King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Prep-Year Math II
MIDTERM EXAM
Semester I, Term 061
Tuesday, November 14, 2006
Net Time Allowed: 120 minutes

"Sourcres of Problems"

MASTER VERSION

1. If for the function $f(x) = -3\sin(\pi x - 2) + 5$, A is its amplitude, P is its period, M is its maximum value and m is its minimum vlaue, then $\dfrac{A+P}{M+m} =$

(a) $\frac{1}{2}$

(b) 2

(c) $\frac{1}{4}$

(d) 4

(e) 1

See examples 2 and 3 p. 514-515
3 and 4 p. 531

See problems 1 to 8 p. 535

2. If f^{-1} is the inverse of the function $f(x) = 2^{-x+1} - 3$, then $f^{-1}(x) =$

(a) $1 - \log_2(x+3)$

(b) $3 + \log_2(x-1)$

(c) $-1 + \log_2(x+3)$

(d) $3 + \log_2(x+3)$

(e) $-1 + \log_2(3-x)$

A direct application of the relation between the exponential and logarithmic functions as inverses of one another.

3. Which one of the following statements is **TRUE** about the function $f(x) = -2|\ln x|$ and its graph?

~~(a)~~ increases on $(0, 1)$

(b) increases on $(1, \infty)$

(c) decreases on $\left(\frac{1}{2}, 2\right)$

(d) f has no maximum value

(e) $x = 2$ is a vertical asymptote

See problems 61, 62 and 67 p. 392

4. The **domain** of $f(x) = \log_{x-1} x$ is

~~(a)~~ $(1, 2) \cup (2, \infty)$

(b) $(1, 2)$

(c) $(1, \infty)$

(d) $(0, 1) \cup (1, \infty)$

(e) $(2, \infty)$

To make sure that the base is a positive number different from 1 and $\log_b x$ is defined only for $x > 0$.

5. Which one of the following statements is **TRUE** for all $x > 0, \quad y > 0, \quad b > 0$ and $b \neq 1$?

(a) $\log_b \sqrt{x} = \dfrac{\ln x}{2 \ln b}$ See problems 31 to 40 p.404

(b) $\log_b(x + y) = \log_b x + \log_b y$

(c) $\log_b x \cdot \log_b y = \log_b x + \log_b y$

(d) $\log_b \dfrac{x}{y} = \dfrac{\log_b x}{\log_b y}, \quad y \neq 1$

(e) $\dfrac{\log_b x}{\log_b y} = \log_b x - \log_b y, \quad y \neq 1$

6. The value of $(\ln 10000)(\log \sqrt{e})(\log_3 \sqrt{5})(\log_5 9)$ is

(a) 2 See problems 41 and 42 p.404

(b) 8

(c) 1

(d) 4

(e) 6

7. $\dfrac{\tan 155° - \cot 35°}{1 + \tan 155° \cot 35°} =$

(a) $-\tan 80°$

(b) $\tan 20°$

(c) $-\tan 75°$

(d) $-\tan 25°$

(e) $\tan 15°$

An application of:

- $\cot \alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$
- $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$.

8. If $\dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} = \dfrac{1}{2}$, then $x =$

(a) $\log\sqrt{3}$

(b) $\log\sqrt[3]{2}$

(c) $\log 3$

(d) $\log 2$

(e) $\dfrac{3}{\log 2}$

See example 4 p.410

See problems 39 to 46 p. 415

9. The number of **intersection** points of the graphs of $y = \sin 2\pi x$ and $y = \cos 2\pi x$, over the interval $0 \le x \le \frac{3}{4}$, is

(a) 2

See problems 63 and 64 p. 519

(b) 1

(c) 5

(d) 3

(e) 4

10. If $x = \frac{3}{2}\sin\theta$, where $0 < \theta < \frac{\pi}{2}$, then the expression $\frac{12x^2}{(9-4x^2)^{3/2}}$ simplifies to

(a) $\tan^2\theta \sec\theta$

(b) $\tan^2\theta \sin\theta$

(c) $\tan^2\theta \cos\theta$

(d) $\cot^2\theta \sec\theta$

(e) $\cot^2\theta \sin\theta$

A direct application of the Fundamentamental Trigonometric Identities of section 6.1

11. The expression $\dfrac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$ simplifies to

(a) $1 + \sin x \cos x$ See problem 59 p. 561

(b) $1 + 2\sin x \cos x$

(c) $1 - 2\sin x \cos x$

(d) $-1 - \sin x \cos x$

(e) $-1 + \sin x \cos x$

12. If $y = 2\cot 2x$, then the number of vertical asymptotes over the interval $\left(-\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$ is equal to

See example 3 p. 523

See problems 31, 32, 40, 45 and 48 p. 527

(a) 2

(b) 1

(c) 3

(d) 4

(e) 0

13. If $\sin\alpha = \frac{3}{5}$, α lies in second quadrant, and $\cos\beta = -\frac{5}{13}$, β lies in third quadrant, then $\sin\left(\frac{\pi}{2} - \alpha + \beta\right) =$

(a) $-\frac{16}{65}$

(b) $\frac{48}{65}$

(c) $-\frac{48}{65}$

(d) $\frac{12}{65}$

(e) $-\frac{36}{65}$

See problems 69 and 70 p.51, then problems 39 to 48 p. 570-571

14. If a wheel of radius 8 centimeters is rotating at 450 revolutions per minute, then the **linear speed** of a point on the edge of the wheel in centimeters per seconds is equal to

(a) 120π

(b) 240π

(c) 110π

(d) 220π

(e) 230π

See example 8 p. 471

See problems 73 and 74 p. 473

15. The line $y = 3$ intersects the graph of $y = -2\csc\frac{x}{3}$ over the interval $\left(-\frac{3\pi}{2}, 6\pi\right)$ at

(a) three points

(b) one point

(c) two points

(d) four points

(e) no point

See example 4 p. 524

See problems 33, 34, 38, 41, and 44 p. 527

16. The reference angle θ', in radians, of the angle $\theta = -1656°$ is equal to

(a) $\frac{\pi}{5}$

(b) $\frac{\pi}{7}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{6}$

(e) $\frac{\pi}{9}$

See example 3 p. 494

See problems 25 to 36 p. 497

17. The expression $\dfrac{\tan t}{1+\sec t}+\dfrac{1+\sec t}{\tan t}$ simplifies to

(a) $2\csc t$

See example 4 p. 558

See problems 49 to 52 p. 560

(b) $2\sec t$

(c) $2\cot t$

(d) $2\tan t$

(e) $2\sin t$

18. If the terminal side of an angle θ lies on the line $3x+4y=0$, where $x>0$, then the value of $\cot\theta+\cos\theta$ is

(a) $-\dfrac{8}{15}$

See example 1 p. 491

See problems 1 to 8 p. 497

(b) $-\dfrac{32}{15}$

(c) $\dfrac{32}{15}$

(d) $\dfrac{1}{15}$

(e) $-\dfrac{1}{5}$

19. Which one of the following is an **odd** function?

(a) $f(x) = \dfrac{3\cos x}{x^2 \tan x + \csc x}$

See Problems 41 to 48 p. 508

(b) $f(x) = x^3 + \tan^2 x$

(c) $f(x) = \dfrac{1 + x\cos x}{\sin x + \tan x}$

(d) $f(x) = \dfrac{x^2}{3 + \cos x}$

(e) $f(x) = x^3 \csc x + 1$

20. The exact value of the expression

$$2\cos\left(-\frac{7\pi}{4}\right)\tan(240^\circ) - \sqrt{6}\csc\left(\frac{7\pi}{6}\right)$$

is

see problems 65 and 66 p. 498

(a) $3\sqrt{6}$

(b) $2\sqrt{6}$

(c) $\sqrt{6}$

(d) $-\sqrt{6}$

(e) $-2\sqrt{6}$

21. The range R and the period P of the function $y = -\left|3\sin\frac{x}{2}\right|$ are given by [Hint: Sketch]

(a) $R = [-3, 0]$, $P = 2\pi$

See problems 55 & 58 p. 527

(b) $R = [-3, 3]$, $P = 2\pi$

(c) $R = [-3, 0]$, $P = 4\pi$

(d) $R = [-3, 0]$, $P = \pi$

(e) $R = [-3, 0]$, $P = \frac{\pi}{2}$

22. If L is the distance between the two points $P_1(\cos\theta, \sin\theta)$ and $P_2(\cos 2\theta, \sin 2\theta)$, then $L^2 =$

(a) $2 - 2\cos\theta$

See the distance between two points given p. 563 of section 6.2

(b) $2 + 2\sin\theta$

(c) $2 + 2\cos 3\theta$

(d) $3 - \cos\theta$

(e) $3 - \cos 3\theta$

23. If, in the given figure, the length of AC is 10 cm, then the length of BD is

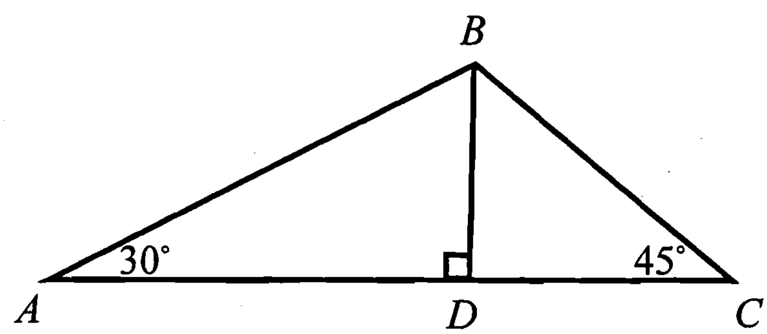
(a) $\dfrac{10}{1+\sqrt{3}}$ cm

(b) $\dfrac{10}{\sqrt{3}-1}$ cm

(c) $\dfrac{10\sqrt{3}}{1+\sqrt{3}}$ cm

(d) $\dfrac{10\sqrt{3}}{\sqrt{3}-1}$ cm

(e) $\dfrac{20}{1+\sqrt{3}}$ cm



An application of the tangent (or cotangent) function.

24. If the points $(-1,3)$ and $(2,81)$ lie on the graph of the exponential function $f(x)=b^{x+c}$, then $b+c=$

(a) 5

(b) 3

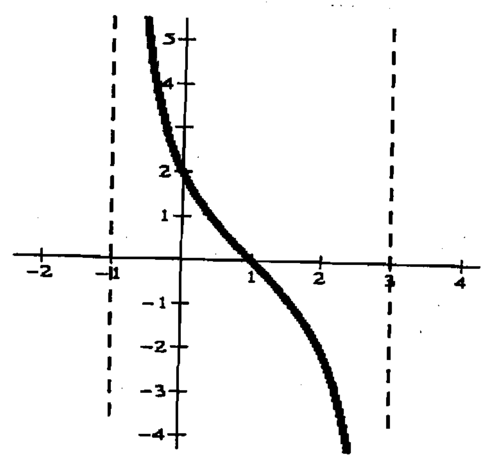
(c) -1

(d) 4

(e) -2

A point lies on the graph of a function $\Longrightarrow$ the point satisfies the equation of the graph.

25. If the given graph represents the function $y = a\tan(bx+c)$ over the interval $(-1, 3)$, then the values of a, b and c are given by



(a) $a = -2,\ b = \dfrac{\pi}{4},$ and $c = -\dfrac{\pi}{4}$

(b) $a = 2,\ b = \dfrac{\pi}{4},$ and $c = \dfrac{\pi}{4}$

(c) $a = -2,\ b = \dfrac{3\pi}{4},$ and $c = -\dfrac{\pi}{4}$

(d) $a = 2,\ b = -\dfrac{\pi}{4},$ and $c = -\dfrac{\pi}{4}$

(e) $a = -2,\ b = 4\pi,$ and $c = -4\pi$

See problem 50 p. 527