

QUIZ # 3 (A)

1) Find all real solutions of $3|x^2 - 2x| + 4 = 28$

$$3|x^2 - 2x| = 24$$

$$|x^2 - 2x| = 8$$

$$x^2 - 2x = 8 \quad \text{or} \quad x^2 - 2x = -8$$

$$x^2 - 2x - 8 = 0 \quad \text{or} \quad x^2 - 2x + 8 = 0$$

$$(x-4)(x+2) = 0 \quad \Delta = 4 - 4(8) = 4 - 32 = -28 < 0$$

$$\boxed{x=4, x=-2}$$

no real
sol^{ns}

2) If $x=k$ is the solution of $2x = 1 - \sqrt{2-x}$. Find the value of $2k-3$

$$\sqrt{2-x} = 1 - 2x$$

$$2-x = 1 - 4x + 4x^2$$

$$\Leftrightarrow 4x^2 - 3x - 1 = 0$$

$$(4x+1)(x-1) = 0$$

$$x = -\frac{1}{4} \quad x = 1$$

Check:

$$x = -\frac{1}{4}$$

$$2\left(-\frac{1}{4}\right) = 1 - \sqrt{2 - \left(-\frac{1}{4}\right)}$$

$$-\frac{1}{2} = 1 - \sqrt{\frac{9}{4}} \quad \checkmark$$

$$= 1 - \frac{3}{2} = -\frac{1}{2}$$

3) Solve the inequality $\frac{3x^2 + 6x - 16}{x-1} \leq 8$

$$\frac{3x^2 + 6x - 16}{x-1} - 8 \leq 0$$

$$\frac{3x^2 + 6x - 16 - 8x + 8}{x-1} \leq 0$$

$$x = 1$$

$$2(1) = 1 - \sqrt{2-1} = 1 - 1 = 0 \quad \times$$

$$2k - 3 = 2\left(-\frac{1}{4}\right) - 3 = \boxed{-4}$$

$$\frac{3x^2 - 2x - 8}{x-1} \leq 0$$

$$\frac{(3x+4)(x-2)}{x-1} \leq 0 \quad \rightarrow \ominus \text{ for } x < 1$$

x	$\frac{3}{4}$	1	2
$3x-4$	- 0 +	+	+
$x-2$	-	-	0 +
$x+1$	-	- 0 +	+
$\frac{(3x-4)(x-2)}{(x-1)}$	- 0 +	+	- 0 +

$$SS = (-\infty, \frac{3}{4}] \cup (1, 2]$$

- 4) Find all values of k such that $x^2 + 2x + |3k-1| = 0$ has at most one real root (one real root or nothing)

$$\Delta \leq 0$$

$$b^2 - 4ac$$

$$4 - 4|3k-1| \leq 0$$

$$\div 4 \quad | - |3k-1| \leq 0$$

$$|3k-1| \geq 1$$

$$3k-1 \geq 1 \quad \text{or} \quad 3k-1 \leq -1$$

$$3k \geq 2 \quad \text{or} \quad 3k \leq 0$$

$$k \geq \frac{2}{3} \quad \text{or} \quad k \leq 0$$

$$(-\infty, 0] \cup [\frac{2}{3}, \infty)$$

- 5) EXTRA (not compulsory)

If $3x^2 + 5x + 2 = 0$ is equivalent to $(x+a)^2 = b$. Find the value of $a+b$

$$3x^2 + 5x = -2$$

$$x^2 + \frac{5}{3}x = -\frac{2}{3}$$

$$x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = -\frac{2}{3} + \frac{25}{36}$$

$$\underbrace{\left(x + \frac{5}{6}\right)^2}_a = \underbrace{-\frac{24}{36} + \frac{25}{36}}_b = \frac{1}{36}$$

$$a+b = \frac{5}{6} + \frac{1}{36} = \frac{30}{36} + \frac{1}{36}$$

$$= \frac{31}{36}$$