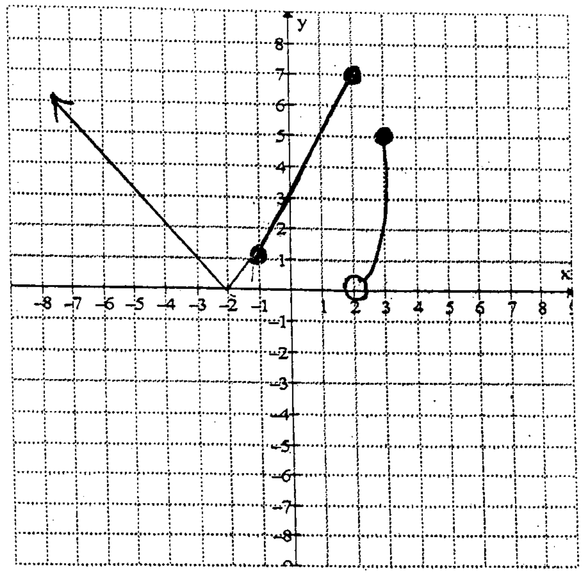
1. Given $f(x)=\begin{cases}|x+2| & \text{for } x\le -1\\ 2x+3 & \text{for } -1<x\le 2\\ x^2-4 & \text{for } 2<x\le 3\end{cases}$

(a) (3 points) Sketch the graph of f.



(b) (2 points) Find

$f(-1)=$ $|-1+2|=1$ 1

$f(2)=$ $2(2)+3=7$ 1/2

$f(3)=$ $9-4=5$ 1/2

(c) (1 point) Find, in interval notation, the Domain of f.

$(-\infty, 3]$ 1

(d) (1 point) Find, in interval notation, the Range of f.

$[0, \infty)$ 1

(e) (1 point) Is f a one-to-one function? Why?

NO, Horizontal Line test 1

(f) (1 point) Find the interval where f is decreasing.

$(-\infty, -2]$ 1

2. (2 points) Find the value of k for which the slope of the line that passes through the points $\left(-4,\frac{k}{2}\right)$ and $\left(k,\frac{7}{2}\right)$ is equal to $-\frac{3}{2}$.

$$\frac{\frac{7}{2}-\frac{k}{2}}{k+4}=\frac{-3}{2}$$ 1

$7-k=-3k-12$ 1/2

$2k=-19$

$k=\frac{-19}{2}$ 1/2

3. (4 points) Find the x-and y-intercepts of the line that passes through the point $(1,3)$ and perpendicular to the line $3x+4y=-24$.

l_1

l_2

$$l_1 \perp l_2 \Rightarrow m_1 \cdot m_2 = -1$$

l_2: $4y = -3x - 24$

$y = \frac{-3}{4}x - 6$

$\Rightarrow m_2 = -\frac{3}{4}$ (1)

l_1: $m_1 = \frac{-1}{-3/4} = \frac{4}{3}$ (1)

$\therefore y - 3 = \frac{4}{3}(x-1)$

$y = \frac{4}{3}x + \frac{5}{3}$ (1)

x-intercept: $\frac{4}{3}x + \frac{5}{3} = 0 \Rightarrow x = -\frac{5}{4} \Rightarrow \left(-\frac{5}{4}, 0\right)$ (1/2)

y-intercept: $y = \frac{5}{3} \Rightarrow \left(0, \frac{5}{3}\right)$ (1/2)

4. The perimeter of a rectangle with length y and width x is 48 inches.

(a) (1 point) Write y as a function of x.

$$2(x+y) = 48$$
$$y = -x + 24$$ (1)

(b) (1 point) Write the area A of the rectangle as a function of x.

$$A = xy = x(-x+24)$$
$$= -x^2 + 24x$$ (1)

(c) (2 points) Find the length and width that produce the greatest area.

the width x that produce the greatest area is

$x = h = \frac{-b}{2a} = -\frac{24}{2(-1)} = 12$ inches. (1)

also $y = -x + 24 = -12 + 24 = 12$ inches. (1

5. (6 points) Answer the following questions about the quadratic function $f(x) = -2x^2 - 6x - 1$

(a) Write the standard form of $f(x)$.

$$f(x) = -2\left(x+\frac{3}{2}\right)^2 + \frac{7}{2}$$ 2

(b) Find the vertex. $\left(-\frac{3}{2}, \frac{7}{2}\right)$ 1

(c) Find the maximum value of f. $\frac{7}{2}$ 1

(d) Find the equation of the axis of symmetry of the graph of f. $x = -\frac{3}{2}$ 1

(e) Find, in interval notation, the range of f. $\left(-\infty, \frac{7}{2}\right]$ 1

6. (4 points) Fill in the spaces with the words "even" or "odd" or "neither even nor odd" for each of the following:

(a) $f(x) = \sqrt{3-x^4}$ is ....EVEN.......... function. 1

(b) $g(x) = \dfrac{x^2+1}{x}$ is ....ODD.......... function. 1

(c) $h(x) = x^3 + x + 7$ is ....NEITHER.......... function. 1

(d) $F(x) = x^5 - \sqrt[3]{x}$ is ....ODD.......... function. 1

7. (6 points) Determine whether the graph of the equation $|x|y - x^3y^2 = 0$ has symmetry with respect to the (Show your steps)

(a) x-axis: Replace y by $-y$

$|x|(-y) - x^3(-y)^2 = 0 \Rightarrow -|x|y - x^3y^2 = 0$

Not Symmetric with respect to X-axis.

(b) y-axis: Replace x by $-x$

$|-x|y - (-x)^3y^2 = 0 \Rightarrow |x|y + x^3y^2 = 0$

Not Symmetric with respect to Y-axis

(c) origin:

Replace (x, y) by $(-x, -y)$

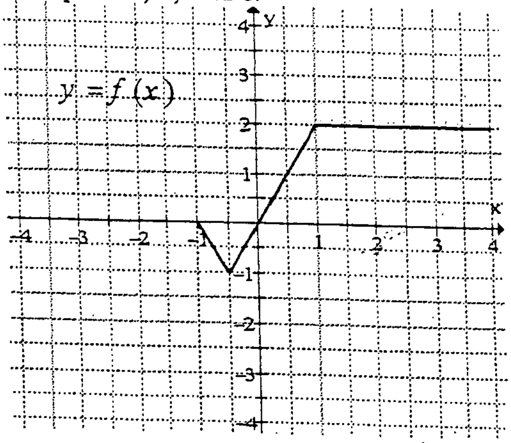
$|-x|(-y) - (-x)^3(-y)^2 = 0 \Rightarrow -|x|y + x^3y^2 = 0$

Multiply by (-1):

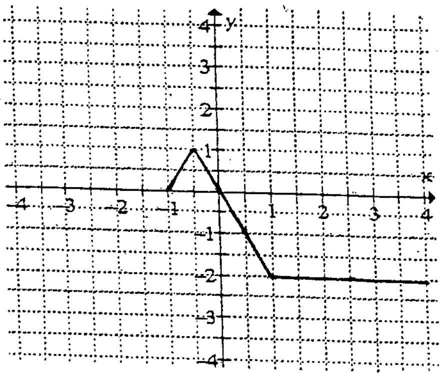
$\Rightarrow |x|y - x^3y^2 = 0$ 2

$\therefore$ Symmetric with respect to the origin

8. (6 points) If the adjacent graph represents the function $y = f(x)$, then sketch the graph of the functions in parts a, b, and c.

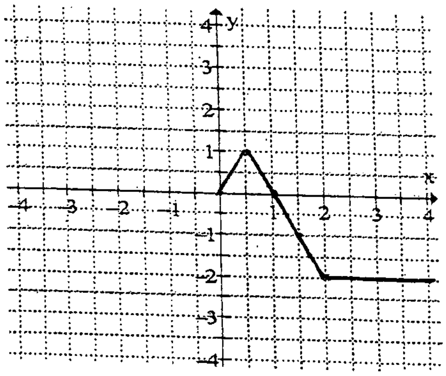


(a) $y = -f(x)$: Reflection across the x-axis.



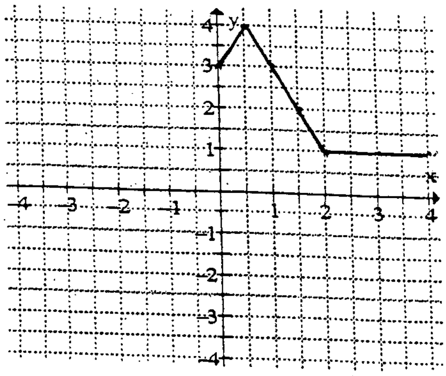
(2) pts.

(b) $y = -f(x-1)$: Then horizontal shift 1 unit to the right.



(2) pts

(c) $y = -f(x-1)+3$: Then vertical translation up 3 units.



(2) pts.

9. (3 points) If $f(x)=3x-2$, then find each of the following:

(a) $f\left(-\frac{1}{3}c+2\right)= \quad 3\left(-\frac{1}{3}c+2\right)-2 = -c+6-2 = -c+4$ (1)

(b) $f^{2}(-1)= \quad f(-1)\cdot f(-1) = (-5)(-5) = 25$ (1/2)

(c) $f(f(-1))= \quad f(-5) = -15-2 = -17$ (1/2 + 1/2 = 1)

10. (3 points) If $f(x)=\sqrt{1-x^{2}}$ and $g(x)=2x+1$, then find, in interval notation, the domain of the function $\frac{f}{g}$.

$D_f: -1 \leq x \leq 1 \;;\; [-1,1]$ (1/2)

$D_g: (-\infty, \infty)$ (1/2)

$D_f \cap D_g : [-1,1]$ (1/2)

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{1-x^2}}{2x+1}$ (1/2) $\Longrightarrow$ $D_{f/g}: [-1,-\frac{1}{2}) \cup (-\frac{1}{2},1]$

11. (4 points) If $f(x)=1+2x$ and $(f\circ g)(x)=3-\frac{x}{2}$, then find

(a) the function g.

$(f\circ g)(x) = f(g(x)) = 1+2g(x)$ (1/2)

$1+2g(x) = 3-\frac{x}{2}$ (1/2)

$2g(x) = 2-\frac{x}{2}$ (1/2)

$g(x) = 1-\frac{x}{4} = \frac{4-x}{4}$ (1/2)

(b) the value of $(g\circ f)\left(-\frac{9}{2}\right)$.

$(g\circ f)\left(-\frac{9}{2}\right) = g\left(f\left(-\frac{9}{2}\right)\right)$ (1/2)

$= g\left(1+2\left(-9/2\right)\right)$ (1/2)

$= g(-8)$ (1/2)

$= 1-\frac{(-8)}{4} = 3$ (1/2)