

QUIZ # 2 (A)

1) Determine which is a polynomial and which is not, giving the reason.

- 1)  $x^2 + 3x + 2x^{-1}$  not a polynomial because of  $x^{-1}$   
 2)  $\sqrt{x^2 + 3}$  " " " " of  $\sqrt{x^2 + 3}$   
 3)  $\frac{1}{2x^3 - x + 3}$  " " " " of  $x$  in denominator  
 4)  $5x^4 - \sqrt{2}x^2 + \sqrt[3]{x^3} = 5x^4 - \sqrt{2}x^2 + x$  is a polynomial

2) Factor  $4y^2 - 4yz + z^2 - 9$

$$= (4y^2 - 4yz + z^2) - 9 = (2y - z)^2 - 9$$

$$= (2y - z + 3)(2y - z - 3)$$

3) Factor if possible or show that it is non-factorable over the integers

$$10x^2 - 4x - 5$$

$$b^2 - 4ac = 16 - 4(10)(-5) = 16 + 200 = 216 =$$

not a perfect square ( $\because 14^2 = 196, 15^2 = 225$ )  
 $\Rightarrow$  non-factorable over the integers

4) Simplify  $\frac{1 - (1-x)^{-1}}{x^{-1} + (x-1)^{-1}}$

$$= \frac{1 - \left(\frac{1}{1-x}\right)}{\frac{1}{x} + \frac{1}{x-1}} = \frac{1 + \frac{1}{x-1}}{\frac{1}{x} + \frac{1}{x-1}} \quad \text{LCD} = x(x-1)$$

$$= \frac{x(x-1) \left( \frac{x(x-1) + x}{x-1 + x} \right)}{x(x-1) \left( \frac{x-1 + x}{x-1 + x} \right)} = \frac{x(x-1) + x}{x-1 + x}$$

$$= \frac{x^2 - x + x}{2x - 1} = \boxed{\frac{x^2}{2x - 1}}$$

5) Write in standard form the complex number  $\frac{\sqrt[3]{-8+i^3} - \sqrt{-2}\sqrt{-8}}{1-i}$

$$= \frac{(-2) + i^3 - (2i)(2\sqrt{2}i)}{1-i} = \frac{-2 + i + 4}{1-i} = \frac{2+i}{1-i}$$

$$= \frac{2+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+3i-1}{2} = \frac{1+3i}{2} = \frac{1}{2} + \frac{3}{2}i$$

6) Factor completely the polynomial  $x^4 - 2x^2 - 8$

$$= (x^2 - 4)(x^2 + 2) = (x-2)(x+2)(x^2 + 2)$$

7) If the coefficient of  $x^3$  in the product  $(x^3 - kx^2 + x - 5)(3x^2 - 4x + k)$  is 15, find the value of  $k$

The term with  $x^3$  is  $kx^3 + 4kx^3 + 3x^3 = (5k + 3)x^3$

$$5k + 3 = 15$$

$$5k = 12 \Rightarrow \boxed{k = \frac{12}{5}}$$

8) Simplify  $\frac{x}{(x-3)^2} + \frac{x^2+2x}{x+5} \div \frac{(x+2)(x-3)}{x+5}$

$$\frac{x}{(x-3)^2} + \frac{x(x+2)}{\cancel{x+5}} \cdot \frac{\cancel{(x+5)}}{(x+2)(x-3)} = \frac{x}{(x-3)^2} + \frac{x}{x-3}$$

$$= \frac{x}{(x-3)^2} + \frac{x(x-3)}{(x-3)^2} = \frac{x^2 - 2x}{(x-3)^2} = \boxed{\frac{x(x-2)}{(x-3)^2}}$$