KEY

1. Consider the function $f(x) = 3^{-x+1} - 1$.

(a) (1 point) Find the y-intercepts, if any.

Let $x = 0 \Rightarrow f(0) = 3 - 1 = 2$ 0.5

$\Rightarrow$ the y-intercept is $(0, 2)$. 0.5

(b) (2 points) Find the x-intercepts, if any.

Let $f(x) = 0 \Rightarrow 3^{-x+1} - 1 = 0 \Rightarrow 3^{-x+1} = 1$ 1

$\Rightarrow -x+1 = 0 \Rightarrow x = 1 \Rightarrow$ the x-intercept is $(1, 0)$ 1

(c) (1 point) Find the horizontal asymptote of the graph of f, if any.

As $x \to \infty$, $f(x) \to 0 - 1 = -1 \Rightarrow$ the line $y = -1$ is the horizontal asymptote. 1

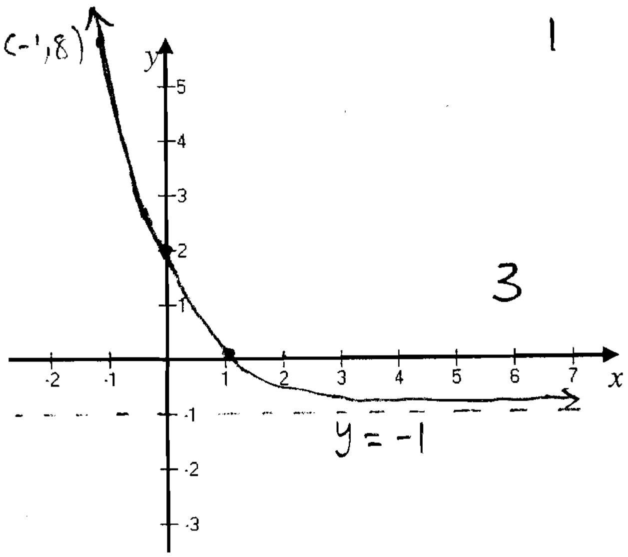
(d) (1 point) Find the domain of f in interval notation.

$(-\infty, \infty)$ 1

(e) (1 point) Find the range of f in interval notation.

$(-1, \infty)$ 1

(f) (3 points) Sketch the graph of f.



3

(g) (2 points) Find the inverse function $f^{-1}(x)$.

Let $y = 3^{-x+1} - 1 \Rightarrow 3^{-x+1} = y + 1 \Rightarrow$ 0.5

$-x+1 = \log_3(y+1) \Rightarrow x = 1 - \log_3(y+1)$. 1

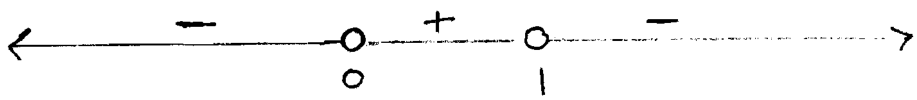
$\Rightarrow f^{-1}(x) = 1 - \log_3(y+1)$. 0.5

KEY

2. Consider the function $f(x) = \log\left(\frac{x}{1-x}\right)$.

(a) (2 points) Find the domain of f in interval notation.

We must have $\frac{x}{1-x} > 0 \Rightarrow$ 1



0.5

$\Rightarrow$ the domain $= (0, 1)$ 0.5

(b) (2 points) Find the x-intercepts, if any.

Let $f(x) = 0 \Rightarrow \log\left(\frac{x}{1-x}\right) = 0 \Rightarrow \frac{x}{1-x} = 1$ 1

$\Rightarrow x = 1 - x \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2} \Rightarrow$ 0.5

$(\frac{1}{2}, 0)$ is the x-intercept 0.5

(c) (1 point) Find the y-intercepts, if any.

0 is not in the domain of f 0.5

$\Rightarrow$ there is no y-intercept 0.5

3. Find the value of each of the following:

(a) (2 points) $(\sqrt{2})^{\log_2 9}$. $= (2^{\frac{1}{2}})^{\log_2 9} = 2^{\frac{1}{2}\log_2 9}$ 0.5

$= 2^{\log_2 \sqrt{9}} = 2^{\log_2 3}$ 0.5

$= 3$ 1

(b) (2 points) $\log\left(\ln \frac{1}{e^{-10}}\right) = \log(\ln e^{10})$ 0.5

$= \log 10$ 1

$= 1$ 0.5

KEY

4. (5 points) Solve the exponential equation

$$\left(\frac{1}{9}\right)^{\frac{1-2x}{2}} = (\sqrt{5})^{2x+4}.$$ [Write your answer in terms of $\ln 3$ and $\ln 5$]

$\Rightarrow (3^{-2})^{\frac{1-2x}{2}} = (5^{\frac{1}{2}})^{2x+4} \Rightarrow 3^{-1+2x} = 5^{x+2}$ 1

$\Rightarrow (-1+2x)\ln 3 = (x+2)\ln 5$ 2

$\Rightarrow -\ln 3 + 2x\ln 3 = x\ln 5 + 2\ln 5$ 0.5

$\Rightarrow 2x\ln 3 - x\ln 5 = 2\ln 5 + \ln 3$ 0.5

$\Rightarrow x(2\ln 3 - \ln 5) = 2\ln 5 + \ln 3$ 0.5

$\Rightarrow x = \dfrac{2\ln 5 + \ln 3}{2\ln 3 - \ln 5}$. 0.5

5. (5 points) Solve the logarithmic equation

$$\log(3x+8) = \log(2x+2) + \log(x-2).$$

$\Rightarrow \log(3x+8) = \log[(2x+2)(x-2)]$ 1

$\Rightarrow 3x+8 = 2x^2 - 2x - 4$ 1

$\Rightarrow 2x^2 + 5x + 12 = 0 = (2x+3)(x-4)$ 1

$\Rightarrow x = -3/2$ or $x = 4$ are possible solutions 1

A check will show that 4 is a solution, but $-\frac{3}{2}$ is not. 1

6. (3 points) Write the expression $\dfrac{\cos\theta + \sin\theta\tan\theta}{\cos\theta}$ in terms of $\sec\theta$ only in simplest form.

$= \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\tan\theta = 1 + \tan^2\theta$ 2

$= \sec^2\theta$ 1

KEY

7. (a) (3 points) Each tire on a car has a radius $\frac{60}{\pi}$ cm and rotates at 1500 revolutions per minute. Find the speed of the car in kilometers per hour.

The radius r of a tire $= \frac{60}{\pi(100)(1000)}$ kilometers. 1

The angular speed $\omega = 1500 \times 2\pi \times 60$ radians per hour — 1

$\Rightarrow$ The speed V of the car $= \omega \cdot r = \frac{(1500)(2\pi)(60)(60)}{\pi(100)(1000)} = 108$ km/h. 1

(b) (1 point) Find the supplement of the angle $57^0\,36'\,27''$.

$= 179^\circ\,59'\,60'' - 57^\circ\,36'\,27'' = 122^\circ\,23'\,33''$ 1

(c) (1 point) Find the positive angle less than 360° that is coterminal to −827°.

$-827^\circ = -720^\circ - 107^\circ$ coterminal with -107°.

$\Rightarrow$ -107° is coterminal with $360^\circ - 107^\circ = 253^\circ$ which is the required angle. 1

(d) (1 point) Find the measure of the arc of a circle with radius $\frac{30}{7}$cm and central angle 42°.

The required measure $= \left(\frac{30}{7}\right)\left(42 \frac{\pi}{180}\right)$

$= \pi$ cm 1

(e) (1 point) Convert $\frac{11\pi}{18}$ radians to degrees.

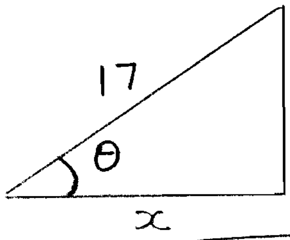
$\frac{11\pi}{18}$ radians $= \frac{11\pi}{18}$ radians $\left(\frac{180^\circ}{\pi \text{ radians}}\right) = 110^\circ$ 1

(f) (1 point) Convert 540° to radians.

$540^\circ = 540^\circ \left(\frac{\pi \text{ radians}}{180^\circ}\right) = 3\pi$ radians 1

8. (a) (2 points) Let θ be an acute angle of a right triangle for which $\csc\theta = \dfrac{17}{8}$. Find the value of $\tan\theta + \sec\theta$.

$$\left.\begin{aligned}\tan\theta + \sec\theta &= \frac{8}{15} + \frac{17}{15}\\ &= \frac{25}{15} = \frac{5}{3}\end{aligned}\right\} (1)$$



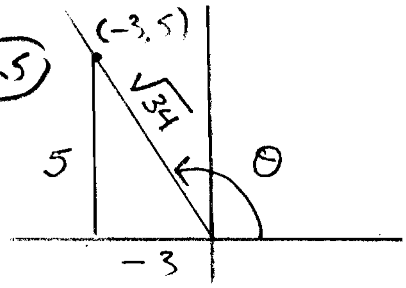
(1)

$$x = \sqrt{289-64} = \sqrt{225} = 15$$

(b) (2 points) If the terminal side of an angle θ in standard position contains the point $P(-3,5)$, find the value of $\cot(-\theta) + \sec(-\theta)$.

$$\cot(-\theta) + \sec(-\theta) = -\cot\theta + \sec\theta \quad (0.5)$$

$$\left.\begin{aligned}&= \frac{3}{5} - \frac{\sqrt{34}}{3}\\ &= \frac{9 - 5\sqrt{34}}{15}.\end{aligned}\right\} (0.5)$$



(1)

9. (3 points) If in the given figure the length of $BC = 5\,\text{cm}$, find the length of AB.

The length of $DC = 5 \tan 60^\circ = 5\sqrt{3}$ — (1)

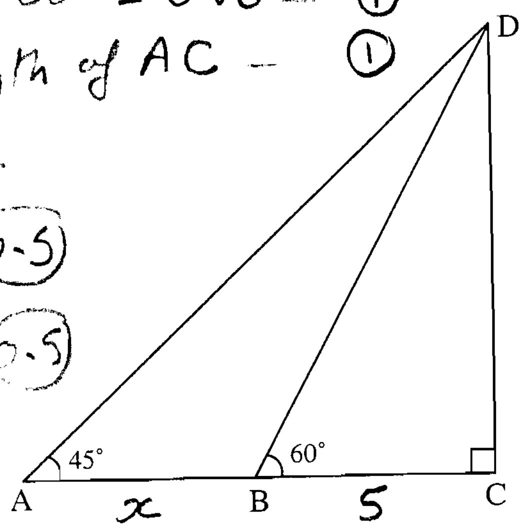
= The length of AC — (1)

If the length of $AB = x$

$\Rightarrow x + 5 = 5\sqrt{3}$ (0.5)

$\Rightarrow x = 5\sqrt{3} - 5$ cm. (0.5)

= The length of AC.



KEY

10. (3 points) Let W be the wrapping function and let t be a real number with $\pi < t < \frac{3\pi}{2}$. If $W(t) = P\left(x, -\frac{2}{3}\right)$, find the value of x.

$W(t) = (\cos t, \sin t) = (x, -\frac{2}{3})$ — 1

$\Rightarrow \quad x^2 + \frac{4}{9} = 1$ 0.5

$\Rightarrow \quad x^2 = \frac{5}{9} \Rightarrow x = \pm \frac{\sqrt{5}}{3}$ 0.5

But $\pi < t < \frac{3\pi}{2} \Rightarrow x = \cos t < 0 \Rightarrow x = -\frac{\sqrt{5}}{3}$. 1

11. (5 points) Find the exact value of $\cot\left(-\frac{19\pi}{6}\right)\sin\left(\frac{13\pi}{3}\right) - \tan 660^\circ \cos 120^\circ$.

$= \cot(-3\pi - \frac{\pi}{6}) \sin(4\pi + \frac{\pi}{3}) - \tan(720^\circ - 60^\circ)\cos(180^\circ - 60^\circ)$ 1.5

$= \cot(-\frac{\pi}{6}) \sin\frac{\pi}{3} - \tan(-60^\circ)(-\cos 60^\circ)$. 1.5

$= -\cot\frac{\pi}{6} \sin\frac{\pi}{3} - \tan 60^\circ \cos 60^\circ$ 1

$= -\sqrt{3} \cdot \frac{\sqrt{3}}{2} - \sqrt{3} \cdot \frac{1}{2} = -\frac{3}{2} - \frac{\sqrt{3}}{2}$ 1

12. (4-points) Write the following expressing as a single logarithm with a coefficient of 1 and with base 2, where $t > 0$:

$$3\log_2 t - \frac{1}{3}\log_4 t^{12} + \frac{1}{2}\log_{1/2} t^6.$$

$= \log_2 t^3 - \log_4 t^{12/3} + \log_{1/2} t^{6/2}$ 1

$= \log_2 t^3 - (\log_2 t^4 / \log_2 4) + (\log_2 t^3 / \log_2 \frac{1}{2})$ 1

$= \log_2 t^3 - \frac{1}{2}\log_2 t^4 - \log_2 t^3$ 1

$= \log_2 (t^3 / (t^{4/2} t^3)) = \log_2 (1/t^2)$ 1