King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Code: 002

Prep-Year Math II
EXAM I
Term 063
Wednesday, July 18, 2007
Net Time Allowed: 110 minutes

Code: 002

EXAM SOLUTION.

Student's Name: ....

ID #: .... **Section #:** ....

Important Instructions:

1. All types of CALCULATORS, PAGERS, OR MOBILES ARE NOT ALLOWED to be with you during the examination.
2. Use an HB $2\frac{1}{2}$ pencil.
3. Use a good eraser. Do not use the eraser attached to the pencil.
4. Write your name, ID number and Mathematics Section on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Math Section number, be sure that bubbles match with the number that you write.
6. The test Code Number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
9. Check that the exam paper has 22 questions.

Q1.
Which one of the following functions has its graph symmetric with respect to the y-axis?

Sol^n: symmetry w.r.t the y-axis $\iff$ function is even.

A) $y = \dfrac{\sin x}{x - \tan x}$

B) $y = \sin x \sec x$

C) $y = \tan x + \sin x$

D) $y = -4\sin x$

E) $y = \sin x + \cos x$

A) $\dfrac{\sin x}{x - \tan x}$ → odd / odd ⟶ even.

B) $\sin x \sec x$ = (odd)(even) = odd

C) $\tan x + \sin x$ = odd + odd = odd

D) $-4\sin x$ = (nbr)(odd = odd

E) $\sin x + \cos x$ = odd + even = neither

Q2.

The expression $(\log_3 5)(\log_5 7)(\log_7 81)\left(\sqrt{2}\right)^{-\log_2 \frac{1}{25}}$ simplifies to

$\cancel{\log_3 5}\left(\dfrac{\cancel{\log_3 7}}{\cancel{\log_3 5}}\right) \cdot \dfrac{\log_3 81}{\cancel{\log_3 7}} \cdot 2^{-\frac{1}{2}\log_2\left(\frac{1}{25}\right)}$

$\log_3 81 \,,\; 2^{\log_2 (25)^{\frac{1}{2}}} =$

$\log_3 81 \cdot 25^{1/2} = 4 \cdot 5 = 20$

A) 10

B) 24

C) 15

D) 20

E) 25

Q3.
Which one of the following equations has no solution?

A) $\tan x = -0.5$ $\quad \in \text{Range}(\tan x) = (-\infty, \infty)$

B) $\sec x = \pi$ $\quad \simeq 3.14 \in \text{Range}(\sec x) = (-\infty, -1] \cup [1, \infty)$

✓ C) $3\csc x = -2$ $\quad \Leftrightarrow \csc x = -\frac{2}{3} \notin (-\infty, -1] \cup [1, \infty) = \text{Range}(\csc x)$

D) $4\cos x = \pi$ $\quad \Leftrightarrow \cos x = \frac{\pi}{4} \in [-1, 1] = \text{Range}(\cos x)$

E) $5\sin x = 3$ $\quad \Leftrightarrow \sin x = \frac{3}{5} \in [-1, 1] = \text{Range}(\sin x)$

Q4.
If the graph below represents the function $y = a\tan(bx + c)$, where $b>0$, over $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$, then $a+b+c$ is equal to

A) $\frac{-6+\pi}{4}$

B) $\frac{3-\pi}{2}$

C) $\frac{6-\pi}{4}$

✓ D) $\frac{-6-\pi}{4}$

E) $\frac{-3-\pi}{2}$

graph decreasing

$\Rightarrow \boxed{a = -2}$

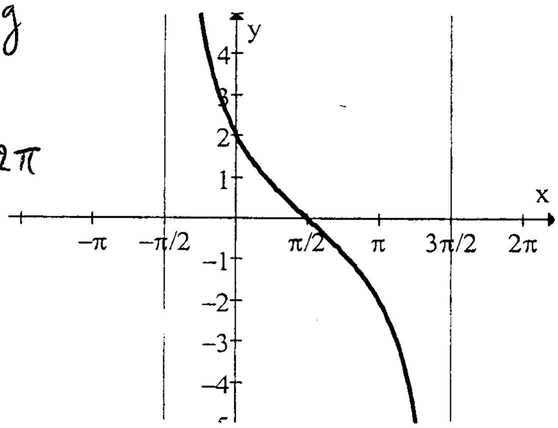
$P = \frac{3\pi}{2} - \left(-\frac{\pi}{2}\right) = 2\pi$

$P = 2\pi = \frac{\pi}{b}$

$\Rightarrow \boxed{b = \frac{1}{2}}$

$PS = \frac{\pi}{2} = -\frac{c}{b} = -2c$

$\Rightarrow c = -\frac{\pi}{4}$



$$a + b + c = -2 + \frac{1}{2} - \frac{\pi}{4}$$
$$= -\frac{8}{4} + \frac{2}{4} - \frac{\pi}{4} = \boxed{\frac{-6-\pi}{4}}$$

Q5.
Which one of the following statements about the solutions of the equation $4^x - 2(2^x) = 8$ is true?

$2^{2x} - 2(2^x) - 8 = 0$
$(2^x)^2 - 2(2^x) - 8 = 0$
$y^2 - 2y - 8 = 0$
$(y-4)(y+2) = 0$
$2^x = 4 \Rightarrow x = 2$
$2^x = -2$ No solution

A) There are two solutions, one positive and one negative.

B) There is only one solution and it is an odd number.

C) The sum of the solutions is 3

D) There is only one solution and it is not an integer.

✓ E) There is only one solution and it is an even number.

Q6.
The linear speed, (in cm/second) of a point on the edge of a wheel of radius 3 cm that is turning at a speed of 50 revolutions per minute is

$v = \omega \cdot r$

$= \left(\frac{5\cancel{0} \cdot \cancel{2}\pi \text{ rad}}{\cancel{6}\cancel{0} \text{ sec}} \right) \cdot \cancel{3} \text{ cm} = 5\pi \text{ cm/sec}$

A) 5 cm/sec

✓ B) 5π cm/sec

C) $\frac{5\pi}{2}$ cm/sec

D) 150 cm/sec

E) $\frac{5\pi}{3}$ cm/sec

Q7.

The exact value of $\csc(-\frac{17\pi}{6})$ is

csc odd

$$\csc\left(-\frac{17\pi}{6}\right) = -\csc\left(\frac{17\pi}{6}\right)$$

A) $\frac{\sqrt{3}}{2}$

$$\frac{17\pi}{6} = \frac{12\pi}{6} + \frac{5\pi}{6} = 2\pi + \frac{5\pi}{6} \rightarrow \text{II} \rightarrow \csc(+)$$

B) -2

& $\theta' = \pi - \frac{5\pi}{6} = \pi/6$

C) 2

D) $-\frac{2}{\sqrt{3}}$

$$-\csc\left(\frac{17\pi}{6}\right) = -\csc\left(\frac{\pi}{6}\right) = -\frac{1}{\sin \pi/6} = \boxed{-2}$$

E) -1

Q8.

If $\tan\theta = \frac{3}{4}$ and $\cos\theta < 0$, then $\sin\theta + \cos\theta$ is equal to

$\tan\theta > 0 \Rightarrow$ I or III
$\cos\theta < 0 \Rightarrow$ II or III $\Rightarrow$ III

A) 7

B) $\frac{2}{5}$

$\tan\theta = \frac{3}{4} = \frac{y}{x}$

C) 1

$\xRightarrow{\text{III}}$ $x = -4$, $y = -3$

D) $-\frac{3}{5}$

$r = 5$

$$\sin\theta + \cos\theta = \frac{-3}{5} + \frac{-4}{5} = \boxed{-\frac{7}{5}}$$

E) $-\frac{7}{5}$

Q9.

If $y = -2 - 3\sin(2x + \frac{2\pi}{3})$, M is the maximum, P the period and S the phase shift then

$M + \dfrac{P+S}{\pi}$ is equal to

$M = Max = |a| + d = 3 + (-2) = 1$

$P = \dfrac{2\pi}{b} = \dfrac{2\pi}{2} = \pi$

$S = -\dfrac{c}{b} = -\dfrac{\left(\frac{2\pi}{3}\right)}{2} = -\dfrac{\pi}{3}$

$M + \dfrac{P+S}{3} = 1 + \dfrac{\pi - \frac{\pi}{3}}{\pi} = 1 + 1 - \frac{1}{3}$

$= \dfrac{6}{3} - \dfrac{1}{3} = \boxed{\dfrac{5}{3}}$

A) $\frac{7}{3}$

B) $\frac{-1}{3}$

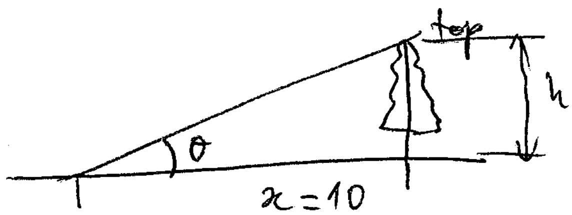
C) $\frac{4}{3}$

D) $\frac{17}{3}$

E) $\frac{5}{3}$

Q10.

If the angle of elevation from a point 10 feet from the base of a tree to the top of the tree is θ, and if $\sin\theta = \frac{3}{5}$, then the height of the tree is



A) $\frac{15}{2}$ feet.

B) $\frac{20}{3}$ feet.

C) $\frac{25}{4}$ feet

D) 6 feet.

E) 15 feet.

$\tan\theta = \dfrac{h}{10} \Rightarrow h = 10 \tan\theta$

$\sin\theta = 3/5 = \dfrac{y}{r} \Rightarrow x = \sqrt{25-9} = 4$

$\Rightarrow h = 10 \cdot \tan\theta = 10 \cdot \dfrac{3}{4} = \dfrac{30}{4} = \boxed{\dfrac{15}{2}}$

Q11.

The graph of the function $f(x)=\left|5^{-x}-4\right|$ intersects the line $y=\frac{1}{3}x+2$ in

$f(x) = 5^{-x} - 4 = \left(\frac{1}{5}\right)^{x} - 4$

$y=\frac{1}{3}x+2$

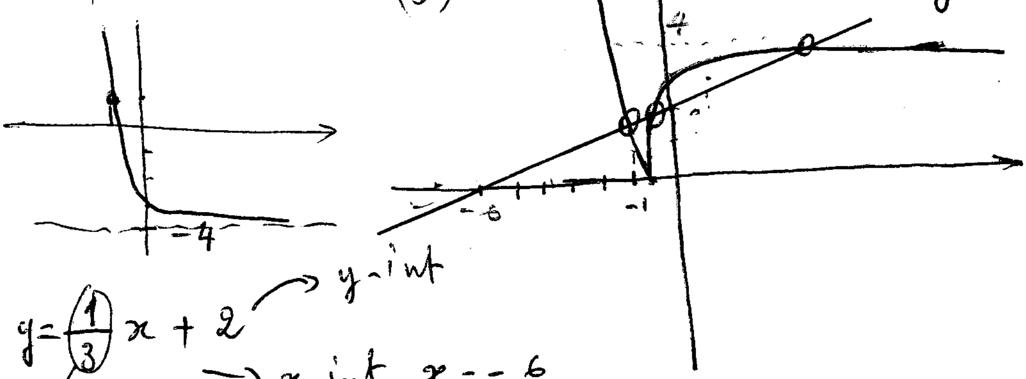
A) 2 points

B) 3 points

C) No points

D) 4 points

E) 1 point



$y=\frac{1}{3}x+2$ → y-int

slope → x-int $x=-6$

Q12.

The graph of the function $f(x)=-2\sin(3x)$ over $[0,2\pi]$ intersects the line $y=\frac{3}{2}$ at

$P=\frac{2\pi}{3}$, $[0,2\pi]$ contains 3 periods

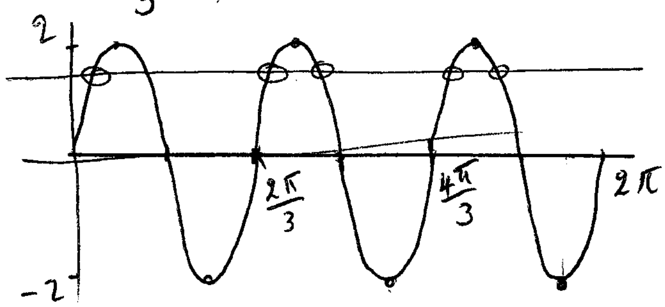
A) 3 points.

B) 7 points.

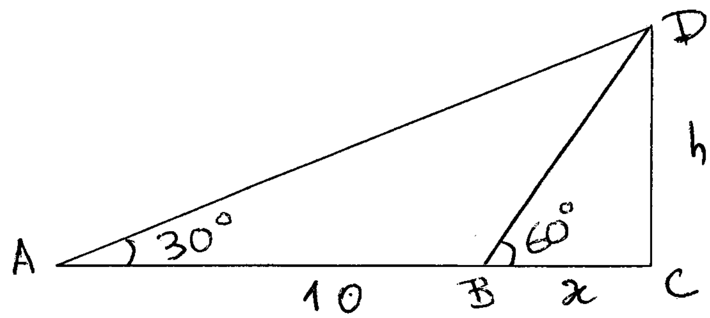
C) 6 points, ✓

D) 2 points.

E) 5 points.



Q13.
If the length of AB in the following figure is 10 cm, then the length of DC is



A) $5\sqrt{3}$ cm

B) $\frac{15}{2}\sqrt{3}$ cm

C) 15 cm

D) 5 cm

E) $10\sqrt{3}$

$\sqrt{3} = \tan 60° = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \tan 30° = \frac{h}{x+10} = \frac{h}{\frac{h}{\sqrt{3}}+10} = \frac{h\sqrt{3}}{h+10\sqrt{3}}$

$\frac{(h+10\sqrt{3})}{\sqrt{3}} = h\sqrt{3} \Rightarrow h + 10\sqrt{3} = 3h$

$\Rightarrow 10\sqrt{3} = 2h \Rightarrow \boxed{h = 5\sqrt{3}}$

Q14.
The number of vertical asymptotes of $f(x) = 2\cot\frac{3x}{2}$ in the interval $\left(-\frac{\pi}{6}, 3\pi\right)$ is

A) 5

B) 9

C) 3

D) 2

E) 4

V. A: $\frac{3x}{2} = n\pi$

$\Leftrightarrow \boxed{x = \frac{2n\pi}{3}}$

$-\frac{\pi}{6} < \frac{2n\pi}{3} < 3\pi$

$-\frac{\pi}{2} < 2n\pi < 9\pi$

$-\frac{1}{2} < 2n < 9$

$-\frac{1}{4} < n < \frac{9}{2} = 4.5$

The integers are
0, 1, 2, 3, 4

We have $\overline{5}$ V.A

Q15.

The equation of the function whose part of its graph is drawn below is

A) $y = -\cos 4x$

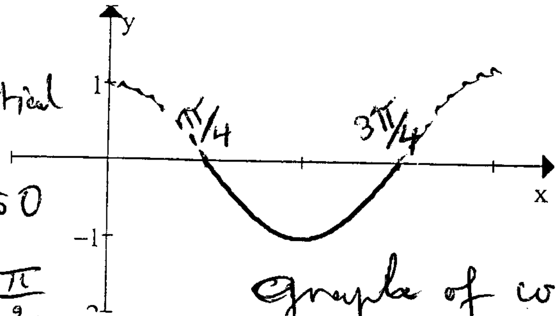
B) $y = -\sin 3x$

C) $y = -\sin 2x$

D) $y = \cos 2x$

E) $y = -\cos 2x$

All the choices are without phase shift & Vertical shift. So extend it to 0



$\frac{P}{2} = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$

$\Rightarrow P = \pi = \frac{2\pi}{b} \Rightarrow \boxed{b = 2}$

graph of cosine $\Rightarrow y = \cos 2x$

Q16.

The graph of $f(x) = 3 - 2^{-x}$ is above the x-axis on the interval

A) $(-\log_2 3, \infty)$

B) $(-1, \infty)$

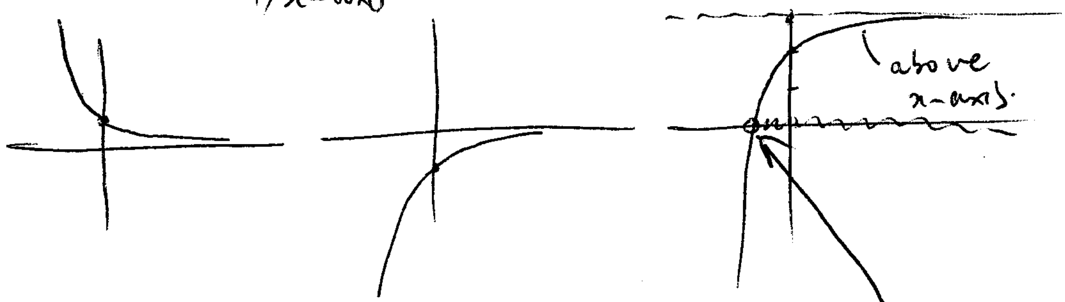
C) $(-\infty, 3)$

D) $\left(-\infty, \log_2 \frac{1}{3}\right)$

E) $(-\log_3 2, \infty)$

$f(x) = 3 - \left(\frac{1}{2}\right)^x$

$\left(\frac{1}{2}\right)^x \xrightarrow[\text{Refl/x-axis}]{} -\left(\frac{1}{2}\right)^x \xrightarrow[\text{3 up}]{} -\left(\frac{1}{2}\right)^x + 3$



$3 - \left(\frac{1}{2}\right)^x = 0 \Rightarrow \left(\frac{1}{2}\right)^x = 3 \Rightarrow$

$2^{-x} = 3 \Rightarrow -x = \log_2 3 \Rightarrow x = -\log_2 3$

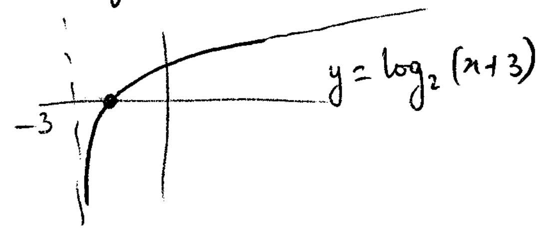
So the answer is

$(-\log_2 3, \infty)$

Q17.

The function $f(x) = \log_2\left(\frac{3+x}{8}\right)$ is

Sol^n $f(x) = \log_2(x+3) - \log_2 8$
$= \log_2(x+3) - 3$



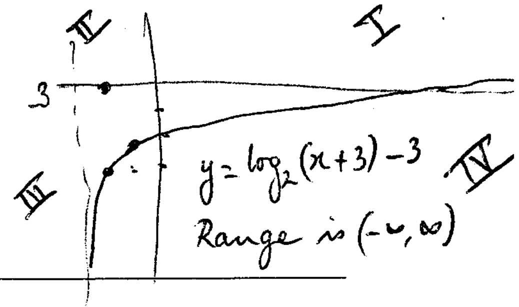
A) increasing and passing through the quadrants I and IV.

B) decreasing and passing through the quadrants III and IV.

C) increasing and passing through the quadrants II and III.

D) decreasing and passing through the quadrants II, III and IV.

E) increasing and passing through the quadrants I, III and IV.



Q18.

Which statement about the graph of the function $f(x) = -3\sec\frac{\pi}{4}x$ over the interval $\left[\frac{5}{2}, 5\right]$ is true?

A) There is a minimum but no maximum for the function.

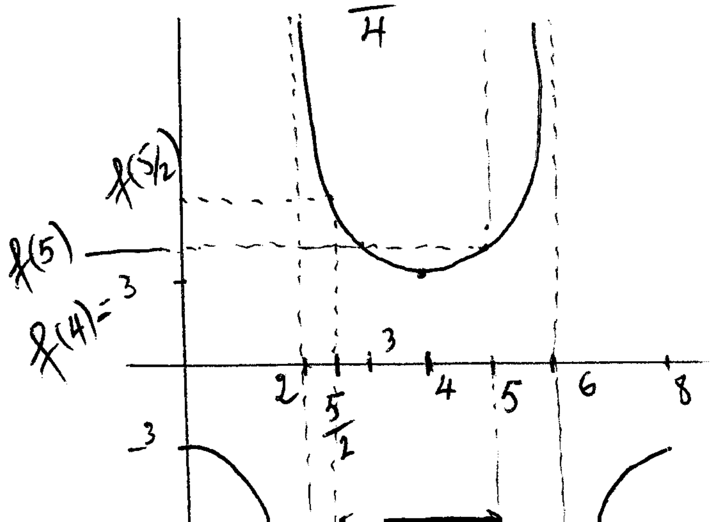
B) The maximum value of the function is $f\left(\frac{5}{2}\right)$

C) The maximum value of the function is $f(5)$

D) The graph has neither a minimum nor a maximum for the function.

E) The maximum value of the function is $f(4)$

$P = \frac{2\pi}{\frac{\pi}{4}} = 8$, $a = -3$ So the graph is



There is a max & a min

max $= f(5/2)$ min $= 3$

Q19.

The domain in interval notation of the function $f(x) = \ln\left|\frac{x-5}{x}\right|$

.

.

A) $(-\infty,0)\cup(5,\infty)$

B) $(-\infty,0)\cup(0,5)\cup(5,\infty)$

.

C) $(-\infty,0)\cup[5,\infty)$

.

D) $(-\infty,\infty)$

.

E) $(5,\infty)$

.

$\left|\frac{x-5}{x}\right| > 0 \iff \left|\frac{x-5}{x}\right| \neq 0$

So $x \neq 5$ & $x \neq 0$

Dom : $(-\infty, 0) \cup (0,5) \cup (5,\infty)$

Q20.

If α is the complementary angle of $26^o\, 25'21"$ and $\beta = 32^o\, 31'41"$ then $\alpha + \beta$ is equal to

.

.

A) 96° 6' 20"

.

B) 96° 16' 59"

.

C) 95° 6' 62"

.

D) 95° 56' 2"

.

E) 96° 59' 40".

.

$$\begin{array}{r} 89^\circ\ 59'\ 60'' \\ -\ 26^\circ\ 25'\ 21'' \\ \hline \alpha = 63^\circ\ 34'\ 39'' \end{array}$$

$$\begin{array}{r} 63^\circ\ 34'\ 39'' \\ 32^\circ\ 31'\ 41'' \\ \hline 95^\circ\ 65'\ 80'' \\ = 96^\circ\ 6'\ 20'' \end{array}$$

Q21.
The solution set of $\log(-x-2)+\log(1-x)=1$ consists of

A) One positive number only.

B) One negative number only.

C) Two positive numbers only.

D) Two negative numbers only.

E) One positive and one negative numbers.

$\log(-x-2)(1-x) = 1$

$\log(x^2+x-2) = 1 = \log 10$

$x^2+x-2 = 10$

$x^2+x-12 = 0$

$(x+4)(x-3) = 0$

$x=-4,\ x=3$

Check: $x=3$ rejected, $x=-4$ accepted

Q22.
If $\log_3 2 = a$ and $\log_3 5 = b$, then $\log 9$ in terms of a and b is equal to

A) $\frac{2}{a+b}$

B) $\frac{1}{a}+\frac{2}{b}$

C) $\frac{ab}{2}$

D) $\frac{a+b}{2}$

E) $\frac{2}{ab}$

$$\log 9 = \frac{\log_3 9}{\log_3 10} = \frac{2}{\log_3(5.2)}$$

$$= \frac{2}{\log_3 5 + \log_3 2} = \boxed{\frac{2}{a+b}}$$