KEY

1. (5 points) Given the sets

$A = \{x | x \text{ is an even positive integer } < 10\}$
$B = \{x | x \text{ is a composite number } < 12\}$
$C = \{y | y = x^2 - 1, \text{ where } x \text{ is an integer with } 0 \leq x \leq 4\}$

(a) List all elements in each of the above sets to get:

$A = \{2, 4, 6, 8\}$ – 1

$B = \{4, 6, 8, 9, 10\}$ 1

$C = \{-1, 0, 3, 8, 15\}$ 1

(b) Find the set $(A \cap B) \cup C$.

$A \cap B = \{4, 6, 8\}$ 1

$(A \cap B) \cup C = \{-1, 0, 3, 4, 6, 8, 15\}$ 1

2. (2 points) Write the polynomial $(7-5x)(5+8x-3x^2)$ in standard form.

$= 35 + (-25 + 56)x + (-40 - 21)x^2 + 15x^3$ 1

$= 15x^3 - 61x^2 + 31x + 35$ 1

3. (2 points) Find the polynomial, in standard form, of the third degree for which: the leading coefficient $= -5$, coefficient of $x^2 = 0$, coefficient of $x = 3$, and constant term $= -4$.

$-5x^3 + 0x^2 + 3x - 4$ 1.5

$= -5x^3 + 3x - 4$ 0.5

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4. (3 points) Write the number $\dfrac{(4000)(8.1\times 10^{-2})}{0.0018}$ in scientific notation. [Show your steps].

$= \dfrac{(4\times 10^{3})(81\times 10^{-3})}{18\times 10^{-4}}$ 1

$= 18\times 10^{4}$ 1

$= 1.8\times 10^{5}$ 1

5. (4 points) Simplify $\dfrac{[(-3)^{0}\,x^{-2/3}\,y^{5/3}]^{-1/2}}{x^{7/6}\,y^{-7/2}}$, where $x>0,\ y>0,$ then write your answer in radical form.

$= \dfrac{(x^{-2/3})^{-1/2}\cdot(y^{5/3})^{-1/2}}{x^{7/6}\cdot y^{-7/2}} = \dfrac{x^{1/3}\cdot y^{-5/6}}{x^{7/6}\cdot y^{-7/2}}$ 1.5

$= \dfrac{y^{7/2-5/6}}{x^{7/6-1/3}} = \dfrac{y^{16/6}}{x^{5/6}} = \dfrac{y^{8/3}}{x^{5/6}}$ 1.5

$= \dfrac{y^{2}\cdot y^{2/3}}{x^{5/6}} = y^{2}\,\dfrac{\sqrt[3]{y^{2}}}{\sqrt[6]{x^{5}}}$ 1

6. Factor each of the following as completely as possible over the set of integers:

(a) (3 points) $x^2-12xy+36y^2-z^2$.

$= (x^{2}-12xy+36y^{2})-z^{2}$ .5

$= (x-6y)^{2}-z^{2}$ 1

$= [(x-6y)-z]\,[(x-6y)+z]$

$= (x-6y-z)(x-6y+z)$ 1.5

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(b) (3 points) $125x^3 - 8y^6$.

$= (5x)^3 - (2y^2)^3$ 1.5

$= (5x - 2y^2)\left[(5x)^2 + (5x)(2y^2) + (2y^2)^2\right]$ 1.5

$= (5x - 2y^2)(25x^2 + 10xy^2 + 4y^4]$ 1

(c) (3 points) $-3x^{3n} + 6x^{2n} - 3x^n$.

$= -3x^n(x^{2n} - 2x^n + 1)$ 1.5

$= -3x^n(x^n - 1)^2$ 1.5

(d) (2 points) $x^4 - 3x^2 - 4$.

$= (x^2 - 4)(x^2 + 1)$ 1

$= (x-2)(x+2)(x^2+1)$ 1

7. (4 points) If $A = \dfrac{2+\sqrt{5}}{2-\sqrt{5}}$ and $B = \dfrac{\sqrt[4]{32}}{\sqrt{\sqrt{2}}}$, then write $A + B$ in simplest form.

$A = \dfrac{(2+\sqrt{5})(2+\sqrt{5})}{(2-\sqrt{5})(2-\sqrt{5})} = \dfrac{4+5+4\sqrt{5}}{4-5}$ 1.5

$= -9 - 4\sqrt{5}$ 0.5

$B = \dfrac{\sqrt[4]{2^4 \cdot 2}}{\sqrt[4]{2}} = 2\dfrac{\sqrt[4]{2}}{\sqrt[4]{2}} = 2$ 1.5

$\Rightarrow A + B = (-9 - 4\sqrt{5}) + 2 = -7 - 4\sqrt{5}$ 0.5

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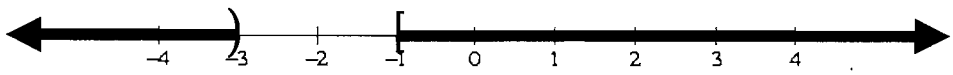
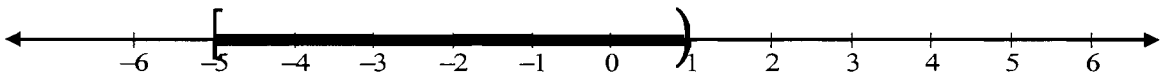
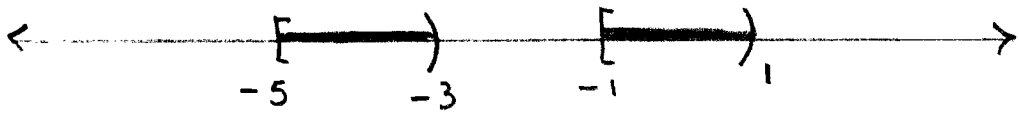
8. (4 points) The graph of a subset A of real numbers is given by



and the graph of a subset B of real numbers is given by



(a) Sketch the graph of $C = A \cap B$.



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(b) Write C using interval notation.

$C = [-5, 3) \cup [-1, 1)$ 1

(c) Write C using inequality notation.

$C = \{x \mid -5 \leq x < 3 \text{ or } -1 \leq x < 1\}$ 1

9. (2 points) Find the product $(2x - 3y)^3$.

$= (2x)^3 - 3(2x)^2(3y) + 3(2x)(3y)^2 - (3y)^3$ 1

$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$ 1

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10. Simplify each of the following expressions:

(a) (3 points) $\dfrac{5}{(y+3)^2} + \dfrac{2y-5}{y^2-4y-21} \div \dfrac{2y^2+y-15}{y-7}$.

$= \dfrac{5}{(y+3)^2} + \dfrac{2y-5}{(y-7)(y+3)} \cdot \dfrac{y-7}{(2y-5)(y+3)}$ 2

$= \dfrac{5}{(y+3)^2} + \dfrac{1}{(y+3)^2} = \dfrac{6}{(y+3)^2}$ 1

(b) (4 points) $\dfrac{x^{-1}y^3 - xy}{x^{-1}y - xy^{-1}}$.

$= \dfrac{\frac{y^3}{x} - xy}{\frac{y}{x} - \frac{x}{y}} = \dfrac{\frac{y^3 - x^2y}{x}}{\frac{y^2 - x^2}{xy}}$ 2

$= \dfrac{y(y^2-x^2)}{x} \cdot \dfrac{xy}{y^2-x^2} = y^2$. 2

(c) (4 points) $\dfrac{-3}{2(x-1)(x+2)} + \dfrac{5}{2(x+2)(x-3)}$.

$= \dfrac{-3(x-3) + 5(x-1)}{2(x-1)(x+2)(x-3)} = \dfrac{2x+4}{2(x-1)(x+2)(x-3)}$ 2

$= \dfrac{2(x+2)}{2(x-1)(x+2)(x-3)} = \dfrac{1}{(x-1)(x-3)}$ 2

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11. Write each of the following complex numbers in standard form (where $i = \sqrt{-1}$).

(a) (4 points) $\dfrac{(2-2i)^2}{3+4i}$.

$= \dfrac{4-8i+4i^2}{3+4i} = \dfrac{4-8i-4}{3+4i}$ 1

$= \dfrac{-8i(3-4i)}{(3+4i)(3-4i)} = \dfrac{-24i+32i^2}{9+16}$ 2

$= -\dfrac{32}{25} - \dfrac{24}{25}i$ 1

(b) (4 points) The complex conjugate $\overline{z}$ of the complex number $z = \sqrt{-2}\sqrt{-8} + \sqrt[3]{-27} + i^{2007}$.

$= (\sqrt{2}i)(\sqrt{8}i) - 3 + i^7$ 1

$= 4i^2 - 3 + i^3$ 1

$= -4 - 3 - i = -7 - i$ 1

$\Rightarrow \overline{z} = -7 + i$ 1

12. (4 points) Write the following expression in simplest form and without the absolute value symbols:

$$\frac{\sqrt{9-12x+4x^2}}{\left|x-\frac{3}{2}\right|}, \quad \text{where } x \neq \frac{3}{2}.$$

$= \dfrac{\sqrt{(3-2x)^2}}{|x-3/2|}$ 1

$= \dfrac{|3-2x|}{|x-3/2|}$ 2

$= \dfrac{2\,|x-\frac{3}{2}|}{|x-3/2|} = 2$ 1