King Fahd University of Petroleum and Minerals
College of Sciences
Prep-Year Math Program

Code 002

Math 001 Exam I

Term 021 (2002-2003)

Saturday, October 19, 2002

Time Allowed: 90 Minutes

Code 002

Student's Name: ______________________________

ID #: ________________ **Section #:** ____________________

This exam consists of Two parts

Part I : Multiple Choice Bubble the correct answer on the OMR sheet.
Part II : Written Questions Provide neat and complete solutions.
Show all necessary steps for full credit.

Calculators, Pagers, or Mobiles are NOT allowed during this exam.

Question	Points	Student's Score
Part I: (1 - 8)	12	
Part II:		
1	3	
2	2	
3	3	
4	4	
5	4	
6 (a)	2	
6 (b)	2	
7	3	
8	5	
9	4	

Total	
	44

Part I: (12-points) Multiple Choice Questions (MCQ).
Bubble the the correct answer on the OMR sheet

1. Which one of the following statements is FALSE?

   (a) If $x \neq 3$, then $\dfrac{x^3 - 27}{x - 3} = x^3 + 3x + 9$ is an identity.

   (b) The equation $|3x - 5| = -8$ is a contradiction.

   (c) The equation $\dfrac{x^2 - 4}{x - 2} = 4$ has a real solution.

   (d) The equation $5x + 7 = 3$ is a conditional equation.

2. If $A = \frac{1}{2}(B + x)y$ and $y \neq 0$, then $B =$

   (a) $\dfrac{A + 2xy}{y}$

   (b) $\dfrac{2A - x}{y}$

   (c) $2A - \frac{1}{2}xy$

   (d) $\dfrac{2A - xy}{y}$

$2A = By + xy$

$2A - xy = By$

$B = \dfrac{2A - xy}{y}$

3. The distance between the two points whose coordinates on a number line are $-\pi$ and 3, is equal to

   (a) $|-\pi + 3|$

   (b) $\pi + 3$

   (c) $|\pi - 3|$

   (d) $-(-\pi + 3)$

$|(-\pi) - 3| = |\pi + 3| = \pi + 3$

4. The coefficient of xy^2 in the expression$(3x - 2y)^3$ is equal to

   (a) 36

   (b) -36

   (c) 18

   (d) -12

$(3x)^3 - 3(3x)^2 2y + 3(3x)(2y)^2 - (2y)^3$

$27x^3 - 54x^2y + 36xy^2 - 8y^3$

5. Which one of the following statements is TRUE?

(a) The sum of two composite numbers is a composite number. F $8+9=17$

(b) The sum of two irrational numbers is an irrational number. F $\sqrt{2}+(-\sqrt{2})=0$

(c) The product of two irrational numbers is an irrational number. F $\sqrt{2}\cdot\sqrt{2}=2$

(d) The product of two composite numbers is a composite number. T

6. Which one of the following statements is TRUE for any real number x?

(a) $\sqrt[3]{-x^3}=-x$ T $\sqrt[3]{-x^3}=\sqrt[3]{(-x)^3}=-x$

(b) $\sqrt{16x^2}=4x$ F $4|x|$

(c) $\sqrt{(-2x)^2}=-2x$ $|2x|$ 2 even

(d) $\sqrt[3]{64x^3}=4|x|$ $4x$ 3 odd

7. The expression $\dfrac{x+y}{x-y}\cdot\dfrac{x^{-1}-y^{-1}}{x^{-1}+y^{-1}}$ is equal to

(a) 0

(b) -1

(c) $\dfrac{0}{0}$

(d) x^2-y^2

$$\frac{x+y}{x-y}\cdot\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}=\frac{x+y}{x-y}\cdot\frac{\frac{y-x}{xy}}{\frac{y+x}{(xy)}}=$$

$$=\frac{\cancel{x+y}}{x-y}\cdot\frac{y-x}{\cancel{x+y}}=\frac{-(x-y)}{x-y}=-1$$

8. If $i=\sqrt{-1}$, then $i^{50}+i^{51}+i^{52}=$

(a) i $i^2+i^3+i^0=-1-i+1=\boxed{-i}$

(b) -1

(c) $-i$

(d) 0

Part II: Written Questions.
[Provide neat and complete solution. Show necessary steps for full credit.]

1. (3-points) Given the sets

$$A = \{z | z = -|x| + x, \text{ where } x \text{ is an integer with } -4 < x \leq 0\},$$
$$\text{and } \quad B = \{z | z = 2x - 2, \text{ where } x \text{ is an integer with } -3 \leq x < 0\}.$$

List the elements of the sets A, B, and $A \cap B$.

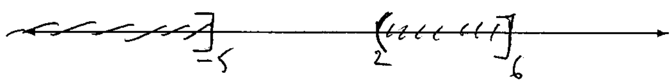
$A = \{z / z = -|x| + x \quad x = -3, -2, -1, 0\}$
$= \{-6, -4, -2, 0\}$

$B = \{2x - 2 / x = -3, -2, -1\} = \{-8, -6, -4\}$

$A \cap B = \{-4\}$

2. (2-points) Given the inequality $x \leq -5$ or $2 < x \leq 6$.

(a) Graph the given inequality on a number line:



(b) Write the given inequality using interval notation.

$(-\infty, -5] \cup (2, 6]$

3. (3 points) Simplify $(3x - 5)(2x^2 + 4x - 6)$. Write the result in standard form.

$6x^3 + 12x^2 - 18x - 10x^2 - 20x + 30$

$6x^3 + 2x^2 - 38x + 30$

4. (4-points) Given that $0 < x < \frac{1}{8}$, write the expression $\left|\frac{|x-\frac{1}{4}|}{|x-\frac{1}{8}|+|x+\frac{1}{8}|}\right|$ without absolute value symbols and in the simplest form.

$$= \frac{|x - 1/4|}{|x - 1/8| + |x + 1/8|} = \frac{1/4 - x}{(\frac{1}{8} - x) + (x + \frac{1}{8})}$$

$$= \frac{\frac{1}{4} - x}{1/4} = \boxed{1 - 4x}$$

5. (4-points) Simplify $\left[\frac{(-2y)^0 y^{-1}(2y)^3}{(2y^{-2})^{-1}y^{-4}}\right]^{-1/2}$, where $y \neq 0$. Write the result using positive exponents only.

$$\left[\frac{y^{-1}\, 2^3 y^3}{2^{-1} y^2\, y^{-4}}\right]^{-1/2} = \frac{y^{1/2}\, 2^{-3/2}\, y^{-3/2}}{2^{1/2}\, y^{-1}\, y^2} = \frac{2^{-3/2}\, \cancel{y^{-1}}}{2^{1/2}\, \cancel{y^{-1}}\, y^2}$$

$$= \frac{1}{2^2 y^2} = \frac{1}{4y^2}$$

6. (a) (2-points) Factor $9x^2 - 24xy + 16y^2 - 100z^2$.

$$(3x - 4y)^2 - (10z)^2$$
$$= (3x - 4y + 10z)(3x - 4y - 10z)$$

(b) (2-points) Rationalize the **numerator** $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$. Write your answer in the simplest form.

$$\frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}} \cdot \frac{(\sqrt{2} + \sqrt{3})}{(\sqrt{2} + \sqrt{3})} = \frac{2 - 3}{2 + 2\sqrt{6} + 3} = \frac{-1}{5 + 2\sqrt{6}}$$

7. (3-points) Simplify $-3x\sqrt[3]{54x^4} + 2\sqrt[3]{16x^7}$. Write the result in the simplest form.

$$= -3x\sqrt[3]{2 \cdot 3^3 x^4} + 2\sqrt[3]{2^4 x^7}.$$
$$= -3x^2 \cdot 3\sqrt[3]{2x} + 2 \cdot 2x^2\sqrt[3]{2x}$$
$$= -5x^2\sqrt[3]{2x}$$

8. (5-points) Simplify $\frac{x}{2x-1} - \frac{1}{2x^2-7x-4} \div \frac{x+3}{x^2-x-12}$. Write the result in the simplest form.

$$\frac{x}{2x-1} - \frac{1}{(2x+1)\cancel{(x-4)}} \cdot \frac{\cancel{(x-4)}\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$= \frac{x(2x+1) - (2x-1)}{(2x-1)(2x+1)} = \frac{2x^2 - x + 1}{(2x-1)(2x+1)}$$

$$= \frac{\cancel{(2x+1)}(x-1)}{(2x-1)\cancel{(2x+1)}} = \boxed{\frac{x-1}{2x-1}}$$

9. (4-points) Write the conjugate of the complex number $\frac{1}{(2+i)^2-8i}$ in standard form.

$$z = \frac{1}{4+4i+i^2-8i} = \frac{1}{3-4i} = \frac{1}{3-4i} \cdot \frac{3+4i}{3+4i}$$

$$= \frac{3+4i}{9+16} = \frac{3}{25} + \frac{4}{25}i$$

$$\boxed{\bar{z} = \frac{3}{25} - \frac{4}{25}i}$$