1. [8-points] Let $f(x) = \left(\frac{1}{4}\right)^x - 2.$ $= 2^{-2x} - 2$

(a) Find the y-intercept of the graph of f.

$f(0) = 1-2 = -1 \Rightarrow$

y-intercept is $(0,-1)$

1 Point

(b) Find the x-intercept of the graph of f.

$f(x) = 0 \Rightarrow 2^{-2x} = 2 \Rightarrow x = -\frac{1}{2}$

x-intercept is $(-\frac{1}{2}, 0)$

1 point

(c) Write the equation of the horizontal asymptote of the graph of f.

$y = -2$

1 point

(d) Write the range of f using interval notation.

$(-2, \infty)$

1 Point

(e) Complete the following table:

x	-2	-1	0	1	2
$f(x)$	14	2	-1	$-7/4$	$-31/16$

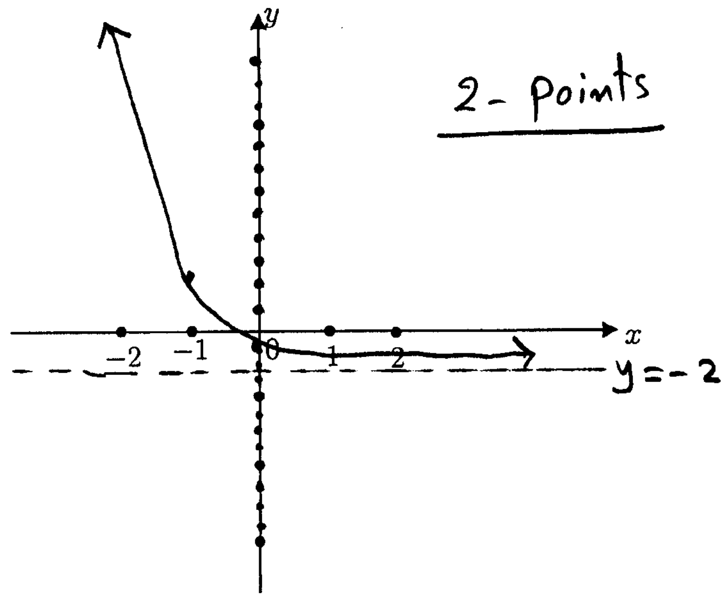
2 - Points

$f(-2) = 2^4 - 2 = 14$, $f(-1) = 2$, $f(0) = -1$

$f(1) = \frac{1}{4} - 2 = -\frac{7}{4}$, $f(2) = \frac{1}{16} - 2 = -\frac{31}{16}$

(f) Use all of the above to sketch the graph of f.

See examples 2,3 P. 371 and Problems 15,21-26 P. 377.



2 - points

2. [6-points] Let $f(x) = 2 - \log_3(x+3)$.

See examples 5, 6 P. 388 and Problems 39 to 56 P. 391

(a) Find the y-intercept of the graph of f.

$x = 0 \Rightarrow f(0) = 2 - \log_3 3 = 2 - 1 = 1$

$\Rightarrow$ y-intercept is $(0, 1)$ 1 point

(b) Find the x-intercept of the graph of f.

$f(x) = 0 \Rightarrow \log_3(x+3) = 2 \Rightarrow x+3 = 9$

$\Rightarrow x = 6$ 2-points

$\Rightarrow$ The x-intercept is $(6, 0)$

(c) Find the domain of f using interval notation.

$x+3 > 0 \Rightarrow x > -3$ 1 point

Domain $= (-3, \infty)$

(d) Find the interval on which the graph of f lies completely above the x-axis.

$2 - \log_3(x+3) > 0 \Rightarrow \log_3(x+3) < 2$

$\Rightarrow 0 < x+3 < 9 \Rightarrow -3 < x < 6$ 2-points

$\Rightarrow (-3, 6)$

3. [4-points] Write the expression $1 + 2\log_{1/9}(xy) - \frac{1}{2}\log_{\sqrt{3}}\left(\frac{x}{y}\right)$ as a single logarithm with base 3.

$= \log_3 3 + \dfrac{2\log_3 xy}{\log_3 \frac{1}{9}} - \dfrac{1}{2}\dfrac{\log_3 \frac{x}{y}}{\log_3 \sqrt{3}}$ $\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)$pt

$= \log_3 3 - \log_3 xy - \log_3 \frac{x}{y}$ $\left(\frac{1}{2}+\frac{1}{2}\right)$pt

$= \log_3\left(\dfrac{3}{(xy)\left(\frac{x}{y}\right)}\right)$ 1 point

$= \log_3 \dfrac{3}{x^2}$ $\frac{1}{2}$ point

An application on properties of logarithmic functions and change of base formula. See Ex. 2 P. 369 and Ex. 3 P. 397 and problems 403-40

4. [4-points] Use natural logarithm to find the exact solution of the equation

$$2^{x+1} = 3^{2x-1}.$$

$\Rightarrow \ln 2^{x+1} = \ln 3^{2x-1}$ $\quad \frac{1}{2}$ pt

$\Rightarrow (x+1)\ln 2 = (2x-1)\ln 3$ $\quad \frac{1}{2}$ pt

$\Rightarrow x\ln 2 + \ln 2 = 2x\ln 3 - \ln 3$ $\quad \frac{1}{2}+\frac{1}{2}$ pt

$\Rightarrow 2x\ln 3 - x\ln 2 = \ln 2 + \ln 3$ $\quad \frac{1}{2}+\frac{1}{2}$ pt

$\Rightarrow x(2\ln 3 - \ln 2) = \ln 2 + \ln 3$ $\quad \frac{1}{2}$ pt

$\Rightarrow x = \dfrac{\ln 2 + \ln 3}{2\ln 3 - \ln 2}$ $\quad \frac{1}{2}$ pt

See example 3 P. 410 and Problems 17-20 P. 415.

5. [4-points] Find the solution set of the logarithmic equation
$\ln x = 1 + \ln(x+1)$.

$\Rightarrow \ln x - \ln(x+1) = 1$ $\quad \frac{1}{2}$ pt

$\Rightarrow \ln \dfrac{x}{x+1} = 1$ $\quad \frac{1}{2}$ pt

$\Rightarrow \dfrac{x}{x+1} = e$ $\quad \frac{1}{2}$ pt

$\Rightarrow x = ex + e$

$\Rightarrow x - ex = e$ $\quad \frac{1}{2}$ pt

$\Rightarrow x(1-e) = e$ $\quad \frac{1}{2}$ pt

$\Rightarrow x = \dfrac{e}{1-e}$ $\quad \frac{1}{2}$ pt

See examples 5-7 P. 411-412 and Problems 21-30 P. 415

But $e > 1 \Rightarrow x < 0 \Rightarrow$ The solution set $= \phi$

1 point

6. [3-points] Find the degree measure of the supplement of the angle whose radian measure is $\frac{13\pi}{18}$.

The supplement $= \pi - \frac{13\pi}{18}$ 1 point

$= \frac{5\pi}{18}$ $\frac{1}{2}$ point

In degree measure $= \left(\frac{5\pi}{18} \cdot \frac{180}{\pi}\right)^{\circ}$ 1 point

See example 1 P. 463 $= 50^{\circ}$ $\frac{1}{2}$ point

and Problems 1 to 12 P.472

7. [3-points] Determine the degree measure of the positive angle with measure less than 360° that is coterminal with the angle −610°.

$-720^{\circ} = (-2) \cdot 360^{\circ} = -610^{\circ} - 110^{\circ}$ 1 pt

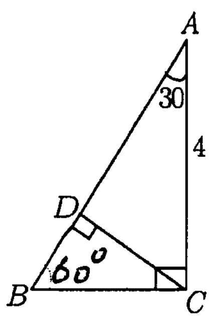
$\Rightarrow -610^{\circ} = 110^{\circ} + (-2) \cdot 360^{\circ}$ 1 pt

$\Rightarrow$ The required angle $= 110^{\circ}$ 1 pt

See example 2 P. 464

and problems 13-18 P. 472

8. [4-points] In the right triangle shown in the figure, if the length of AC is 4 cm, find the exact length of BD.



$BC = 4 \tan 30^{\circ}$ 1 pt

$= 4 \left(\frac{1}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}}$ 1 pt

Now $\angle B = 60^{\circ}$

$\Rightarrow BD = BC \cos 60^{\circ}$ 1 pt

$\Rightarrow BD = \left(\frac{4}{\sqrt{3}}\right)\left(\frac{1}{2}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ 1 pt

acceptable answer

See p. 479 for special angles 30° and 60°

and see problems 1-14 p. 484

9. [3-points] Find the exact value of $\sin\frac{7\pi}{6} - (\cos 660°)\tan\frac{39\pi}{4}$.

$= \sin\left(\pi + \frac{\pi}{6}\right) - \cos(720° - 60°) \cdot \tan\left(9\pi + \frac{3\pi}{4}\right)$ 1 pt

$= -\sin\frac{\pi}{6} - \cos 60° \tan\frac{3\pi}{4}$ 1 pt

$= -\frac{1}{2} - \left(\frac{1}{2}\right)(-1)$

$= 0$ 1 pt

See problems 61-66 P. 498

10. [3-points] For $\pi < \theta < \frac{3\pi}{2}$, write $\cot\theta$ in terms of $\cos\theta$.

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\pm\sqrt{1-\cos^2\theta}}$ 1 pt

But $\pi < \theta < \frac{3\pi}{2} \Rightarrow \sin\theta < 0$ 1 pt

$\Rightarrow \cot\theta = \frac{-\cos\theta}{\sqrt{1-\cos^2\theta}}$ 1 pt

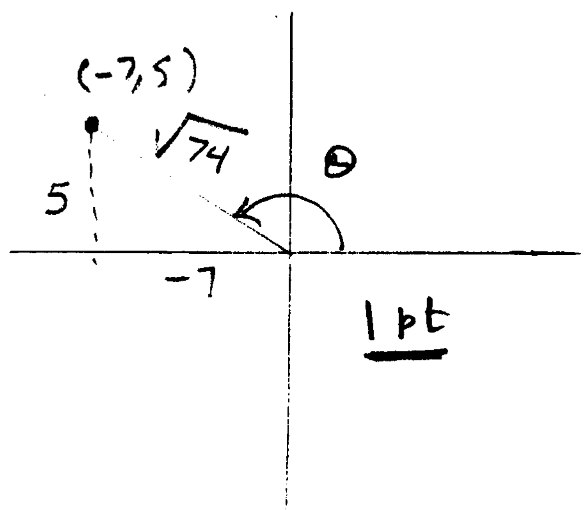
See example 6 P. 506
and Problems 73 -76 P. 509

11. [3-points] Let θ be an angle in standard position whose terminal side contains the point $P(-7, 5)$. Find the value of $25\csc\theta + 21\sec\theta$.

$= 25\left(\frac{\sqrt{74}}{5}\right) + 21\left(-\frac{\sqrt{74}}{7}\right)$ 1 pt

$= 5\sqrt{74} - 3\sqrt{74}$

$= 2\sqrt{74}$ 1 pt



See example 1 P. 491
and problems 1-8 P. 497