King Fahd University of Petroleum and Mine: ls
Prep-Year Math Program

Prep-Year Math I
SECOND EXAM
Semester I, Term 061
Tuesday, December 05, 2006
Net Time Allowed: 75 minutes

Sources of Problems

MASTER VERSION

1. If $M(h, 6)$ is the midpoint of the line segme from $P(3, k)$ to $Q(-5, 4)$, then $h + k =$

~~(a)~~ 7

(b) -1

(c) 9

(d) 1

(e) 5

2.1

See example 1 p. 56

See Problems 19 to 24 p. 174-175

$(h, \cdot) = \left(\frac{3-5}{2}, \frac{k+4}{2}\right) \iff$

$h = \frac{-2}{2} = -1$, $6 = \frac{k+4}{2} \iff 12 = k + 4$

$\iff k = 8$ So $h + k = -1 + 8 = 7$

2. The center $C(h, k)$ and the radius r of the ci le $\frac{1}{2}x^2 + \frac{1}{2}y^2 - 3x + 2y - \frac{3}{2} = 0$ are

~~(a)~~ $C(3, -2)$, $r = 4$

(b) $C(2, -3)$, $r = 4$

(c) $C(3, -2)$, $r = \sqrt{15}$

(d) $C(\frac{3}{2}, -1)$, $r = \frac{\sqrt{2}}{2}$

(e) $C(3, -2)$, $r = \sqrt{17}$

2.1

See example 7 p. 173

See Problems 65 to 72 p. 175

Multiply by the LCD (2)

$x^2 + y^2 - 6x + 4y - 3 = 0$

$(x^2 - 6x) + (y^2 + 4y) = 3$

$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 3 + 9 + 4$

$(x - 3)^2 + (y + 2)^2 = 16$

$C(3, -2)$ $r = 4$

3. If $x < 0$, then the distance between the points $P(2x, -7x)$ and $Q(-2x, -4x)$ is equal to

2.1

(a) $-5x$

(b) $7x$

(c) $-7x$

(d) $5x$

(e) $-3x$

See Problems 15 and 16 p. 174

Sec 2.1

$d((2x, -7x), (-2x, -4x))$

$= \sqrt{(2x - 2x)^2 + (-4x + 7x)^2} = \sqrt{(4x)^2 + (3x)^2}$

$= \sqrt{25x^2} = 5\sqrt{x^2} = 5|x| = -5x$

$x < 0$

4. If the graph of the function $g(x)$ is obtained from the graph of $f(x) = \sqrt{x}$ by means of a reflection across the x-axis, a horizontal shift 2 units left and a vertical shift 1 unit up then $g(x) =$

2.5

(a) $-\sqrt{x+2} + 1$

(b) $\sqrt{x+2} - 1$

(c) $-\sqrt{x-2} + 1$

(d) $\sqrt{1-x} + 2$

(e) $-\sqrt{x+2} - 2$

See examples 4 and 5 p. 233-234

See problems 5 to 60 p.239

Sec 2.5

$y = \sqrt{x} \xrightarrow[/x\text{-axis}]{\text{Refl}} -y = \sqrt{x} \xrightarrow[2\text{ left}]{\text{shift}} -y = \sqrt{x+2}$

$y = -\sqrt{x+2}$

shift up

$y = -\sqrt{x+2} + 1$

$\boxed{g(x) = -\sqrt{x+2} + 1}$

5. Identify the set of ordered pairs (x, y) or the relation which defines y as a function of x

(a) $5y + x = 2y + \sqrt{x^2 - 5}$

(b) $\left\{\left(\frac{1}{2}, 0\right), (2, -1), (3, 3), \left(\frac{1}{2}, \frac{1}{4}\right)\right\}$

(c) $(x-1)^2 + (y-2)^2 = 25$

(d) $|y| = x + 5$

(e) $y^2 = x^2$

See example 3 p.181

See problems 11 to 26 p.191

Sec 2.2

No $(\frac{1}{2}, 0)$, $(\frac{1}{2}, \frac{1}{4})$ have same x

No $x = 1 \pm \sqrt{25 - (y-2)^2}$ 2 soln y

d) No $y = \pm(x+5)$

e) No $y = \pm\sqrt{x^2} = \pm|x|$

a) Yes $3y = -x + \sqrt{x^2 - 5} \Rightarrow y = \frac{1}{3}(-x + \sqrt{x^2 - 5})$

6. A ball is thrown vertically upward. If the height h in feet of the ball is given by the equation $h(t) = -16t^2 + 80t + 100$ where time t is in seconds, then the maximum height that the ball attains is

Sec 2.4

(a) 200 feet

(b) 150 feet

(c) 300 feet

(d) 100 feet

(e) 250 feet

See example 7 p.221

See Problems 69 and 70 p. 225

$h(t) = (-16t^2 + 80t) + 100$

$= -16\left(t^2 + \frac{5}{16}t\right) + 100$

$= -16\left(t^2 - 5t + \left(\frac{5}{2}\right)^2\right) + 100 + 16\left(\frac{25}{4}\right)$

$= -16\left(t - \frac{5}{2}\right)^2 + \underbrace{200}_{k}$

The max height is then $k = 200$ ft

7. The equation of th line that passes through th point $(3,5)$ and perpend ılar to the line $2x+5y-4=0$ is

~~(a)~~ $5x-2y-5=($

(b) $5x+2y-25=$

(c) $5x-2y+15=$

(d) $2x-5y+19=$

(e) $5x-2y-30=$

2.3

See problems 73 to 76 P. 211

L_1

L_2 : $5y = -2x+4$

$y = -\frac{2}{5}x + \frac{4}{5} \Rightarrow m_2 = -\frac{2}{5}$

$\Rightarrow m_1 = -\frac{1}{m_2} = -\frac{-5}{2} = \frac{5}{2}$

By the point slope form

$y-5 = \frac{5}{2}(x-3) \Leftrightarrow 2y-10 = 5x-15$

$5x-2y-15+10=0 \Rightarrow \boxed{5x-2y-5=0}$

8. The x-intercept of e line passing through the points $(5,-6)$ and $(2,-8)$ is

~~(a)~~ $(14,0)$

(b) $(18,0)$

(c) $(10,0)$

(d) $\left(-\frac{28}{3},0\right)$

(e) $\left(\frac{2}{3},0\right)$

2.3

See Problems 3 to 38 p. 208

and 4 to 48 P. 208

First find the eq of the line

$m = \frac{-8-(-6)}{2-5} = \frac{-2}{-3} = \frac{2}{3}$

Take $(x_1,y_1) = (5,-6)$

Point-Slope form

$y+6 = \frac{2}{3}(x-5)$

$3y+18 = 2x-10$

x-int, $y=0$ $\Rightarrow$ $18 = 2x-10$

$9 = x-5$

$x = 14 \Rightarrow x\text{-int } (14,0)$

9. If the lines $kx + 4y = 24$ and $y = -\frac{3}{k+1}x + \frac{5}{4}$ are parallel, then the set f values of k consists of

~~(a)~~ one positive and one negative integers

2.3

(b) two positive int ers

(c) two negative in gers

(d) one positive int er only

(e) one negative in ger only

This is an application of the concept that two lines are parallel if they have the same slope.

L_1 : $y = -\frac{k}{4}x + \frac{24}{4} \Rightarrow m_1 = -$

L_2 : $m_2 = -\frac{3}{k+1}$

$\Rightarrow m_1 = m_2$

$-\frac{k}{4} = -\frac{3}{k+1}$

$\Rightarrow k^2 + k - 12 = 0 \quad (k+4)(k-3) = 0 \quad \boxed{k = 3,\ k = -4}$

10. The graph of the eq tion

$$y^3 = -x^3y^2 + \frac{x}{|x|}$$

is symmetric with re pect to

See example 1 p. 229 and example 2 p. 230

See problems 13 & 30 p. 238

2.

~~(a)~~ the origin only

(b) the x-axis only

(c) the y-axis only

(d) the x-axis and e origin

(e) the y-axis and e origin

$(-y)^3 = -x^3(-y)^2 + \frac{x}{|x|}$

$-y^3 = -x^3y^2 + \frac{x}{|x|} \Rightarrow$ Not sym/x-axis

$y^3 = -(-x)^3y^2 + \frac{(-x)}{|-x|}$

$y^3 = +x^3y^2 - \frac{x}{|x|} \Rightarrow$ Not sym/y-axis

$(-y)^3 = -(-x)^3(-y)^2 + \frac{(-x)}{|-x|}$

$-y^3 = +x^3y^2 - \frac{x}{|x|}$

$\times -1 \quad y^3 = -x^3y^2 + \frac{x}{|x|}$ same eqn $\Rightarrow$ Sym/origin

11. Which one of the fol wing numbers is in the ran of the quadratic fuction f) $= -2x^2 + x - \frac{3}{8}$?

~~(a)~~ $-\frac{1}{3}$

(b) 2

(c) $-\frac{1}{8}$

(d) 0

(e) $-\frac{1}{16}$

See example 3 P. 217

See problems 33 and 34 P.223

2.4

Le find the ra

$= -2(x^2 - \frac{x}{2}) - \frac{3}{8}$

$= -2(x^2 - \frac{x}{2} + (\frac{1}{4})^2) - \frac{3}{8} + \frac{1}{8}$

$= -2(x - \frac{1}{4})^2 - \frac{1}{4}$, $a < 0$

Ra $(-\infty, -\frac{1}{4}]$

Only

12. If $[y]$ denotes the eatest integer less than or ual to y, then the domain) and the range R of the nction $f(x) = |[x]| + 1$ are iven by

~~(a)~~ $D = (-\infty, \infty)$, $R =$ all natural numbers

(b) $D = R = [1, \infty$

(c) $D = [0, \infty)$, R $[1, \infty)$

(d) $D = R = (\infty, o$

(e) $D = (-\infty, \infty)$, $R =$ all nonnegative integer

2.2

See the de-imition of the greatest i-teger function P. 185

See Problems 43 and 44 P. 191

Domain is $\mathbb{R}$

Range$([\![x]\!]) = \mathbb{Z}$

Range$(|[x]| + 1$ $\{1, 2, 3\}$ all natural numbers

13. The domain, in interval notation, of the function $f(x) = \sqrt{2 - x - x^2}$ is

2.2

(a) $[-2, 1]$

(b) $(-\infty, -2] \cup [1, \infty)$

(c) $(-\infty, -1] \cup [2, \infty)$

(d) $(-\infty, 1]$

(e) $[-2, \infty)$

See example 4 P. 182

See problems 27 to 38 P. 191

$2 - x - x^2 \geq 0$

$x^2 + x - 2 \leq 0$

$(x + 2)(x - 1) \leq 0$

$\therefore S = [-2, 1]$

$x+2$	$-$
$x-1$	$-$
	$+$

14. The function $f(x) = \begin{cases} -2x+1 & \text{if } x < -2 \\ -x^2 & \text{if } -2 \leq x \leq 2 \\ -4 & \text{if } x > 2 \end{cases}$

is **increasing** on the interval [Hint: sketch]

(a) $[-2, 0]$

(b) $(-\infty, -2] \cup [2, \infty)$

(c) $[-2, \infty)$

(d) $(-\infty, -2]$

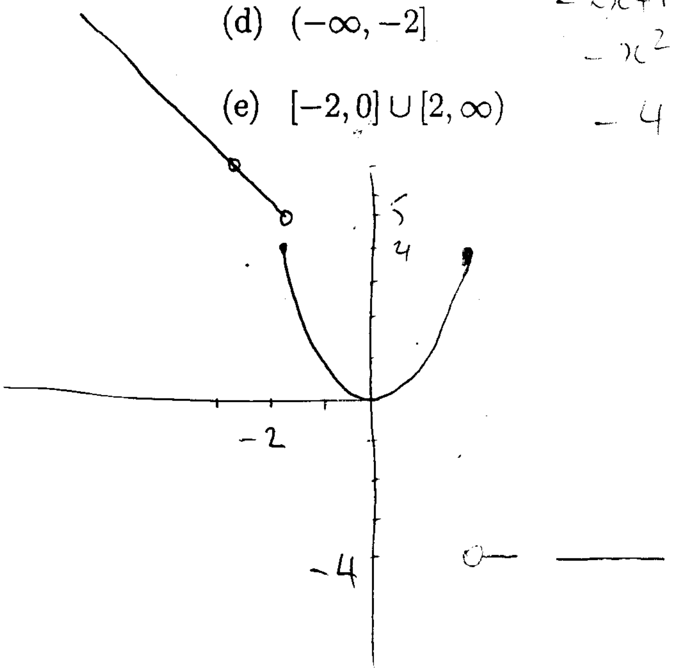
(e) $[-2, 0] \cup [2, \infty)$

See example P. 183

See problems 39 to 42 P. 191

	-3	-2	0	2	3
$-2x+1$	7	5			
$-x^2$		$\boxed{-4}$	0	$\boxed{-4}$	
-4				-4	-4

f is increasing on $[-2, 0]$



15. Let f be a functic such that $f(-1) = 3$ and $f(2) = -4$. The coordinates of two p nts on the graph of $y = 3f$ -$x) - 2$ are

~~(a)~~ (1, 7), (−2, −14

(b) (1, 1), (−2, −14

(c) (1, 7), (2, 2)

(d) (−1, 1), (2, 6)

(e) (1, 7), (2, 4)

See Problem 61 and 62 P. 239

(−1, 3) (2 −4) are points of the graph

$y = f(x) \xrightarrow[/y\text{-axis}]{\text{Reflt}} = f(-x) \xrightarrow[\text{by } 3]{\text{Vert stretching}} y = 3 \;\; -x)$

Shift 2 units down

$y = 3$ $x) - 2$

$(-1, 3) \xrightarrow[/y\text{-axis}]{\text{Refl}}$ (3) → (1, 3( ) (1, 9)

2 its

(1, 9) $\boxed{(1, 7)}$

OR

$y = 3f($ $) - 2$

$y = 3($ $- 2$

2nd pts

$y = 3$ $f(-$ $) - 2$ −14

$\Rightarrow \boxed{(x, y)}$ − 14)