

QUIZ # 4 (A)

1) $f(x) = -3\sin(2x) + 3\sqrt{3}\cos(2x)$

$= k \sin(2x + \alpha)$

Find

a) The amplitude

$k = \sqrt{a^2 + b^2} = \sqrt{(-3)^2 + (3\sqrt{3})^2} = \sqrt{9 + 9 \cdot 3} = \sqrt{36} = 6$

b) The smallest positive phase shift

$\cos \alpha = \frac{-3}{6} = -\frac{1}{2} < 0$

$\sin \alpha = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} > 0$

$\Rightarrow \alpha \in Q II \quad \left| \quad \begin{matrix} \alpha \leq \frac{\pi}{3} \\ \Rightarrow \alpha = \frac{2\pi}{3} \text{ or } -\frac{4\pi}{3} \end{matrix} \right.$

PS = $-\frac{c}{b} = -\frac{2\pi}{3} = -\frac{\pi}{3}$

or $-\left(\frac{-\frac{4\pi}{3}}{2}\right) = \boxed{\frac{2\pi}{3}}$

2) Evaluate

a) $\sin\left(\sin^{-1}\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$

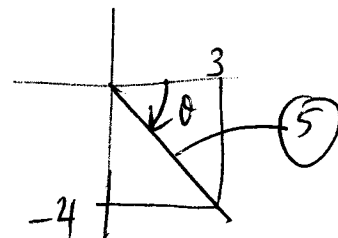
b) $\cos\left(\cos^{-1}\left(-\frac{2}{3}\right)\right) = -\frac{2}{3}$

c) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \theta' = \frac{\pi}{6}$
 \downarrow
 $II \rightarrow \sin \oplus$

d) $\cos^{-1}\left(\cos\frac{9\pi}{5}\right) = \frac{\pi}{5}$
 \downarrow
 $IV \rightarrow \cos \oplus$

e) $\csc\left(\tan^{-1}\left(-\frac{4}{3}\right)\right) = \csc \theta = \frac{r}{y} = \frac{5}{-4} = \boxed{-\frac{5}{4}}$

$\theta \in Q IV$ & $\tan \theta = -\frac{4}{3} = \frac{y}{x}$



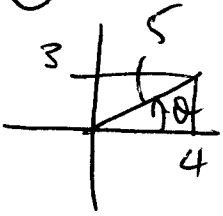
3) Solve

$$\sin^{-1}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{3}{4}\right)$$

(4)

$$x = \sin\left(\frac{\pi}{2} - \underbrace{\tan^{-1}\left(\frac{3}{4}\right)}_{\theta}\right) = \sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$

$$= \frac{4}{5}$$



4) a) Give the general solutions of $\cos 2x = \frac{1}{2}$ (don't use double angle formula)

(3)

$$y = 2x, \quad \cos y = \frac{1}{2} > 0 \quad \begin{matrix} \text{I} \\ \text{IV} \end{matrix} \quad \& \quad y' = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{3} \text{ or } y = \frac{5\pi}{3}$$

$$\Rightarrow y = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad y = \frac{5\pi}{3} + 2n\pi \quad \Rightarrow \quad \boxed{x = \frac{\pi}{6} + n\pi \quad \text{or} \quad x = \frac{5\pi}{6} + n\pi}$$

b) Give the solutions in $\left[0, \frac{3\pi}{2}\right]$

$$0 \leq x \leq \frac{3\pi}{2} \Rightarrow 0 \leq y \leq 3\pi \quad (\text{1 period \& half})$$

(2)

$$n=0 \quad x = \frac{\pi}{6}, \quad x = \frac{5\pi}{6}$$

$$n=1 \quad x = \frac{\pi}{6} + \pi = \frac{7\pi}{6}, \quad x = \frac{5\pi}{6} + \pi = \frac{11\pi}{6} \quad \times$$

5) Solve $\sin 2x - \cos x = 0$, in $[0, 2\pi]$ (use formula and factor)

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

(4)

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\text{or} \quad \sin x = \frac{1}{2} > 0 \quad \begin{matrix} \text{I} \\ \text{II} \end{matrix}$$

$$x' = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6}$$

$$SS = \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$$