King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Midterm 052

Prep-Year Math I
MIDTERM EXAM
Semester II, Term 052
Tuesday March 28, 2006
Net Time Allowed: 120 minutes

Sources of Problems

MASTER VERSION

1. If $A = \{x|x \leq -2\} \cup \{x|x \geq 1\}$ and $B = \{x|x > -3\} \cap \{x|x < 5\}$, then the set $A \cap B$ in interval notation is equal to

(a) ~~(a)~~ $(-3, -2] \cup [1, 5)$

(b) $[-2, 5)$

(c) $(-\infty, -3) \cup [1, \infty)$

(d) $(-3, 1]$

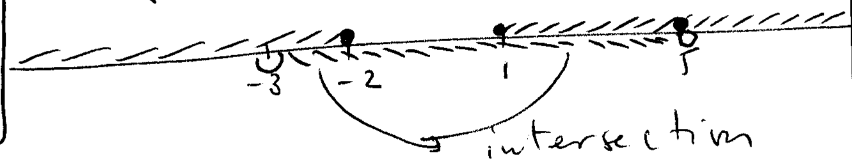
(e) the empty set ϕ

See example 6 P. 9

and problems 51 to 66

$A = (-\infty, -2] \cup [1, \infty)$

$B = (-\infty, 5) \cap (-3, \infty) = (-3, 5)$



$A \cap B = (-3, -2] \cup [1, 5)$

2. If the equation $18x - 12 = 3(ax + b) - 6x$ is an identity, then $a + b =$

(a) ~~(a)~~ 4

(b) -1

(c) 0

(d) 3

(e) -3

See example 4 p. 85

and problems 23 to 32 P. 88

$3(ax + b) - 6x = 3ax + 3b - 6x$

$= (3a - 6)x + 3b$

Hence the equation becomes

$18x - 12 = (3a - 6)x + 3b$

$\div 3 \quad 6x - 4 = (a - 2)x + b$

to be an identity

$a - 2 = 6$ & $b = -4$

$a = 8$ $\quad$ $b = -4$

$\Rightarrow \boxed{a + b = 4}$

3. One factor of $(x-5)^3+8$ is

(a) $x^2-12x+39$

(b) $x^2-12x+19$

(c) $x^2+12x-39$

(d) $x^2+12x+19$

(e) $x^2-12x-39$

Similar to problems 61, 62 P. 54

$(x-5)^3+2^3=$ (sum of 2 cubes) [redacted]

$=[(x-5)+2][(x-5)^2-2(x-5)+4]$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(x-3)(x^2-10x+25-2x+10+4)$

$=(x-3)(x^2-12x+39)$

4. If $t=\frac{3}{2}x(5y-7z)$, then $z=$

(a) $\dfrac{15xy-2t}{21x}$

(b) $\dfrac{5xy-2t}{3}$

(c) $\dfrac{15xy+2t}{7x}$

(d) $\dfrac{2t}{15xy+7}$

(e) $\dfrac{15xy-2t}{7x}$

See example 1 p. 91

and problems 1 to 10 p. 98

Solve for z.

$2t=3x(5y-7z)$

$2t=15xy-21xz$

$21xz=15xy-2t$

$\boxed{z=\dfrac{15xy-2t}{21x}}$

5. The **sum** of all solutions of the equation $3|2x+1|+4=28$ is equal to

see example 5 p. 86 and problems 33 to 48 p. 88-89

(a) -1

(b) 1

(c) 0

(d) -3

(e) 3

$3|2x+1|+4=28$

$3|2x+1| = 28-4 = 24$

$|2x+1| = \frac{24}{3} = 8$

$\Leftrightarrow 2x+1=8$ or $2x+1=-8$

$2x = 7$ $\quad$ $2x=-9$

$x_1 = 7/2$ $\quad$ $x_2 = -\frac{9}{2}$

Sum $= 7/2 + \left(-\frac{9}{2}\right) = -\frac{2}{2} = \boxed{-1}$

6. The expression $2(x+y)^3-3(x-y)^3$ simplifies to

(a) $-x^3+15x^2y-3xy^2+5y^3$

(b) $-x^3-3x^2y-3xy^2+5y^3$

(c) $-x^3+15x^2y-xy^2+5y^3$

(d) $-x^3+3x^2y-3xy^2+5y^3$

(e) $-x^3+5y^3$

similar to problems 84 to 89 p. 43

$= 2\left(x^3+3x^2y+3xy^2+y^3\right)$
$\quad -3\left(x^3-3x^2y+3xy^2-y^3\right)$
$= \boxed{-x^3+15x^2y-3xy^2+5y^3}$

7. The solutions of the equation

$$\frac{3}{4}x^2 - \frac{3}{2}x + \frac{9}{4} = 0$$

are the complex numbers

See example 6 p. 109
and problems 31, 32 p. 113

(a) $1 \pm \sqrt{2}\,i$

(b) $-1 \pm \sqrt{2}\,i$

(c) $2 \pm 2\sqrt{2}\,i$

(d) $-2 \pm \sqrt{2}\,i$

(e) $-2 \pm 2\sqrt{2}\,i$

Multiply by LCD = 4

$3x^2 - 6x + 9 = 0$

$\iff x^2 - 2x + 3 = 0$

$\Delta = (-2)^2 - 4(1)(3) = 4 - 12 = -8$

$x = \dfrac{2 \pm \sqrt{-8}}{2} = \dfrac{2 \pm 2i\sqrt{2}}{2}$

$= \boxed{1 \pm \sqrt{2}\,i}$

8. The expression $6 - 12\left[-\frac{3}{4}x - \left(\frac{5}{6}x - \frac{1}{3}\right)\right]$ simplifies to

See example 9 p. 13
and problems 99 to 106 p. 17

(a) $19x + 2$

(b) $\frac{49}{6}x + \frac{19}{3}$

(c) $-x + 10$

(d) $\frac{59}{6}x - \frac{17}{3}$

(e) $19x + 10$

$6 - 12\left[-\frac{3}{4}x - \frac{5}{6}x + \frac{1}{3}\right]$

$6 - (-9x - 10x + 4)$

$= 6 + 9x + 10x - 4$

$= \boxed{19x + 2}$

9. $\dfrac{3}{\sqrt{5}-\sqrt{2}} - \dfrac{2}{2\sqrt{5}-3\sqrt{2}} =$

(a) $-\sqrt{5}-2\sqrt{2}$

(b) $6+2\sqrt{10}$

(c) $-\sqrt{5}+6\sqrt{2}$

(d) $-6-2\sqrt{10}$

(e) $\sqrt{5}+2\sqrt{2}$

See example 9 p.31

and problems # 107 to 112 p.33

$$\frac{3}{(\sqrt{5}-\sqrt{2})} \cdot \frac{\sqrt{5}+\sqrt{2}}{(\sqrt{5}+\sqrt{2})} = \frac{3\sqrt{5}+3\sqrt{2}}{5-2} = \frac{\cancel{3}(\sqrt{5}+\sqrt{2})}{\cancel{3}} = \boxed{\sqrt{5}+\sqrt{2}}$$

$$\frac{2}{2\sqrt{5}-3\sqrt{2}} \cdot \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{2(2\sqrt{5}+3\sqrt{2})}{(2\sqrt{5})^2-(3\sqrt{2})^2}$$
$$= \frac{2(2\sqrt{5}+3\sqrt{2})}{20-18} = \boxed{2\sqrt{5}+3\sqrt{2}}$$

$$\Rightarrow \frac{3}{\sqrt{5}-\sqrt{2}} - \frac{2}{2\sqrt{5}-3\sqrt{2}} = \sqrt{5}+\sqrt{2}-2\sqrt{5}-3\sqrt{2} = \boxed{-\sqrt{5}-2\sqrt{2}}$$

10. If the equation $9x^2+(3x+1)k=0$ has two equal roots, where $k\neq 0$, then $k=$

(a) 4

(b) -4

(c) 3

(d) -3

(e) -2

See example 6 p.109

and problems 47 to 56 p.113

put equation in Standard form

$9x^2+(3k)x+k=0$

two equal roots $\Leftrightarrow$ $\Delta=0$

$b^2-4ac=(3k)^2-4(9)k = 9k^2-36k=0$

$9k(k-4)=0 \Leftrightarrow k=0$ or $\boxed{k=4}$

$k=0$ rejected

13. The standard form of the complex number $i^{153} + \dfrac{i}{1-i}$ is

(a) $-\dfrac{1}{2} + \dfrac{3}{2}i$

(b) $\dfrac{1}{2} - \dfrac{3}{2}i$

(c) $-\dfrac{1}{2} - \dfrac{3}{2}i$

(d) $\dfrac{1}{2} + \dfrac{3}{2}i$

(e) $-\dfrac{1}{2} - \dfrac{i}{2}$

See examples 4 and 5 p. 70-71 and problems 37 to 62 p. 72

$$i^{153} + \frac{i}{1-i} = i^{1} + \frac{i\cdot(1+i)}{(1-i)(1+i)}$$

$$= i + \frac{i-1}{1+1} = i + \frac{i}{2} - \frac{1}{2}$$

$$= \boxed{-\frac{1}{2} + \frac{3}{2}i}$$

14. The expression $\dfrac{x}{x^2-1} - \dfrac{3}{x^2+4x-5}$ simplifies to

(a) $\dfrac{x+3}{(x+1)(x+5)}$

(b) $\dfrac{x+3}{(x-1)(x+5)}$

(c) $\dfrac{x^2-x+3}{(x-1)(x+1)(x+5)}$

(d) $\dfrac{x+3}{(x-1)(x+1)(x+5)}$

(e) $\dfrac{x^2+3}{(x-1)(x+1)}$

See example 3 p. 59 and problems 31, 32 p. 63

$$\frac{x}{(x-1)(x+1)} - \frac{3}{(x+5)(x-1)} =$$

LCD $= (x-1)(x+1)(x+5)$

$$= \frac{x(x+5) - 3(x+1)}{LCD}$$

$$= \frac{x^2+5x-3x-3}{LCD} = \frac{x^2+2x-3}{LCD}$$

$$= \frac{(x+3)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)(x+5)} = \frac{x+3}{(x+1)(x+5)}$$

15. The number of real roots of the equation $5x^{-4} + 2x^{-3} = 0$ is

~~(a)~~ 1

(b) 2

(c) 3

(d) 4

(e) 0

An application of solving by factoring

$x^{-3}(5x^{-1} + 2) = 0$

$\Leftrightarrow x^{-3} = 0$ or $5x^{-1} = 2$

$\frac{1}{x^3} = 0$ or $\frac{5}{x} = 2$

No sol^n $\boxed{x = 5/2}$

1 real sol^n

16. The solution set, in interval notation, of the compound inequality

$|2x - 1| \leq 3$ and $x^2 + 2x \geq 0$

is

See examples 1 to 4 p. 130-133

and problems 9 to 28 p. 140

~~(a)~~ $[0, 2]$

(b) $[-2, -1] \cup [0, \infty)$

(c) $(-\infty, -1] \cup [0, \infty)$

(d) $(-\infty, -2] \cup [0, 2]$

(e) $[-2, -1] \cup [2, \infty)$

$|2x - 1| \leq 3 \Leftrightarrow -3 \leq 2x - 1 \leq 3$

$-2 \leq 2x \leq 4$

$\boxed{-1 \leq x \leq 2}$

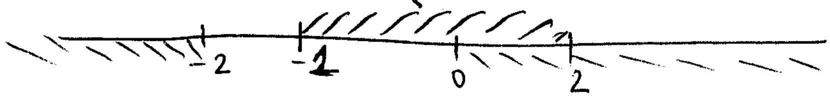
$x^2 + 2x \geq 0 \Leftrightarrow x(x+2) \geq 0$

Critical values $x = 0$ & $x = -2$

$(-\infty, -2] \cup [0, \infty)$

The Solution set of compound inequality is

$SS = [-1, 2] \cap ((-\infty, -2] \cup [0, \infty)) = [0, 2]$



17. The solution set of the equation

$$(x+1)^{2/3} - 2(x+1)^{1/3} - 3 = 0$$

contains

See example 9 p. 124 and problems 47–50 p. 126

(a) one positive and one negative integers

(b) two positive integers

(c) two negative integers

(d) one positive and one negative irrational numbers

(e) two positive irrational numbers

Sol^n:
Let $y = (x-1)^{1/3}$
$y^2 - 2y - 3 = 0$
$(y-3)(y+1) = 0$

$y = +3$ or $y = -1$
$(x+1)^{1/3} = 3$ or $(x+1)^{1/3} = -1$
$x + 1 = 27$ $\quad$ $x + 1 = -1$
$\boxed{x = 26}$ $\quad$ $\boxed{x = -2}$

One positive & one neg integers.

18. The solution, in interval notation, of the inequality $\dfrac{3x^2+6x-16}{x-1} \geq 8$ is

Similar to problems 47–50 p. 140

(a) $\left[-\frac{4}{3}, 1\right) \cup [2, \infty)$

(b) $[-2, 0) \cup (2, \infty)$

(c) $\left(-\frac{2}{3}, 0\right) \cup \left(\frac{5}{2}, \infty\right)$

(d) $(-\infty, -3) \cup (2, \infty)$

(e) $\left(-\frac{4}{3}, 0\right] \cup [1, 2]$

$$\frac{3x^2+6x-16}{x-1} - 8 \geq 0$$

$$\frac{3x^2+6x-16-8x+8}{x-1} \geq 0$$

$$\frac{3x^2-2x-8}{x-1} \geq 0 \iff \frac{(3x+4)(x-2)}{x-1} \geq 0$$

$\oplus, 0$

$$SS = \left[-\tfrac{4}{3}, 1\right) \cup [2, \infty)$$

		$-4/3$		1		2	
$3x+4$	−	0	+		+		+
$x-2$	−		−		−	0	+
D: $x-1$	−		−	0	+		+
$\frac{(3x+4)(x-2)}{(x-1)}$	−	0	+	U	−	0	+

19. $\dfrac{x^{-1}-y^{-1}}{x^{-2}y^{-2}} \div \dfrac{x^{-2}-y^{-2}}{x^{-3}y^{-3}} =$

(a) $\dfrac{1}{x+y}$

(b) $\dfrac{x}{x+y}$

(c) $\dfrac{y}{x+y}$

(d) $\dfrac{xy}{x+y}$

(e) $xy(x+y)$

See examples 4 and 5 p. 60-61

and problems 41 – 62 p. 63-64

$$\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2y^2}} = \frac{\frac{y-x}{xy}}{\frac{1}{x^2y^2}} = \frac{y-x}{xy} \cdot x^2y^2 = xy(y-x)$$
$$= xy(y-x)$$

$$\frac{x^{-2}-y^{-2}}{x^{-3}y^{-3}} = \frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x^3y^3}} = \frac{\frac{y^2-x^2}{x^2y^2}}{\frac{1}{x^3y^3}} = (y^2-x^2)xy$$

$$xy(y-x) \div (y^2-x^2)xy$$
$$= \frac{\cancel{xy}\,\cancel{(y-x)}}{\cancel{xy}\,\cancel{(y-x)}(y+x)} = \boxed{\frac{1}{x+y}}$$

20. If the expression $3x^2+5x+2$ is written in the form $3(x+a)^2+b$, then the product ab is equal to

(a) $-\dfrac{5}{72}$

(b) $\dfrac{49}{12}$

(c) $-\dfrac{25}{36}$

(d) $\dfrac{5}{12}$

(e) $-\dfrac{5}{36}$

See example 3 and 4 p. 106–107

and problems 21 to 32 p. 113

Completing the square.

$$3\left(x^2+\frac{5}{3}x\right)+2$$

$$3\left(x^2+\frac{5}{3}x+\left(\frac{5}{6}\right)^2\right)+2-3\left(\frac{5}{6}\right)^2$$

$$=3\left(x+\frac{5}{6}\right)^2+2-3\,\frac{25}{36} = 3\left(x+5/6\right)^2-\frac{25}{12}+\frac{24}{12}$$

$$=3\left(x+5/6\right)^2-\frac{1}{12} \Rightarrow a=5/6 \quad b=-\frac{1}{12}$$

$$\boxed{ab=\frac{-5}{72}}$$

21. Three students decided to share the cost of a car. By bringing in an additional student, they can reduce the cost of each student by 4000 Saudi Riyals. The total cost of the car is

(a) 48000 Saudi Riyals

(b) 64000 Saudi Riyals

(c) 72000 Saudi Riyals

(d) 44000 Saudi Riyals

(e) 52000 Saudi Riyals

See problem # 52 p. 101

x = cost of the car

$$\frac{x}{3} = \frac{x}{4} + 4000 \qquad \times 12$$

$$4x = 3x + 48000$$

$$4x - 3x = 48000$$

$$\boxed{x = 48000}$$

22. The expression $\dfrac{\sqrt[3]{x^2}\sqrt{\sqrt{x}}}{\sqrt[4]{x^3}}$ in simplest form is equal to

(a) $\sqrt[6]{x}$

(b) $\sqrt[12]{x^5}$

(c) $\dfrac{1}{\sqrt{x^3}}$

(d) $\sqrt[6]{x^5}$

(e) $\sqrt[12]{x}$

An application of

$$\sqrt[n]{b^m} = b^{m/n} \quad \text{p. 26}$$

and $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ p. 28

rationalize first

$$\frac{\sqrt[3]{x^2}\,\sqrt{\sqrt{x}}}{\sqrt[4]{x^3}} \cdot \frac{\sqrt[4]{x}}{\sqrt[4]{x}} = \frac{x^{2/3}\,x^{1/4}\,x^{1/4}}{(\sqrt[4]{x})^4}$$

$$= \frac{x^{2/3}\,x^{2/4}}{x} = \frac{x^{2/3}x^{1/2}}{x} = \frac{x^{4/6+3/6}}{x}$$

$$= \frac{x^{7/6}}{x} = x^{1/6} = \boxed{\sqrt[6]{x}}$$

23. If $-5 < x < -2$, then the expression

$$||x+5| + |x-2| + \sqrt{x^2} + \sqrt[3]{x^3}|$$

simplifies to

(a) 7

(b) $-2x - 3$

(c) $2x + 3$

(d) 3

(e) $2x + 7$

See problems 31 to 40 p. 16 together with $\sqrt{x^2} = |x|$.

$|x+5| = x+5$, $|x-2| = 2-x$

$\sqrt{x^2} = |x| = -x$, $\sqrt[3]{x^3} = x$

$$||x+5| + |x-2| + \sqrt{x^2} + \sqrt[3]{x^3}|$$

$$= |(\cancel{x}+5) + (2-\cancel{x}) + (\cancel{-x}) + \cancel{x}| = |7| = 7.$$

24. If $z = 2 - 3i$, then **the imaginary part** of the complex number $z^2 - 3z + 5$ is equal to

(a) -3

(b) -15

(c) 21

(d) -11

(e) 6

See p. 66 (for the real and imaginary part of a complex number)

and problems 75 & 76 p. 72

$z^2 = (2-3i)^2 = 4 - 2(2)(3i) + \underbrace{3i^2}_{-3}$
$= 1 - 12i$

$z^2 - 3z + 5 = (1 - 12i) - 3(2 - 3i) + 5$
$= 1 - 12i - 6 + 9i + 5 = 0 - 3i$

$\Rightarrow$ Imaginary part is $\boxed{-3}$

11. The coefficient of x^2y in the product $(4x-5y)(2x-y)(3x-4y)$ is equal to

(a) -74

(b) -10

(c) -43

(d) 10

(e) -33

See example 1 p. 36
example 3 p. 37
and Problems # 53, 54 p. 41

$(4x-5y)(2x-y)(3x-4y)$

12. The equation $\dfrac{20x-9}{4} = \dfrac{15x+11}{3}$ is

(a) a contradiction

(b) a conditional

(c) an identity

(d) equivalent to the equation $60x - 27 = 0$

(e) equivalent to the equation $60x + 44 = 0$

See example 1 p. 85
and problems 23 to 32 p. 88

~~Multiply by LCD = 12 = 3·4~~
$\cancel{3(20x-3) = 4(15x+11)}$
$\cancel{20x - 15x = 11 + 9}$
$\cancel{5x = 20}$
$\cancel{x = 4}$

Multiply by LCD = 4·3 = 12
$3(20x-9) = 4(15x+11)$
$60x - 27 = 60x + 44$
$-27 = 44$
Contradiction

25. One factor of $25y^{2m} - (x^{2n} - 2x^n + 1)$ is

See problems 94 & 95 p. 54

~~(a)~~ $5y^m - x^n + 1$

(b) $5y^m + x^n + 1$

(c) $5y^m - x^n - 1$

(d) $5y^{2m} - x^n - 1$

(e) $5y^m + x^{2n} - 1$

$$(5y^m)^2 - (x^n - 1)^2$$

$$(5y^m - (x^n - 1))(5y^m + (x^n - 1))$$

$$= (5y^m - x^n + 1)(5y^m + x^n - 1)$$