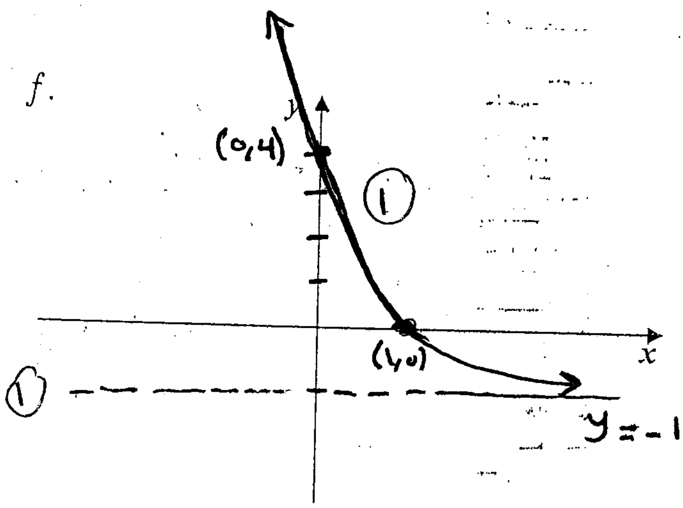
$y\text{-int} \Rightarrow x = 0 \Rightarrow y = -1 + 5 = 4$

$(0, 4)$ 1

   (c) (1 point) Find the asymptote of f.

$y = -1$ 1

   (d) (2 points) Sketch the graph of f.



2. (a) (1 point) Change the equation $\ln(x-1) = 4$ to its exponential form.

$\ln(x-1) = 4 \Leftrightarrow x - 1 = e^4$

1

   (b) (1 point) Change the equation $10^x = 25$ to its logarithmic form.

$10^x = 25 \Leftrightarrow \log 25 = x$

1

3. (4 points) Given that $\log_2 3 = a$ and $\log_2 5 = b$, find $\log 36$ in terms of a and b.

(1) (1)

$$\text{Log } 36 = \frac{\log_2 36}{\log_2 10} = \frac{\log_2 (4)(9)}{\log_2 (5)(2)} = \frac{\log_2 4 + \log_2 9}{\log_2 5 + \log_2 2}$$

(1)

$$= \frac{2 \text{ Log}_2 2 + 2 \log_2 3}{\log_2 5 + \log_2 2} = \frac{2 + 2a}{b+1} = \frac{2+2a}{1+b}$$

$$= \frac{2(1+a)}{1+b} \quad (1)$$

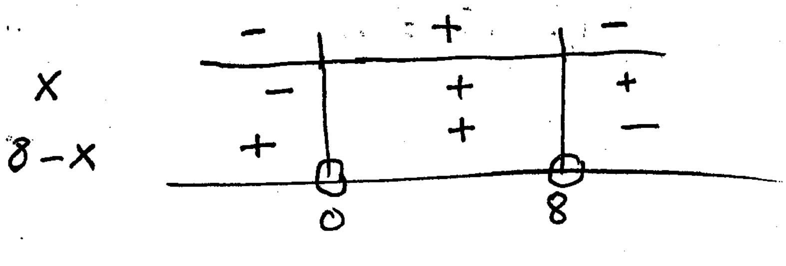
4. (3 points) Express the logarithmic expression $\log \left[\frac{(2x+5)\sqrt{w}}{yz^2}\right]$ in terms of logarithms of $x, y, z,$ and w where $x, y, z, w > 0$.

$$\log \frac{(2x+5)\sqrt{w}}{y\,z^2} = \log(2x+5) + \log\sqrt{w} - \log y\,z^2 \quad (1)$$

$$= \underbrace{\log(2x+5) + \tfrac{1}{2}\log w}_{(1)} \underbrace{- \log y - 2 \text{ Log } z}_{(1)}$$

5. (2 points) Find the domain of the logarithmic function

$f(x) = \log_3 \left(\frac{x}{8-x}\right)$. $D_f: \frac{x}{8-x} > 0$ $(\frac{1}{2})$



(1)

$\therefore D_f = (0, 8)$ $(\frac{1}{2})$

6. Find the solution set of each of the following equations:

(a) (4 points) $\dfrac{e^x - 7e^{-x}}{2} = 3.$

$e^x - 7e^{-x} - 6 = 0$ --- 1 pt

$\Rightarrow e^{2x} - 7 - 6e^x = 0$

$(e^x - 7)(e^x + 1) = 0$ --- 2 pts

$x = \ln 7$ --- 1 pt

(b) (3 points) $\ln x = \dfrac{1}{2}\ln\left(2x + \dfrac{5}{2}\right) + \dfrac{1}{2}\ln 2.$

$\ln x = \ln\left[\left(2x+\frac{5}{2}\right)2\right]^{1/2}$ ... $1/2$ pt

$x = \left[\left(2x+\frac{5}{2}\right)2\right]^{1/2}$ --- $1/2$ pt

$\Rightarrow x^2 = 4x + 5$

$(x-5)(x+1) = 0$ --- 1 pt

$\Rightarrow x = 5$ --- 1 pt

7. (3 points) Find the value of $(\sqrt{3})^{\log 4/\log 3} + \log_{0.3}\dfrac{100}{9}.$

$2 + \log_{0.3} 10 - 2\log_{0.3} 3$ --- 1 pt

$\left.\begin{array}{l}\Rightarrow 2 + 2\log_{0.3}\frac{10}{3} \\ \Rightarrow 2 - 2\log_{\frac{10}{3}}\frac{10}{3}\end{array}\right\}$ --- 1 pt

$= 0$ --- 1 pt

8. (2 points) If $\theta = 0.5$ radian, find the complementary and supplementary angles of θ.

$\theta = \frac{\pi}{2} - \frac{1}{2} = \frac{\pi - 1}{2}$ or $1.57 - 0.5 = 1.07$ ... 1pt

Supplement of $\theta = \pi - \frac{1}{2} = \frac{2\pi - 1}{2}$ --- 1pt

9. (3 points) Find the largest negative angle that is coterminal to $\theta = -975°$, then determine the quadrant where the angle θ terminates.

$-975° + 360k$ --- 1pt

$\Rightarrow -975 + 720$ --- 1pt

$= -255°$ --- 1pt

10. (a) (1 point) Convert $\frac{7\pi}{3}$ radians to degrees.

$\left(7 \times \frac{180}{3}\right) = 420°$ --- 1pt

(b) (2 points) Find the angular speed in radian per second of a wheel rotating at 50 revolutions per minute.

$\left(\frac{50 \times 2\pi}{60}\right)$ --- 1pt

$\frac{5\pi}{3}$ rad / sec --- 1pt

11. Let $f(x) = -\frac{2}{3}\sin\frac{\pi x}{3}$.

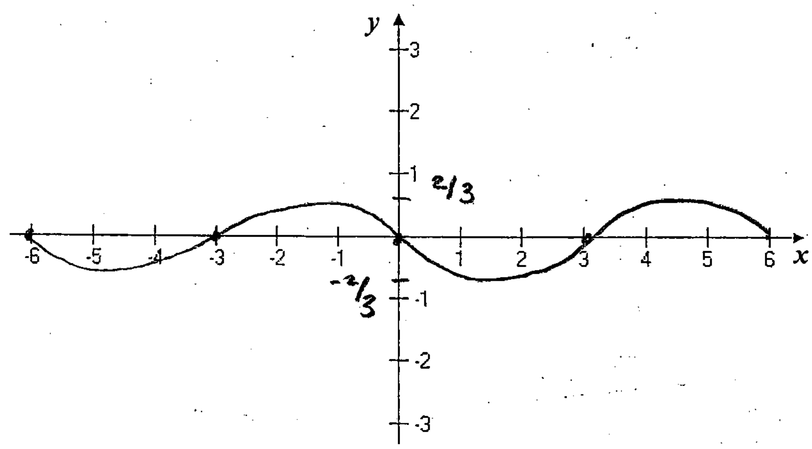
(a) (1 point) Find the period of f.

Period $= \frac{2\pi}{\pi/3} = 6$ --- 1pt

(b) (1 point) Find the amplitude of f.

$\left|-\frac{2}{3}\right| = \frac{2}{3}$ --- 1pt

(c) (2 points) Sketch the graph of f over the interval $-6 \le x \le 6$.



12. (2 points) Find the length of an arc that subtends a central angle of 120° in a circle of radius 10 centimeters.

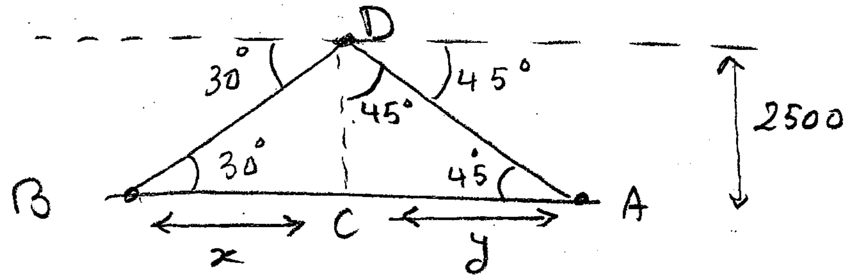
$r = 10\,cm$, $\theta = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ --- 1pt

$S = r\theta = 10 \times \frac{2\pi}{3} = \frac{20\pi}{3}\,cm$ --- 1pt

$\left(-\frac{1}{2}\right.$ for not writing unit$\left.\right)$

13. (4 points) The angle of depression to an object A on one side of a road, measured from a baloon 2500 feet above the road, is $45°$. The angle of depression to an object B on the opposite side of the road is $30°$. Find the distance between A and B.

2 pts ----



$$\tan 30° = \frac{2500}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{2500}{x}$$ ... 1 pt

$$\Rightarrow x = 2500\sqrt{3}$$

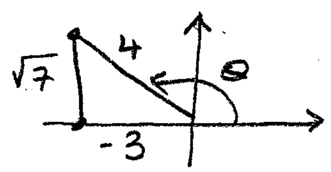
$y = 2500$ feet --- ½ pt

$\Rightarrow$ Dist. $= x + y$

$= 2500(\sqrt{3}+1)$ ... ½

14. (2 points) Given that $\sin\theta = \dfrac{\sqrt{7}}{4}$ and $\tan\theta < 0$, find $\sec\theta$.

θ in Quadrant II --- ½ pt



... 1 pt

$$\sec\theta = -\frac{4}{3}$$ ... 1 pt

15. (3 points) Find $\sin\theta + \cos\theta$ where θ is an angle in standard position whose terminal side contains the point $P(-3, 2)$.

Hense $x = -3,\ y = 2$; $r = \sqrt{13}$ ... ½ pt

$$\sin\theta + \cos\theta = \frac{2}{\sqrt{3}} - \frac{3}{\sqrt{3}}$$ --- 2 pts

$$= \frac{-1}{\sqrt{3}}$$ --- ½ pt

16. (4 points) Find the value of $2\sin 210° + 4\cos 405° + \sqrt{3}\tan(-300°)$.

$= 2(-\sin 30°) + 4\cos 45° - \sqrt{3}\tan 300°$ --- $1\frac{1}{2}$ pts

$= -2\cdot\frac{1}{2} + 4\cdot\frac{\sqrt{2}}{2} - \sqrt{3}(-\sqrt{3})$ --- $1\frac{1}{2}$ pts

$= -1 + 2\sqrt{2} + 3 = 2 + 2\sqrt{2}$ --- 1 pt

17. (3 points) If W is the wrapping function, then find $W\left(-\frac{5\pi}{4}\right)$.

$W\left(-\frac{5\pi}{4}\right) = \left(\cos\left(-\frac{5\pi}{4}\right), \sin\left(-\frac{5\pi}{4}\right)\right)$ --- 1pt

$= \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ --- 2pts.

18. (4 points) Write the expression $\dfrac{\csc t + \cot t}{\sec t + \tan t} \cdot \dfrac{1 + \sin t}{1 + \cos t}$ as a single trigonometric function.

$$\frac{\frac{1}{\sin t} + \frac{\cos t}{\sin t}}{\frac{1}{\cos t} + \frac{\sin t}{\cos t}} \cdot \frac{1 + \sin t}{1 + \cos t}$$ --- 1 pt

$$= \frac{\frac{1 + \cos t}{\sin t}}{\frac{1 + \sin t}{\cos t}} \cdot \frac{1 + \sin t}{1 + \cos t}$$ --- 1pt

$$= \frac{\cancel{1 + \cos t}}{\cancel{1 + \sin t}} \cdot \frac{\cos t}{\sin t} \cdot \frac{\cancel{1 + \sin t}}{\cancel{1 + \cos t}} = \cot t$$ --- 1pt