King Fahd University of Petroleum and Minerals
Prep-Year Math Program

CODE 003 | CODE 003

Prep-Year Math I
FIRST EXAM
Semester I, Term 061
Saturday, October 7, 2006
Net Time Allowed: 75 minutes

Name: ______________________________

ID: ______________ Sec: ______________

Check that this exam has 15 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $i = \sqrt{-1}$ and $z = 1 + i\sqrt{3}$, then the expression $\frac{1}{i}(z^2 - 2z)$ is equal to

(a) $4i$

(b) $6i$

(c) $-2+3i$

(d) $-3i$

(e) $1-3i$

$z^2 = (1+i\sqrt{3})^2 = 1 + 2i\sqrt{3} + (i\sqrt{3})^2 = 1 + 2i\sqrt{3} - 3$
$= -2 + 2i\sqrt{3}$

$\frac{1}{i}(z^2 - 2z) = \frac{1}{i}(-2 + \cancel{2i\sqrt{3}} - 2 - \cancel{2i\sqrt{3}})$

$= \frac{1}{i}(-4) = -\frac{4}{i}\cdot\frac{(-i)}{(-i)} = \frac{4i}{-i^2} = \frac{4i}{-(-1)}$

$= 4i$

2. One of the factors of $4x^2 + 4x + 1 - y^2$ is

(a) $2x+y-1$

(b) $4x-y-1$

(c) $2x+y$

(d) $2x-y$

(e) $2x-y+1$

$((2x)^2 + 2(2x) + 1) - y^2 = (2x+1)^2 - y^2$

$= ((2x+1) - y)((2x+1) + y)$

$= \boxed{(2x - y + 1)}(2x + y + 1)$

3. The degree n and the leading coefficient L of the polynomial $(2 - 3x^2 - x)^3(2x+5)$ are

(a) $n=6$ and $L=-27$

(b) $n=7$ and $L=-54$

(c) $n=7$ and $L=27$

(d) $n=6$ and $L=54$

(e) $n=7$ and $L=-18$

$\deg(2 - 3x^2 - x) = 2$

$\Rightarrow \deg(2 - 3x^2 - x)^3 = 2 \cdot 3 = 6$

$n = \deg(2 - 3x^2 - x)^3(2x+5) = 6 + 1 = 7$

Leading Term $= (-3x^2)^3(2x)$
$= -54x^6 \cdot x$

$\boxed{L = -54 \qquad n = 7}$

4. One of the factors of $8x^6 - 15x^3 - 2$ is

(a) $4x^2 - 2x + 1$

(b) $2x^2 - 4x + 2$

(c) $4x^3 - 2x + 2$

(d) $2x^2 - 4x - 1$

(e) $4x^2 - 2x - 2$

$8(x^3)^2 - 15(x^3) - 2$

$(\ x^3 \quad)(\ x^3 \quad)$

$= (8x^3 + 1)(x^3 - 2)$

cube cube cube not perfect cube

$= ((2x)^3 + 1)(x^3 - 2)$

$= (2x+1)((2x)^2 - (2x)1 + 1^2)(x^3 - 2)$

$= (2x+1)\underbrace{(4x^2 - 2x + 1)}(x^3 - 2)$

5. If $A=\{x|x\leq -3\}\cup\{x|x>1\}$ and $B=\{x|-6\leq x<8\}$, then $A\cap B=$

(a) $\{x|-6\leq x\leq -3\}$

(b) $\{x|-3\leq x<1\}\cup\{x|1<x<8\}$

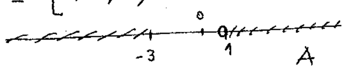
(c) $\{x|-6\leq x\leq -3\}\cup\{x|1<x<8\}$

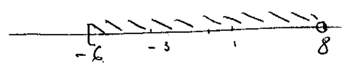
(d) $\{x|-6\leq x<1\}$

(e) $\{x|-3\leq x<8\}$

$A=(-\infty,-3]\cup(1,\infty)$

$B=[-6,8)$





$A\cap B=[-6,-3]\cup(1,8)$

$=\{x/-6\leq x\leq -3\}\cup\{x/\ 1<x<$

6. The **conjugate** of the complex number $\frac{8+i^7}{2+3i^{13}}$ in standard form is

(a) $3-2i$

(b) $\frac{3}{13}-\frac{5}{13}i$

(c) $2-i$

(d) $1+2i$

(e) $\frac{3}{13}+\frac{5}{13}i$

$$z=\frac{8+i^3}{2+3i^1}=\frac{8-i}{2+3i}$$
$$=\frac{(8-i)(2-3i)}{(2+3i)(2-3i)}=$$
$$=\frac{16-24i-2i+3i^2}{4+9}$$
$$=\frac{(16-3)-26i}{13}=\frac{13-26i}{13}$$
$$=1-2i$$

The conjugate is

$$\boxed{\bar{z}=1+2i}$$

7. $\dfrac{\frac{3y}{y-5}-\frac{2}{y-5}}{2(y-2)^{-1}+y^{-1}}=$

This is a complex fraction

$$=\frac{\frac{3y}{y-5}-\frac{2}{y-5}}{\frac{2}{y-2}+\frac{1}{y}}\qquad LCD=y(y-2)(y-5)$$

(a) $y(y-2)(y-5)$

(b) $\frac{y}{(y-2)(y-5)}$

(c) $\frac{y(y+5)}{y-2}$

(d) $\frac{(y-2)(y-5)}{y}$

(e) $\frac{y(y-2)}{y-5}$

$$=\frac{LCD(\quad)}{LCD(\quad)}$$
$$=\frac{3y^2(y-2)-2y(y-2)}{2y(y-5)+(y-2)(y-5)}$$
$$=\frac{3y^3-3y^2-2y^2+4y}{2y^2-10y+y^2-7y+10}=\frac{y(3y^2-5y+4)}{3y^2-17y+10}$$
$$=\frac{y(3y-2)(y-2)}{(3y-2)(y-5)}=\boxed{\frac{y(y-2)}{y-5}}$$

8. $\frac{3x-4}{4x-1}-\frac{3x+6}{(1-4x)(x+2)}=$

(a) $\frac{3x-1}{x+2}$

(b) $\frac{3x-5}{(4x-1)(x+2)}$

(c) $\frac{3x-2}{(4x-1)^2(x+2)}$

(d) $\frac{3x-1}{4x-1}$

(e) $\frac{3x-1}{(4x-1)^2(x+2)}$

$$=\frac{3x-4}{4x-1}+\frac{3\cancel{(x+2)}}{(4x-1)\cancel{(x+2)}}$$
$$=\frac{3x-4+3}{4x-1}$$
$$=\boxed{\frac{3x-1}{4x-1}}$$

9. If $x > 0$, then the expression $\left[\frac{(3x^2)^{-1}(3x^5)^{-2}}{(3^{-1}x^{-2})^2}\right]^{-1}$ is equal to

(a) $27x^4$

(b) $9x^3$

(c) $3x^8$

(d) $9x^6$

(e) $3x^4$

$= \frac{(3^{-1}x^{-2})^2}{(3x^2)^{-1}(3x^5)^{-2}} = \frac{3^{-2}x^{-4}}{3^{-1}x^{-2}3^{-2}x^{-10}}$

$= \frac{3x^2\cancel{3^2}x^{10}}{\cancel{3^2}x^4}$

$= 3\frac{x^{12}}{x^4} = \boxed{3x^3}$

10. $\frac{y^2+7y+12}{y^3-3y^2+9y} \div \frac{y^2+6y+9}{y^3+27} =$

(a) $\frac{y+3}{y+4}$

(b) $\frac{y+3}{y-3}$

(c) $\frac{y+4}{y+3}$

(d) $\frac{y+4}{y}$

(e) $\frac{y+3}{y}$

$\frac{(y+3)(y+4)}{y(y^2-3y+9)} \cdot \frac{y^3+27}{y^2+6y+9}$

$= \frac{\cancel{(y+3)}(y+4)}{y\cancel{(y^2-3y+9)}} \cdot \cancel{\frac{(y+3)(y^2+3y+9)}{(y+3)^2}}$

$= \boxed{\frac{y+4}{y}}$

11. Write without absolute values and simplify

$$|-3x| + \sqrt{(x-3)^2} + 2|x+1|, \quad -3 < x < -2$$

$x < 0 \Rightarrow |x| = -x$

$(x-3) < 0 \rightarrow |x-3| = -(x-3)$

$x+1 < 0 \rightarrow |x+1| = -x-1$

(a) 1

(b) $4x+1$

(c) $5-2x$

(d) $1-6x$

(e) $2x-5$

$= |-3|\cdot|x| + |x-3| + 2|x+1|$

$= 3(-x) + (-(x-3)) + 2(-x-1)$

$= -3x - x + 3 - 2x - 2$

$= -6x + 1 = \boxed{1-6x}$

12. If P and Q are any two different polynomials each of degree $n > 1$, then which one of the following statements is **ALWAYS TRUE**?

(a) $P-Q$ is a polynomial of degree $< n$

(b) $P+Q$ is a polynomial of degree n

(c) $P+P$ is a polynomial of degree $2n$

(d) $P-Q$ is a polynomial of degree $\leq n$

(e) PQ is a polynomial of degree n^2

False, $P = 2x^2$, $Q = x^2$
$2x^2 - x^2 = x^2 \rightarrow$ deg $= n$

False

False deg $= n$

True

False deg $2n$

13. If $2^{x-1} = y$, then $2^{3x-2} =$

$2^x \cdot 2^{-1} = y \Rightarrow \frac{2^x}{2} = y \Rightarrow 2^x = 2y$

$2^{3x-2} = (2^x)^3 \cdot 2^{-2} = (2y)^3 2^{-2} = 8y^3 \cdot \frac{1}{4}$

$= \boxed{2y^3}$

(a) $\frac{y^3}{8}$

(b) $\frac{y^3}{4}$

(c) $\frac{y^3}{2}$

(d) $4y^3$

(e) $2y^3$

14. Which one of the following statements is **ALWAYS TRUE**?

(a) the product of two prime numbers is a composite number T.

(b) the sum of two prime numbers is a prime number F $3+5=8$

(c) every rational number has a multiplicative inverse F 0 has no inverse

(d) the product of two irrational numbers is an irrational number F $\sqrt{2} \cdot \sqrt{2} = 2$

(e) the sum of two irrational numbers is an irrational number F $\sqrt{2} + (-\sqrt{2}) = 0$

15. $\frac{-2}{1+2\sqrt{12}-3\sqrt{3}} = \frac{-2}{1+2\sqrt{2^2 \cdot 3}-3\sqrt{3}} = \frac{-2}{1+4\sqrt{3}-3\sqrt{3}}$

$= \frac{-2}{1+\sqrt{3}} = \frac{-2}{(1+\sqrt{3})} \frac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$= \frac{-2(1-\sqrt{3})}{1-3} = \frac{\cancel{-2}(1-\sqrt{3})}{\cancel{-2}}$

$= \boxed{1-\sqrt{3}}$

(a) $1-2\sqrt{3}$

(b) $5-\sqrt{3}$

(c) $1-\sqrt{3}$

(d) $2+\sqrt{3}$

(e) $-2-\frac{5}{9}\sqrt{3}$