King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Prep-Year Math II
MIDTERM EXAM
Semester II, Term 062
Saturday, April 21, 2007
Net Time Allowed: 120 minutes

Sources of Problems

MASTER VERSION

1. Let $f(x) = a \sin bx$, where $b > 0$. If the period of f is 12 and $f(3) = 4$, then $f(25) =$

(a) 2

See problems 1 to 16 p. 518

(b) 6

(c) 4

(d) 0

(e) 8

2. The adjacent figure represents a part of the graph of

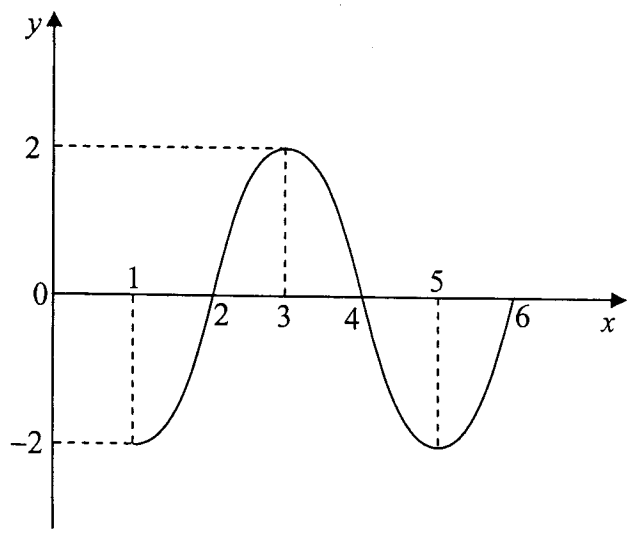
(a) $y = -2 \sin \frac{\pi}{2} x$

(b) $y = -2 \cos \frac{\pi}{2} x - 2$

(c) $y = 2 \sin \frac{\pi}{2} x$

(d) $y = 2 \cos \frac{\pi}{2} x + 2$

(e) $y = 2 \sin \left(\frac{\pi}{2} x - 1 \right)$



See example 2 p. 514
and problem 30 p. 518
and example 7 p. 531
and problems 33 to 44 p. 535

3. $\log_4(\log_2(\log_{\sqrt{3}} 9)) =$

See problems 21 to 30 P. 391

(a) $\frac{1}{2}$

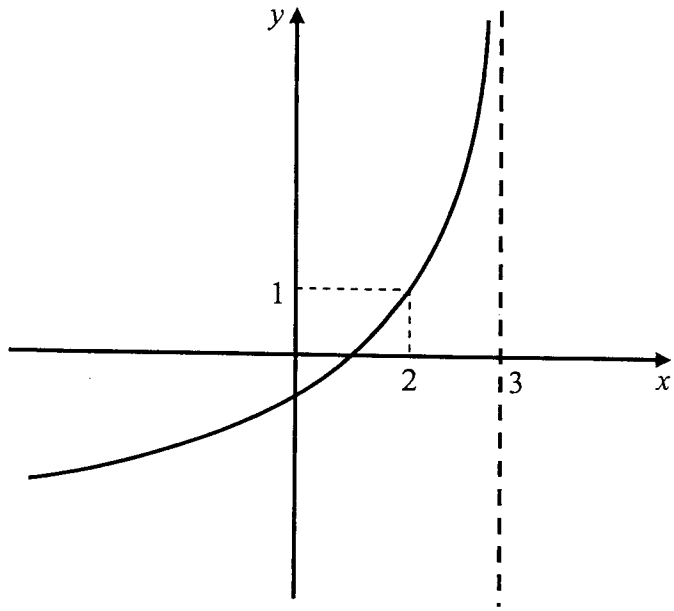
(b) $\frac{1}{4}$

(c) 2

(d) 1

(e) 0

4. The following figure represents the graph of



(a) $f(x) = 1 - \ln(-x+3)$

(b) $f(x) = 1 + \ln(-x+3)$

(c) $f(x) = 1 - \ln|x-3|$

(d) $f(x) = |-1 + \ln(-x+3)|$

(e) $f(x) = -1 - \ln(-x+3)$

See example 6 P. 388

See problems 57, 58 P. 391-392

5. Which one of the following statements is **FALSE**?

(a) $\log_{\sqrt{2}} x = y$ if and only if $y = (\sqrt{2})^x$

(b) $5^{3+x} = y$ if and only if $\log_5 y = x + 3$

(c) $\log_3 \frac{1}{81} = x$ if $3^x = 3^{-4}$

(d) $e^{x+2} = y - 3$ if and only if $\ln(y - 3) = x + 2$

(e) $2^{\frac{1}{2}\log_2 x} = y$ if and only if $y = \sqrt{x}$

See examples 1, 2 p. 384 and problems 1 to 20 p. 391

6. A car has wheels that are 3.6 feet in diameter. If the wheels, rolling without slipping, turn through 72° degrees, then the distance moved by the car is equal to

(a) $\frac{18\pi}{25}$ feet

(b) $\frac{21\pi}{25}$ feet

(c) $\frac{13\pi}{25}$ feet

(d) $\frac{19\pi}{25}$ feet

(e) $\frac{17\pi}{25}$ feet

See discussion opposite to Figure 5.28 p. 471

7. The expression $\dfrac{1-\sin\theta}{1+\sin\theta}$ is identical to

(a) $(\sec\theta - \tan\theta)^2$ See problem 65 p. 561

(b) $(\sec\theta + 1)^2$

(c) $(1+\tan\theta)^2$

(d) $(1-\sec\theta)^2$

(e) $(\csc\theta + 1)^2$

8. If $\sec\theta = -\dfrac{13}{5}$, where $\dfrac{5\pi}{2} < \theta < 3\pi$, then the exact value of $\sin\left(\dfrac{\theta}{2}\right)$ is

See problems 37 to 48 p. 379

(a) $\dfrac{-3\sqrt{13}}{13}$

(b) $\dfrac{-2\sqrt{13}}{13}$

(c) $\dfrac{6}{13}$

(d) $\dfrac{3\sqrt{13}}{13}$

(e) $\dfrac{-1}{13}$

9. Given that $\sin\alpha = \frac{4}{5}$, α in Quadrant I, and $\tan\beta = \frac{5}{12}$, β in Quadrant III, then $\sin\left(\frac{\pi}{2} + \alpha - \beta\right)$ is equal to

See example 4 P. 567
and problems 37 to 48 P. 570-571

(a) $-\frac{56}{65}$

(b) $-\frac{16}{65}$

(c) $-\frac{4}{13}$

(d) $\frac{56}{65}$

(e) $\frac{36}{65}$

10. If we use a trigonometric identity of the difference of two angles, then the exact value of $\sin 165°$ is equal to

See example 1 P. 564
and problems 1 to 6 P. 570

(a) $\frac{1}{4}(\sqrt{6} - \sqrt{2})$

(b) $\frac{1}{4}(\sqrt{6} + \sqrt{2})$

(c) $\frac{1}{4}(\sqrt{3} - 1)$

(d) $-\frac{1}{4}(\sqrt{6} + \sqrt{2})$

(e) $-\frac{1}{4}(\sqrt{3} + 1)$

11. Which one of the following statements is **NOT** an identity?

See example 1 Page 556 and problems 1 to 10 P. 559

(a) $\sqrt{1+\tan^2 x} = \sec x$

(b) $\dfrac{1-\cos x}{\sin x} = \dfrac{\sin x}{1+\cos x}$

(c) $(\sin x + \cos x)^2 = 2\sin x \cos x + 1$

(d) $2\cos^2 x - 1 = \cos^2 x - \sin^2 x$

(e) $\sin\left(x + \dfrac{\pi}{3}\right) = \dfrac{1}{2}\sin x + \dfrac{\sqrt{3}}{2}\cos x$

12. Let θ be an acute angle of a right triangle for which $\sec\theta = \dfrac{3}{2}$, then the exact value of $\dfrac{\tan\theta - \sin\theta}{\csc\theta + \cot\theta}$ is

See example 2 P. 492 and problems 15 to 24 P. 497

(a) $\dfrac{1}{6}$

(b) 0

(c) $\dfrac{1}{30}$

(d) $\dfrac{\sqrt{5}}{6}$

(e) $\dfrac{5}{6}$

13. The sum of the solutions of the equation

$$\log(2x-1) = \log(4x-5) + \log_{1/10}(x-1)$$

is equal to

See example 7 P. 412
and problems 26 to 30 P. 415

(a) $\frac{7}{2}$

(b) 3

(c) $\frac{1}{2}$

(d) 2

(e) $\frac{5}{2}$

14. If the terminal side of an angle θ lies on the line $3x + 4y = 0$, where $x > 0$, then the value of $5\sin\theta + 10\cos\theta$ is equal to

See example 1 p.491
and problems 1 to 8 p.497

(a) 5

(b) 9.5

(c) 11

(d) -4

(e) -3

15. If $\cos^{-1} x + 2\sin^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$, then $x =$

(a) $\frac{24}{25}$

(b) $\frac{18}{25}$

(c) $\frac{4}{5}$

(d) $-\frac{21}{25}$

(e) $\frac{6}{5}$

See example 5 p. 598
and problems 49 to 52 and
63 to 66 p. 602

16. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\sin\frac{5\pi}{7}\right) =$

(a) $-\frac{\pi}{21}$

(b) $\frac{7\pi}{6}$

(c) $\frac{2\pi}{21}$

(d) $-\frac{3\pi}{14}$

(e) $-\frac{5\pi}{21}$

See example 1 p. 593
See example 2(f) p. 596
and problems 1 to 46 p. 601

17. If T is the vertical translation, S is the phase shift and P is the period of the graph of $y = -\frac{3}{4}\left[\cot\left(\frac{x}{4} + 3\pi\right) + 12\pi\right]$, then $T + S + P$ is equal to

See problems 15, 16 P. 535

(a) -17π

(b) -12π

(c) 15π

(d) 18π

(e) -10π

18. The number of the vertical asymptotes of the graph of $y = -3\cot\left(\frac{2x}{3}\right)$ on the interval $\left[-\frac{3\pi}{4}, \frac{15\pi}{4}\right]$ is

See example 3 P. 523 and problems 31, 32, 40, 48 and 49 P. 527

(a) 3

(b) 2

(c) 4

(d) 5

(e) 6

19. If $A = (\log_3 125) \cdot \left(\log_5 \sqrt[3]{3}\right)$ and $B = \left(\sqrt[4]{5}\right)^{-2\log_5 9}$, then $A - B =$

See examples 1 and 2 p. 384
and problems 41, 42 p. 404

(a) $\frac{2}{3}$

(b) $-\frac{1}{3}$

(c) $-\frac{4}{5}$

(d) 0

(e) $\frac{1}{3}$

20. The graph of the function $f(x) = \left|2^{x-1} - 4\right|$ is increasing on the interval

See problem 15 p. 377

(a) $(3, \infty)$

(b) $(-\infty, \infty)$

(c) $(-\infty, 3)$

(d) $(0, \infty)$

(e) $(-\infty, 0)$

21. The expression $4\sin x\cos^3 x - 4\cos x\sin^3 x$ simplifies to

(a) $\sin 4x$

(b) $2\sin 4x$

(c) $2\cos 4x$

(d) $\cos 4x$

(e) $4\sin^2 4x$

$= 4\sin x\cos x\,(\cos^2 x - \sin^2 x)$

$= 2(\sin 2x)(\cos 2x)$

$= \sin 4x$.

#62 p. 579

22. The graph of the function $y = 3\sec\left(\pi x - \frac{\pi}{3}\right)$, **on the interval** $\left[-\frac{1}{6}, \frac{7}{3}\right]$, intersects the line $y = 5$ at [Hint: sketch]

(a) three points

(b) four points

(c) two points

(d) one point

(e) no point

See problems 25, 28 p. 535 and 63 p. 536

23. The exact value of $2\sin^2 5°+2\sin^2 85°+5\sin 217°+5\cos 307°$ is equal to

See example 2 p. 565 and problems 19 & 24 p. 570

(a) 2

(b) 1

(c) -3

(d) 7

(e) 0

24. If from the top of a tower 200 feet high, the angles of depression of the top and bottom of a building opposite to the tower are observed to be 30° and 60°, respectively, then the height of the building is equal to

See examples 5, 6 p. 483 and problem 76 p. 487

(a) $\frac{400}{3}$ feet

(b) $\frac{400\sqrt{3}}{3}$ feet

(c) $100\sqrt{3}$ feet

(d) $\frac{200\sqrt{3}}{3}$ feet

(e) $\frac{350}{3}$ feet

25. $\dfrac{\sin 5^\circ + \cos 5^\circ}{\sqrt{2}} =$

See example 4, 5 p. 585
and problems 59 to 66
p. 588.

(a) $\sin 50^\circ$

(b) $\sin 40^\circ$

(c) $\sin 10^\circ$

(d) $\dfrac{\sqrt{2}}{2} \sin 10^\circ$

(e) $\dfrac{\sqrt{2}}{2} \sin 50^\circ$