KFUPM
Math002
1 Aug 07

Solution.

NAME:
ID:
Sec: Serial:

QUIZ # 4 **(B)**

1) $f(x) = 2\sin(2x) - 2\sqrt{3}\cos(2x)$

$= k \sin(2x + \alpha)$

Find

a) The amplitude

$k = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

b) The smallest positive phase shift

$\cos\alpha = \frac{a}{k} = \frac{2}{4} = \frac{1}{2} > 0$

$\sin\alpha = \frac{b}{k} = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2} < 0$

$\Rightarrow \alpha \in$ Q IV, $\alpha' = \frac{\pi}{3}$

$\Rightarrow \alpha = \frac{5\pi}{3}$

$PS = -\frac{c}{b} = -\frac{\alpha}{b} = -\frac{\frac{5\pi}{3}}{2} = \boxed{-\frac{5\pi}{6}}$

2) Evaluate

a) $\sin\left(\sin^{-1}\left(\frac{\pi}{2}\right)\right)$ undefined

$\frac{\pi}{2} > 1$

b) $\cos\left(\cos^{-1}\left(-\frac{3}{4}\right)\right) = -\frac{3}{4}$

$-\frac{3}{4} \in [-1, 1]$

c) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right) = -\theta' = -\frac{\pi}{6}$

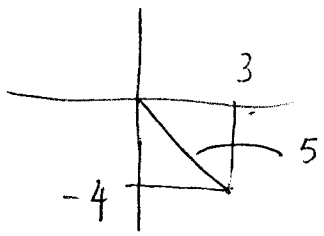
III $\rightarrow \sin < 0$

d) $\cos^{-1}\left(\cos\frac{6\pi}{5}\right) = 2\pi - \theta' = \pi - \frac{\pi}{5} = \frac{4\pi}{5}$

III $\rightarrow \cos < 0$

e) $\csc\left(\tan^{-1}\left(-\frac{4}{3}\right)\right) = \csc\theta = \frac{r}{y} = \frac{5}{-4} = \boxed{-\frac{5}{4}}$

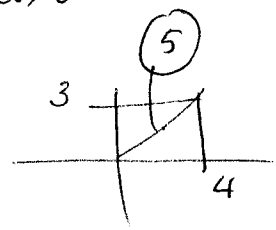
$\theta \in$ Q IV



3) Solve $\sin^{-1}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{3}{4}\right)$

$x = \sin\left(\frac{\pi}{2} - \tan^{-1}(3/4)\right) = \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\theta \in QI$



$= \boxed{\frac{4}{5}}$

4) a) Give the general solutions of $\cos 2x = \frac{\sqrt{3}}{2}$ (don't use double angle formula)

$y = 2x$ $\quad \cos y = \frac{\sqrt{3}}{2} > 0 \Rightarrow y \in QI, IV$ $\quad y' = \frac{\pi}{6}$ $\Rightarrow y = \frac{\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow y = \frac{\pi}{6} + 2n\pi$ or $y = \frac{5\pi}{6} + 2n\pi$

$\rightarrow x = y/2$ $\rightarrow$ $\boxed{x = \frac{\pi}{12} + n\pi}$ or $\boxed{x = \frac{11\pi}{12} + n\pi}$

b) Give the solutions in $\left[0, \frac{3\pi}{2}\right]$

$x \in \left[0, \frac{3\pi}{2}\right] \Rightarrow y = 2x \in [0, 3\pi]$ 1 period & half

$n = 0$ $\quad \boxed{x = \frac{\pi}{12}} \in \left[0, \frac{3\pi}{2}\right]$ ✓ $\quad \boxed{x = \frac{5\pi}{12}} \in \left[0, \frac{3\pi}{2}\right]$ ✓

$n = 1$ $\quad x = \frac{\pi}{12} + \pi = \boxed{\frac{13\pi}{12}}$ ✓ $\quad x = \frac{11\pi}{12} + \pi = \boxed{\frac{23\pi}{12}} \notin \left[0, \frac{3\pi}{2}\right]$

5) Solve $\sin 2x - \sin x = 0$, *in* $[0, 2\pi]$ (use formula and factor)

$2\sin x \cos x - \sin x = 0$

$\sin x (2\cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2} > 0 \rightarrow$ I, IV

$x = 0$ or π $\quad x' = \frac{\pi}{3}$

$\Rightarrow x = \pi/3$ or $x = \frac{5\pi}{3}$

$SS = \left\{0, \pi/3, \frac{5\pi}{3}, \pi\right\}$