1. The solution set of equation

$$\sqrt{2x+3}-\sqrt{x-2}=2$$

contains

$\sqrt{2x+3} = 2+\sqrt{x-2}$, square each side,

(a) two positive integers

(b) only one positive integer

(c) two negative integers

(d) only one negative integer

(e) no real numbers

$2x+3 = 4+4\sqrt{x-2}+(x-2)$

$x+1 = 4\sqrt{x-2}$, square each side,

$x^2+2x+1 = 16(x-2)$

$x^2+2x+1-16x+32=0$

$x^2-14x+33=0$

$(x-11)(x-3)=0$

$\Rightarrow x=11$, $x=3$ $\Rightarrow$ (a)

check ✓ ✓

2. The sum of all solutions of the equation

$$x^{2/3}-4x^{1/3}-32=0$$

is equal to

$(x^{1/3})^2-4x^{1/3}-32=0$, let $u=x^{1/3}$

(a) 448

(b) 528

(c) 128

(d) 4

(e) 256

$\Rightarrow u^2-4u-32=0$

$\Rightarrow (u-8)(x+4)=0$

$\Rightarrow u=8$ or $u=-4$

$\Rightarrow x^{1/3}=8$ or $x^{1/3}=-4$

$\Rightarrow x=8^3$ or $x=(-4)^3$

$\Rightarrow x=512$ or $x=-64$

$\therefore$ Sum $= 512+(-64)$

$= 448.$

3. The solution set of the compound inequality

$$-3|x|+6<12 \text{ and } 8-|2x-1|\geq 6$$

is equal to

$-3|x|< 6$ and $-|2x-1| \geq -2$

$|x|>-2$ $\quad$ $|2x-1| \leq 2$

$\Rightarrow x \in (-\infty,\infty)$ $\quad$ $-2 \leq 2x-1 \leq 2$

$+1 \quad +1 \quad +1$

$-1 \leq 2x \leq 3$

$-\frac{1}{2} \leq x \leq \frac{3}{2}$

(a) $\left[-\frac{1}{2},\frac{3}{2}\right]$

(b) $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{3}{2},\infty\right)$

(c) $\left[-\frac{1}{2},0\right)\cup\left(0,\frac{3}{2}\right]$

(d) $(-\infty,\infty)$

(e) the empty set ϕ

$\therefore$ S.S. $= (-\infty,\infty) \cap \left[-\frac{1}{2},\frac{3}{2}\right]$

$= \left[-\frac{1}{2},\frac{3}{2}\right]$

4. The solution set, in interval notation, of the inequality

$$\frac{9}{x}\geq x-8$$

is equal to

Sol. $\frac{9}{x}-(x-8) \geq 0$

$\frac{9-(x^2-8x)}{x} \geq 0$

$\frac{-x^2+8x+9}{x} \geq 0$

$\frac{-(x^2-8x-9)}{x} \geq 0$

$\Rightarrow \frac{(x-9)(x+1)}{x} \leq 0$

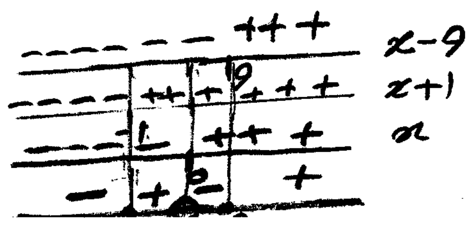
(a) $(-\infty,-1]\cup(0,9]$

(b) $[-1,0)\cup(0,9]$

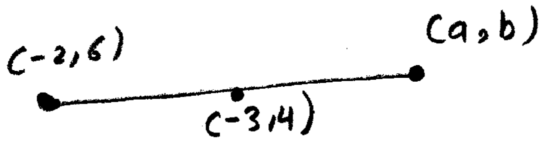
(c) $(-\infty,-1]$

(d) $(-\infty,0)$

(e) $[-1,0)\cup[9,\infty)$



5. If the points $(-2,6)$ and (a,b) are the endpoints of the line segment whose midpoint is $(-3,4)$, then $3a-4b$ is equal to



(a) –20

(b) 25

(c) 20

(d) –25

(e) –35

$-3 = \frac{a+(-2)}{2} \Rightarrow a = -6+2 = -4$

$4 = \frac{6+b}{2} \Rightarrow b = 8-6 = 2$

$\therefore 3a-4b = 3(-4)-4(2) = -12-8 = -20$

6. The radius of the circle $x^2+y^2+2x+y+1=0$ is equal to

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) 2

(d) 4

(e) 1

Complete the Square,

$(x^2+2x) + (y^2+y) = -1$

$(x^2+2x+1) + (y^2+y+\frac{1}{4}) = 1+\frac{1}{4}-1$

$(x+1)^2 + (y+\frac{1}{2})^2 = \frac{1}{4}$

$\therefore r = \frac{1}{2}.$

9. If $g(x)=\begin{cases}\left[x-\frac{3}{2}\right] & \text{if } 0<x<\frac{1}{4}\\ \left|\frac{1}{3}x-1\right| & \text{if } x>2\end{cases}$, where $[x]$ is the greatest integer function, then $2g\left(\frac{1}{8}\right)+g(4)$ is equal to

(a) $-\frac{11}{3}$

(b) $\frac{1}{4}$

(c) $-\frac{2}{3}$

(d) $\frac{1}{3}$

(e) $-\frac{3}{2}$

$$2g\left(\tfrac{1}{8}\right)+g(4)=2\cdot\left[\tfrac{1}{8}-\tfrac{3}{2}\right]+\left|\tfrac{4}{3}-1\right|$$
$$=2\left[\tfrac{-11}{8}\right]+\left|\tfrac{1}{3}\right|$$
$$=2(-2)+\tfrac{1}{3}$$
$$=-4+\tfrac{1}{3}$$
$$=\tfrac{-11}{3}$$

10. A point that lies on the line that is perpendicular to the line $y-2x-1=0$ and passes through the point $(1,3)$ is

(a) $\left(2,\frac{5}{2}\right)$

(b) $(1,4)$

(c) $(0,1)$

(d) $(-1,3)$

(e) $\left(-2,\frac{7}{2}\right)$

$$y=2x+1$$
Eq.: $y-3=\frac{-1}{2}(x-1)$
$$y=\tfrac{-1}{2}x+\tfrac{1}{2}+3$$
$$y=\tfrac{-1}{2}x+\tfrac{7}{2}$$
$$x=2\Rightarrow y=-1+\tfrac{7}{2}=\tfrac{5}{2}.$$

11. The $y-$intercept of the line that passes through the points $\left(\frac{3}{4},2\right)$ and $\left(\frac{1}{8},-\frac{1}{2}\right)$ is equal to

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{-\frac{1}{2}-2}{\frac{1}{8}-\frac{3_{\times 2}}{4_{\times 2}}} = \frac{\frac{-5}{2}}{\frac{-5}{8}}$$

$$= \frac{5}{2}\cdot\frac{8}{5} = 4$$

$\therefore$ Eq.: $y-2 = 4\left(x-\frac{3}{4}\right)$

y-int. $\Rightarrow x=0 \Rightarrow y-2=-3$

$\Rightarrow x = -3+2 = -1$

$\therefore$ x-int.: $(-1,0)$

(a) $(0,-1)$

(b) $\left(0,\frac{1}{4}\right)$

(c) $\left(0,-\frac{5}{4}\right)$

(d) $\left(0,-\frac{3}{4}\right)$

(e) $(0,2)$

12. If the vertex of the parabola $y=-x^2+8x+2c$ is a point on the $x-$axis, then the value of c is equal to

Vertex $= (h,0) = \left(-\frac{b}{2a},0\right) = \left(\frac{-8}{2(-1)},0\right)$

$= (4,0)$

$\Rightarrow 0 = -16+32+2c$

$\Rightarrow 2c = -16$

$\Rightarrow c = -8$

(a) -8

(b) 32

(c) -64

(d) -32

(e) 64

7. The **domain**, in interval notation, of the function

$$f(x) = \frac{\sqrt{x-2}}{x^2 - 3x}$$

is equal to

(a) $[2,3) \cup (3,\infty)$

(b) $[2,\infty)$

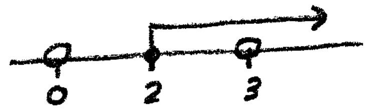
(c) $(-\infty,0) \cup (0,3) \cup (3,\infty)$

(d) $(-\infty,0) \cup [2,\infty)$

(e) $(-\infty,0) \cup (0,2] \cup (3,\infty)$

$x - 2 > 0$ and $x^2 - 3x \neq 0$

$x > 2$ and $x \neq 0$ and $x \neq 3$



$\therefore D = [2,3) \cup (3,\infty)$

8. The **range** of the function

$$f(x) = \begin{cases} 1 & \text{if } x < 0 \\ 2 - x & \text{if } 2 \le x < 4 \end{cases}$$

is equal to

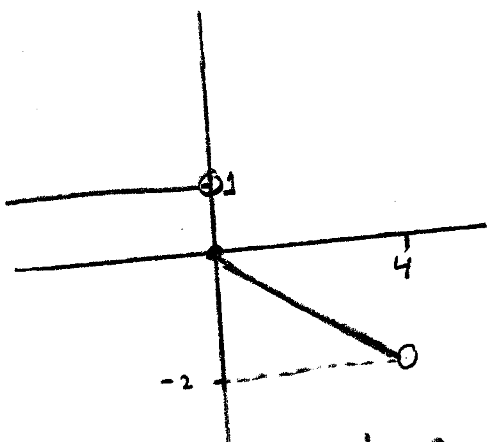
(a) $(-2,0] \cup \{1\}$

(b) $(-\infty,0) \cup [2,4)$

(c) $(-\infty,1]$

(d) $(-2,1]$

(e) $(-4,0] \cup \{1\}$



From the graph $R = (-2,0] \cup \{1\}$

13. Which one of the following statements is **TRUE** about the graph of the function $f(x) = -2x^2 + 2x + \frac{3}{2}$?

(a) The graph is decreasing on $\left[\frac{1}{2}, \infty\right)$

(b) The graph has no $x-$intercept

(c) The vertex is the point $\left(\frac{1}{2}, \frac{3}{2}\right)$

(d) The range is $\left(-\infty, \frac{3}{2}\right]$

(e) The axis of symmetry is the line $y = \frac{1}{2}$

$x = \frac{-b}{2a} = \frac{-2}{2(-2)} = \frac{1}{2}$

Dec. on $[\frac{1}{2}, \infty)$

14. The maximum area of a rectangle that has perimeter 2000 meters equal to

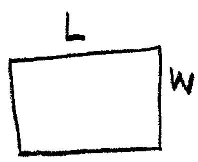
(a) 250000 square meters

(b) 300000 square meters

(c) 200000 square meters

(d) 150000 square meters

(e) 400000 square meters



$2L + 2W = 2000 \Rightarrow L + W = 1000$

$\Rightarrow W = 1000 - L$

$A = LW = L(1000 - L)$

$= 1000L - L^2$

$= -L^2 + 1000L$

$L_{max} = \frac{-b}{2a} = \frac{-1000}{2(-1)} = 500$

$\Rightarrow W_{max} = 500$

$\therefore$ max area $= 500 \cdot 500$

$= 250000$

15. Which one of the following functions is an **odd** function?

(a) $y = x + \frac{1}{x}$

odd: $f(-x) = -f(-x)$

$-x + \frac{1}{(-x)} = -\left(x + \frac{1}{x}\right) = -y$

$\therefore$ odd

(b) $y = 2x^5 - 10$

(c) $y = 4 + \sqrt[3]{x}$

(d) $y = \frac{x^2}{x^3 + 1}$

(e) $y = -|x^3|$

16. The graph of the equation $|xy| - |y-1| = 2x^2 + 1$ is symmetric with respect to

(a) the $y-$axis only

(b) the $x-$axis only

(c) the origin only

(d) the $x-$axis and the origin

(e) the $x-$axis, the $y-$axis and the origin

• x-axis: $\left.\begin{matrix} x \to x \\ y \to -y \end{matrix}\right\} \Rightarrow |x(-y)| - |-y-1| = 2x^2+1$

$|xy - |y+1| = 2x^2+1$

not symm.

• y-axis: $\left.\begin{matrix} x \to -x \\ y \to y \end{matrix}\right\} \Rightarrow |(-x)y| - |y-1| = 2(-x)^2+1$

$\Rightarrow |xy| - |y-1| = 2x^2+1 \Rightarrow$ symm

• Origin not.

17. If the graph of the equation $y = x^2 + x + 1$ is shifted left horizontally 2 units and shifted down vertically 7 units, then the equation of the new graph is equal to

(a) $y = x^2 + 5x$

(b) $y = x^2 + 4x$

(c) $y = x^2 - 3x - 4$

(d) $y = x^2 + 4x - 3$

(e) $y = x^2 - 3x - 6$

$$y_{new} = \left[(x+2)^2 + (x+2) + 1\right] - 7$$
$$= x^2 + 4x + 4 + x + 2 + 1 - 7$$
$$= x^2 + 5x$$

18. If $(f \circ g)(x) = \dfrac{1}{x^2 - 4}$, then which of the following could be functions f and g?

(a) $f(x) = \dfrac{1}{x}$, $g(x) = x^2 - 4$

(b) $f(x) = x^2 - 4$, $g(x) = \dfrac{1}{x}$

(c) $f(x) = \dfrac{1}{x^2}$, $g(x) = x - 4$

(d) $f(x) = \dfrac{1}{x^2}$, $g(x) = -\dfrac{1}{4}$

(e) $f(x) = x - 4$, $g(x) = \dfrac{1}{x^2}$

$$(f \circ g)(x) = f(g(x))$$

$$\left.\begin{array}{l} f(x) = \dfrac{1}{x} \\ g(x) = x^2 - 4 \end{array}\right\} \Rightarrow f(\underbrace{g(x)}_{x^2-4}) = \frac{1}{x^2-4}$$

19. If $f(x) = \sqrt{1-x^2}$, then $(f o f)(x) =$

(a) $|x|$

(b) x

(c) $\sqrt{2-x^2}$

(d) $\sqrt{2}$

(e) $\sqrt{2+x^2}$

$$(f o f)(x) = f(f(x))$$
$$= f\left(\sqrt{1-x^2}\right)$$
$$= \sqrt{1-\left(\sqrt{1-x^2}\right)^2}$$
$$= \sqrt{1-(1-x^2)}$$
$$= \sqrt{x^2}$$
$$= |x|$$

20. If $f(x) = \dfrac{x}{x-3}$ and $g(x)$ is the inverse of $f(x)$, then $g(-2)+g(2)$ is equal to

(a) 8

(b) 10

(c) 0

(d) 6

(e) 12

• Find $g(x) = f^{-1}(x)$

$$x = \frac{y}{y-3} \Rightarrow xy - 3x = y$$
$$\Rightarrow xy - y = 3x$$
$$\Rightarrow y = \frac{3x}{x-1}$$
$$\Rightarrow g(x) = \frac{3x}{x-1}$$

$$g(-2) = \frac{3(-2)}{-2-1} = \frac{-6}{-3} = 2$$
$$g(2) = \frac{3(2)}{2-1} = \frac{6}{1} = 6$$
$$\therefore g(-2) + g(2) = 2 + 6 = 8$$

21. Given $f(x)=\frac{1}{5}x^2-\frac{4}{25}x+k$, where $x\geq 2$. If $f^{-1}(2)=5$, then k is equal to

$f^{-1}(2)=5 \Rightarrow f(5)=2$

$\Rightarrow \frac{1}{5}\cdot 25-\frac{4}{25}\cdot 5+k=2$

$\Rightarrow 5-\frac{4}{5}+k=2$

$\Rightarrow k=2-5+\frac{4}{5}$

$=-3+\frac{4}{5}$

$=\frac{-11}{5}$

(a) $-\frac{11}{5}$

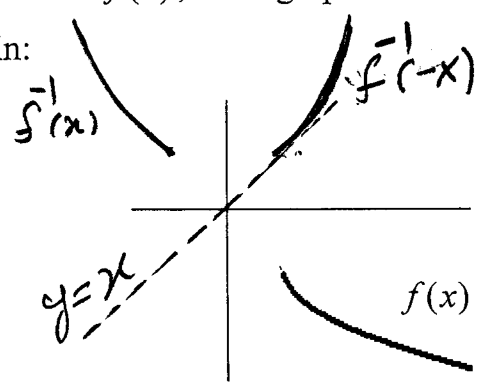
(b) $-\frac{31}{5}$

(c) $\frac{31}{5}$

(d) $\frac{11}{5}$

(e) $\frac{1}{5}$

22. If the graph given below represents $f(x)$, then graph of the function $y=f^{-1}(-x)$ lies completely in:



(a) Quadrant I

(b) Quadrant III

(c) Quadrant II

(d) Quadrant IV

(e) Quadrants II and III